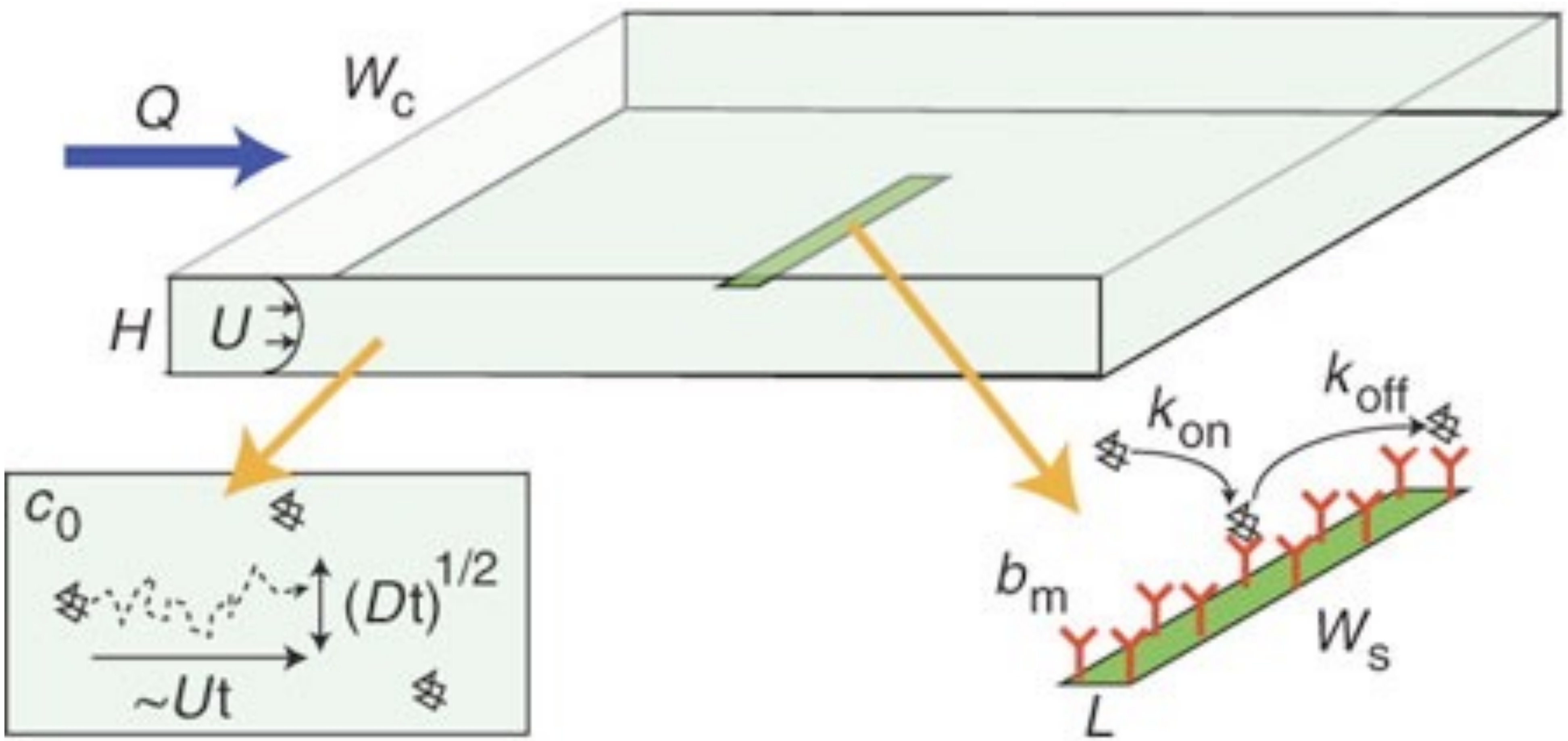
Transport phenomena

Engineering at small scale
Engineering at large scale
Transport in the environment
Transport in living systems

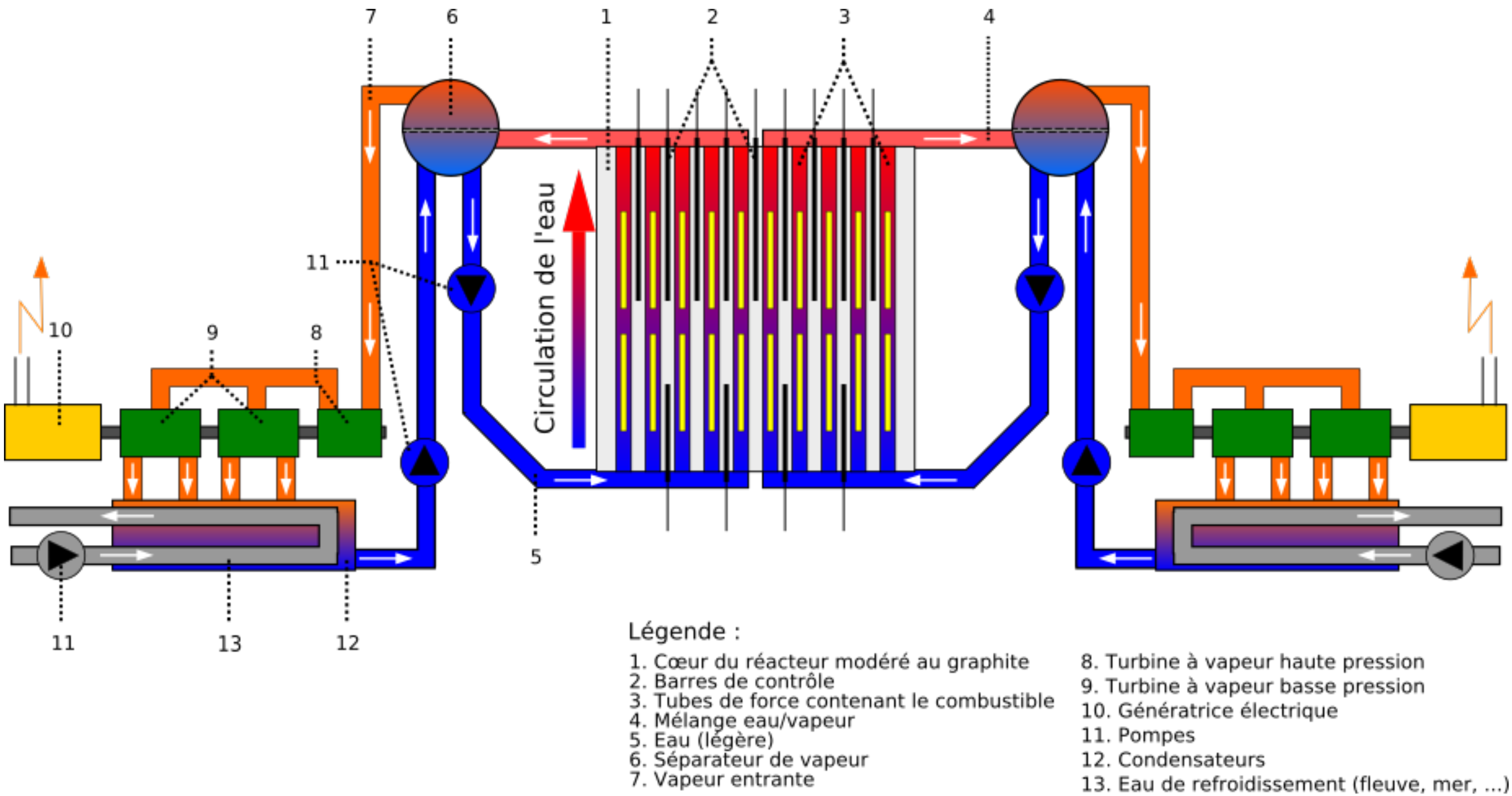
Engineering at small scale : the microchip problem



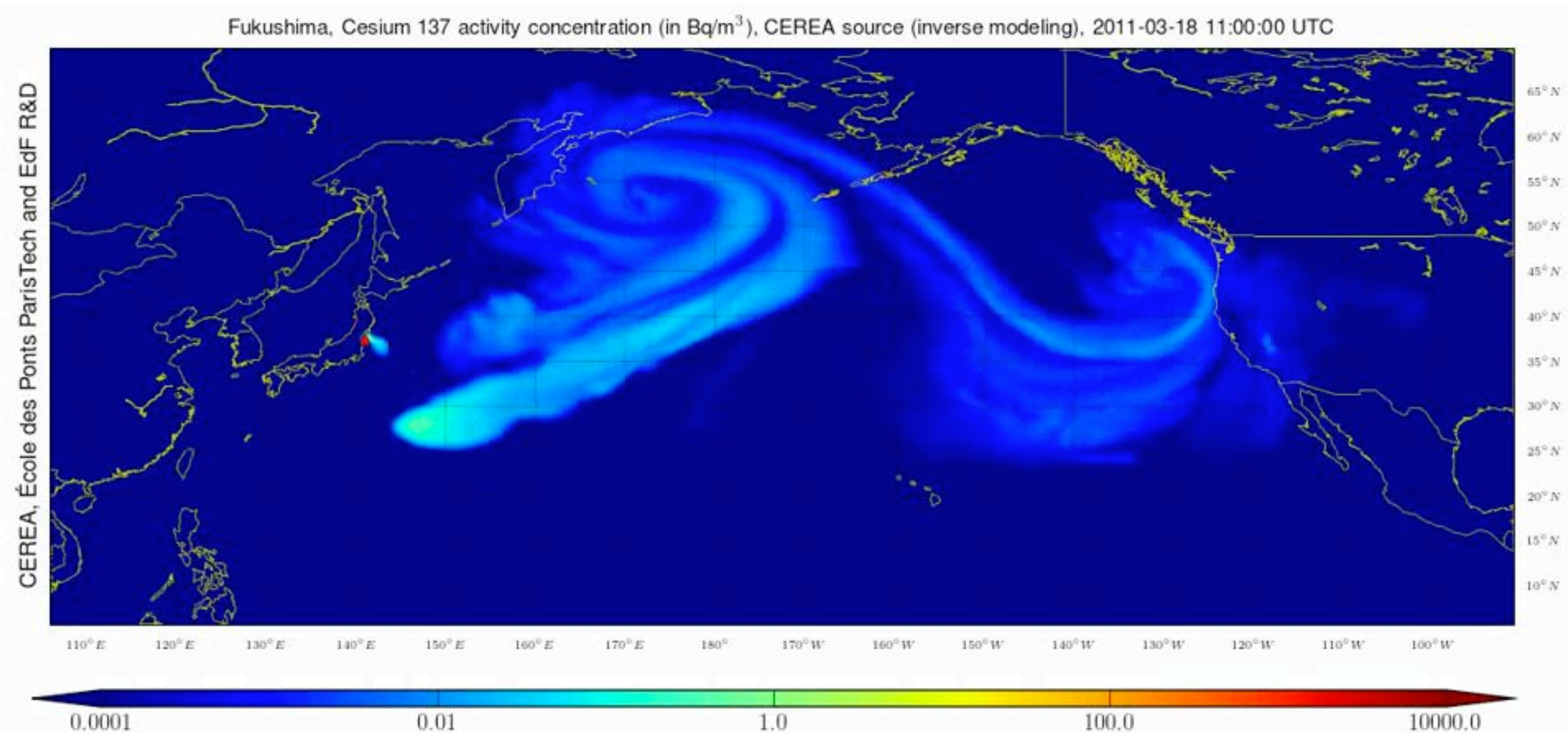
How do you design precisely a biochemical sensor on a microchip ?

T. Squires et al., Nature Biotech. 26, 417 (2008)

Engineering at large scale : a nuclear reactor



Mass transport in the environment

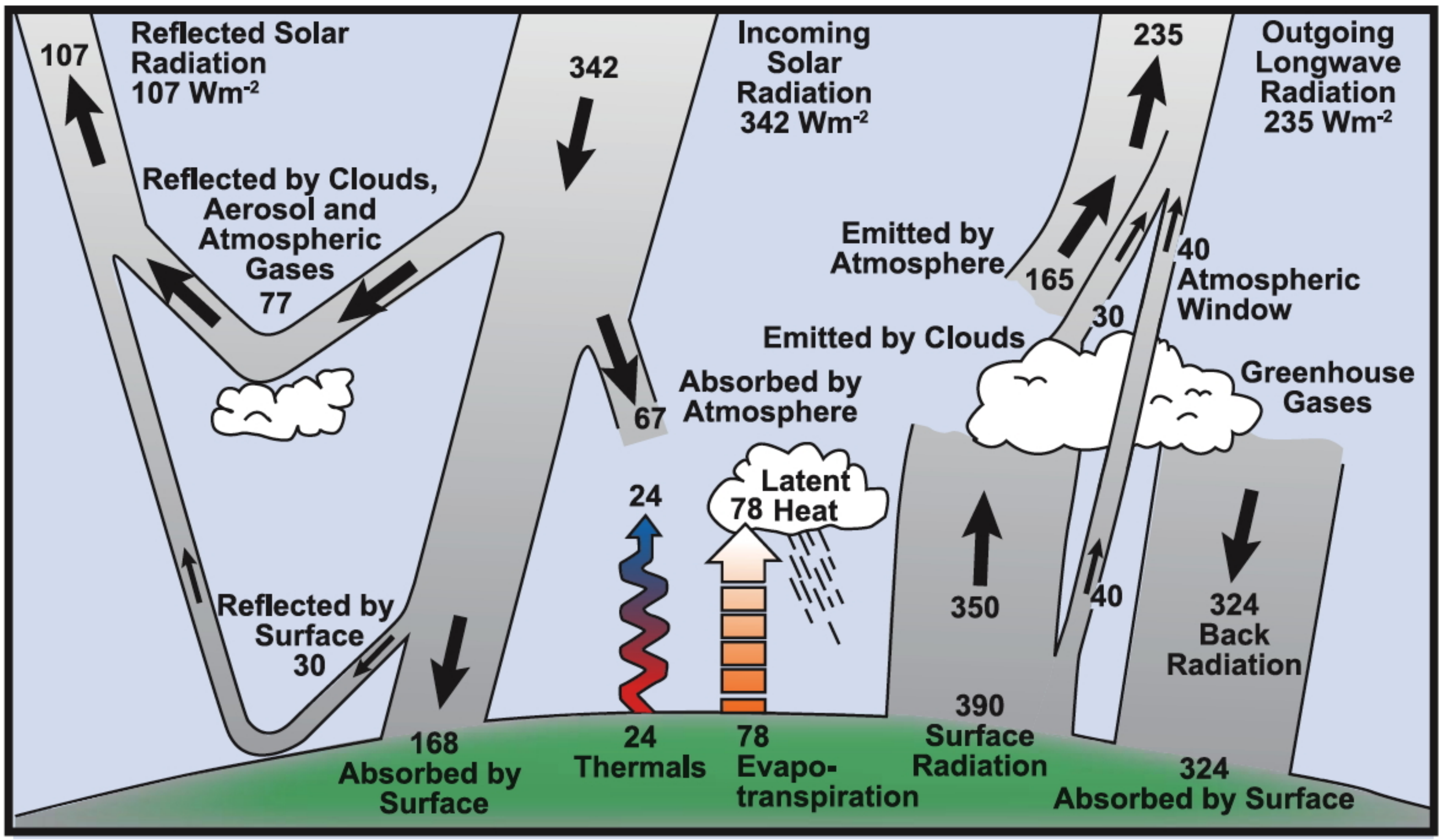


Simulation of Cs^{137} transport in the Pacific Ocean

cerea.enpc.fr/fukushima

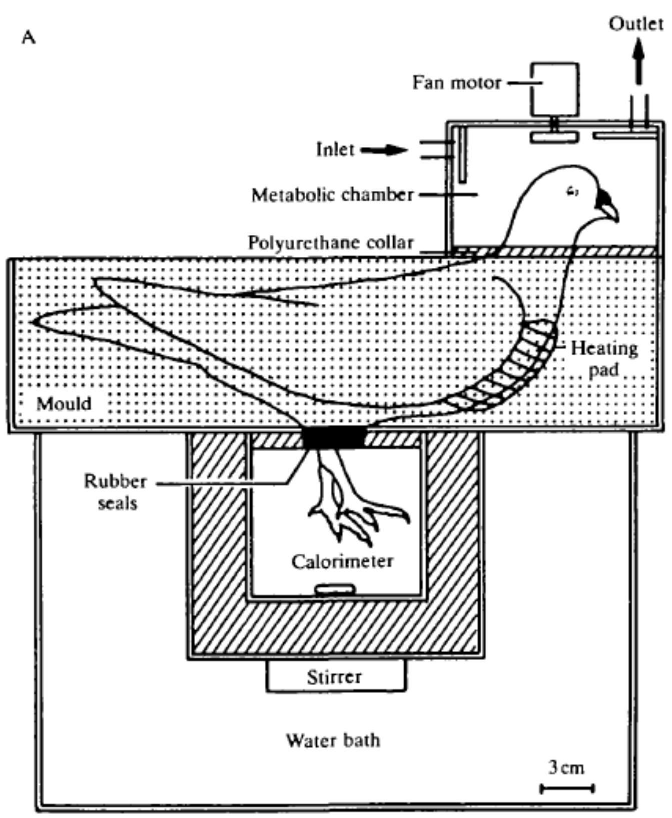
How does turbulence affects heat and mass transport ?

Heat transport in the environment : radiative equilibrium of the Earth

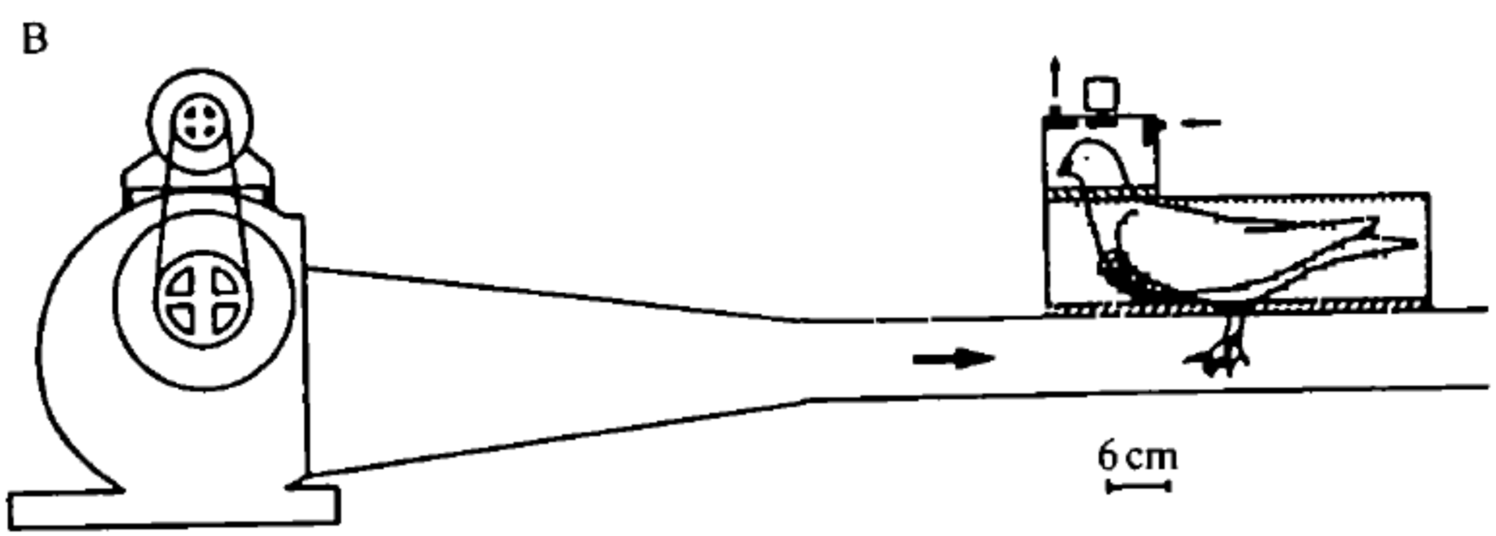


From an IPCC technical report

Heat transport in living systems : thermal regulation of migratory birds



What is the cooling power of pigeon’s legs ?



L. Martineau & J. Larochelle, J. Exp. Biology 136, 193 (1988)

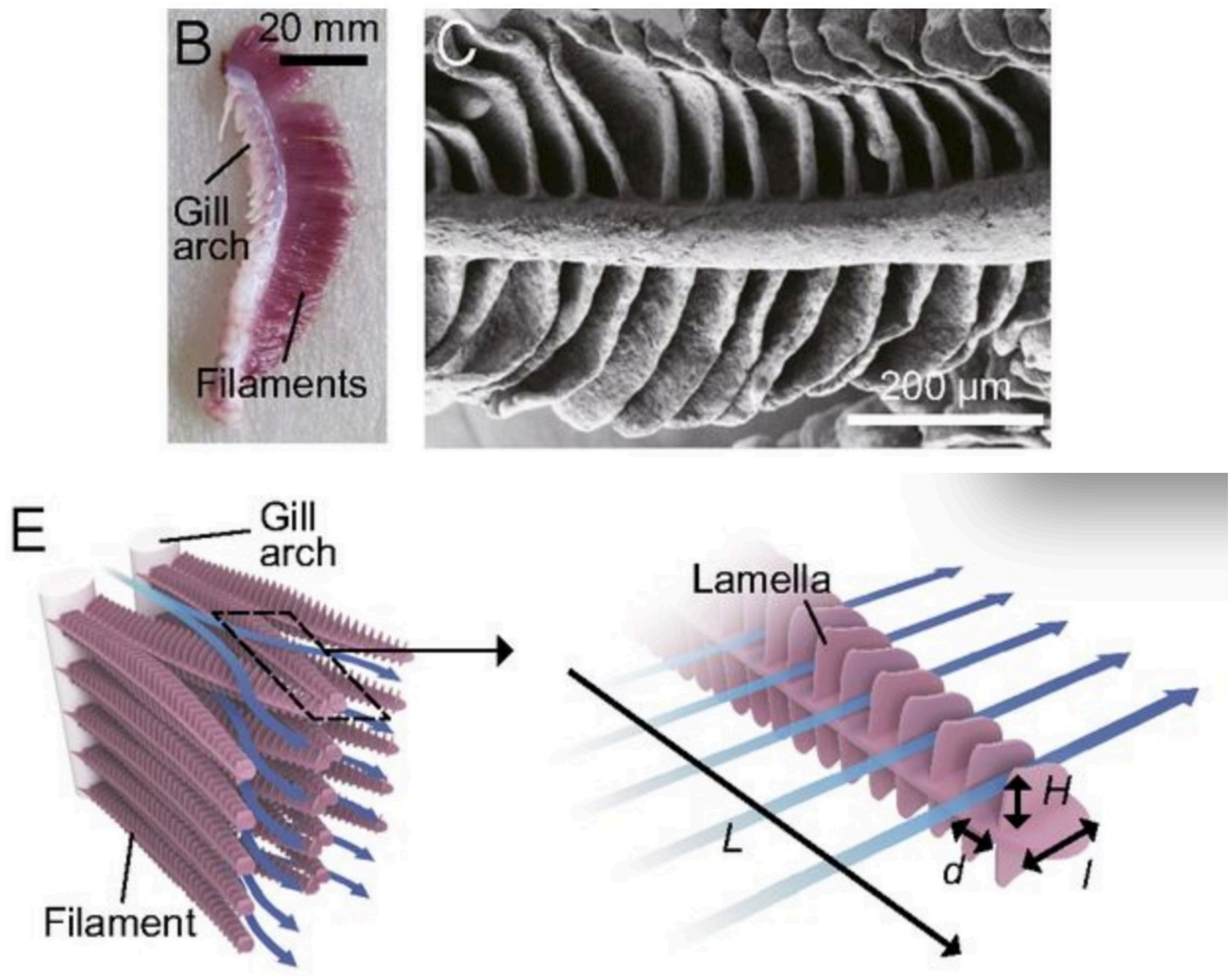
Heat transport in animals : thermal regulation of flying insects

The honeybee problem

T_a from 21 to 45°C

What are the different mechanisms involved in heat exchange ?
What is their relative importance ?
What is the temperature of the body as a function of air temperature ?

Mass transport in living systems : is there an optimal geometry for fish gills ?



K. Park et al., PNAS 111, 8067 (2014)

Outline

- review of diffusive processes
  - 1D steady state diffusion, with sources and phase changes
  - geometrical effects in diffusion
- radiative heat transfer
- transfer by convection (advection)
  - combined convection and diffusion
  - transport boundary layer
- thermal convection, coupling **u** and T
- dispersion in random velocity fields (turbulent flows, porous media)

The « inverted class »

reading material is posted on :
https://blog.espci.fr/marcfermigier/transport-phenomena-2021/

you read and (hopefully) understand it

in (virtual) class we check that ideas and concepts are understood through problem solving

we explain again ideas and concepts that need to be clarified

Transport processes. Basic relations

molecular diffusion

mass flux : Fick's law $\mathbf{J_D} = -D\nabla C$

heat flux : Fourier's law $\mathbf{J_D} = -\lambda\nabla T$

Flux : quantity exchanged through unit surface per unit time

Total Flux : flux integrated over a whole surface

convection
(advection by a macroscopic flow **u**)

mass flux $\mathbf{J_C} = C\mathbf{u}$

heat flux $\mathbf{J_C} = \rho C_p T\mathbf{u}$

Radiative heat transfer

$$\mathbf{J_R} = \sigma T^4$$

Conservation laws

local equations for concentration and temperature

Sources of heat and mass

chemical reactions
phase changes
nuclear reactions
dissipative processes

rate of change = divergence (flux) + source term

$$\frac{\partial T}{\partial t} + \mathbf{u}.\nabla T = \kappa \Delta T + \frac{R}{\rho C}$$

$$\frac{\partial C}{\partial t} + \mathbf{u}.\nabla C = D \Delta C + R$$

convection-diffusion equations

Peclet number Pe = U L/D

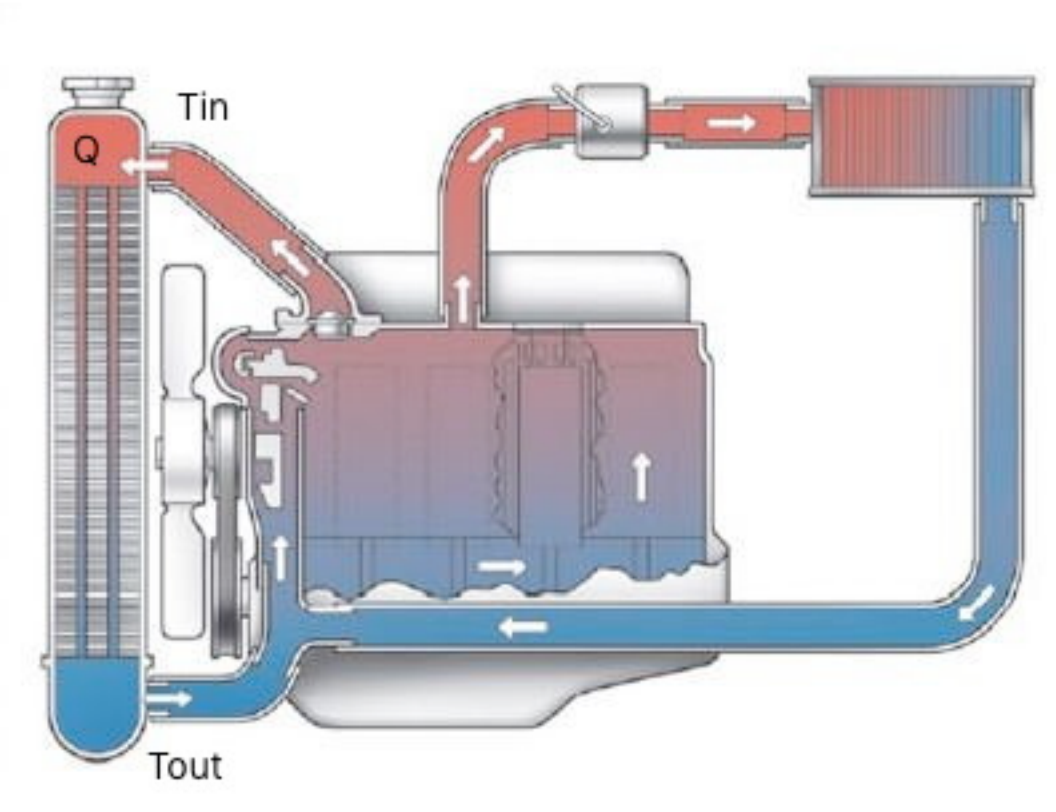
Boundary conditions at interfaces

Continuity of temperature and concentration fields
infinite gradients lead to infinite fluxes

Continuity of heat and mass fluxes
exception : 1st order phase change with latent heat

Boundary conditions for fluid flows
continuity of velocity and stresses

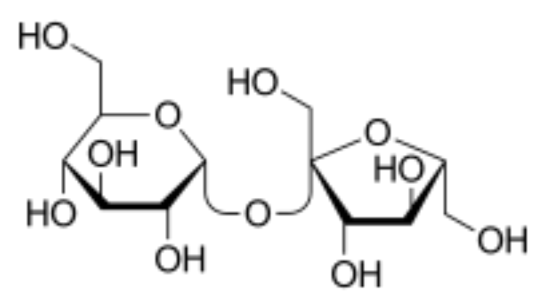
macroscopic balances



The coffee cup problem 1

If you don't stir the sugar in your coffee, why does it get cold way before it is sweetened ?

Coffee (essentially hot water) + sucrose (hydrodynamic radius 0.5 nm)



How long does it take for the sucrose to diffuse to the top ?

The coffee cup problem 1a

What is the molecular diffusion coefficient of sucrose in water ?

$$D = \mu \; k_B T$$

μ : mobility = velocity/applied force

Langevin's equation for the motion of a Brownian object

$$m \frac{d^2\mathbf{x}}{dt^2} = -\frac{1}{\mu}\frac{d\mathbf{x}}{dt} + \mathbf{F_T}(t) \qquad \langle F_T \rangle = 0$$

$$\langle x\dot{x} \rangle = \frac{1}{2}\frac{d}{dt}\langle x^2 \rangle = \mu k_B T + C\exp(-t/\mu m)$$

$$t \gg \mu \, m$$

$$\langle x^2 \rangle = 2\mu \; k_B T \; t$$

Sphere of radius *a* in a viscous fluid

$$\mu = \frac{1}{6\pi\eta a}$$

$$D = \frac{k_B T}{6\pi\eta a}$$

at 300 K, $k_BT = 4 \; 10^{-21}$ J

a = $5 \; 10^{-10}$ m, $\eta = 10^{-3}$ Pa.s

D = $4 \; 10^{-10}$ m^2/s

$t_D = L^2/D \sim 10^6$ s ~10 days

The coffee cup problem 2

How long does it take to cool down to room temperature ?
What if diffusion in air is the only mechanism ?

Physical properties of air :

density ρ = 1 kg/m^3
specific heat C_p = 1000 J/kg.K
thermal conductivity λ = 0.025 W/m.K

$\kappa = \lambda/\rho C_p = 2.5\ 10^{-5}$ m^2/s

Physical properties of coffee :

density ρ = 1000 kg/m^3
specific heat C_p = 4180 J/kg.K
thermal conductivity λ = 0.6 W/m.K

$t_\kappa = L^2/\kappa \sim 100$ s