The Maurice Herzog problem 1

How long can you stand on top of Annapurna without gloves ? (no wind)

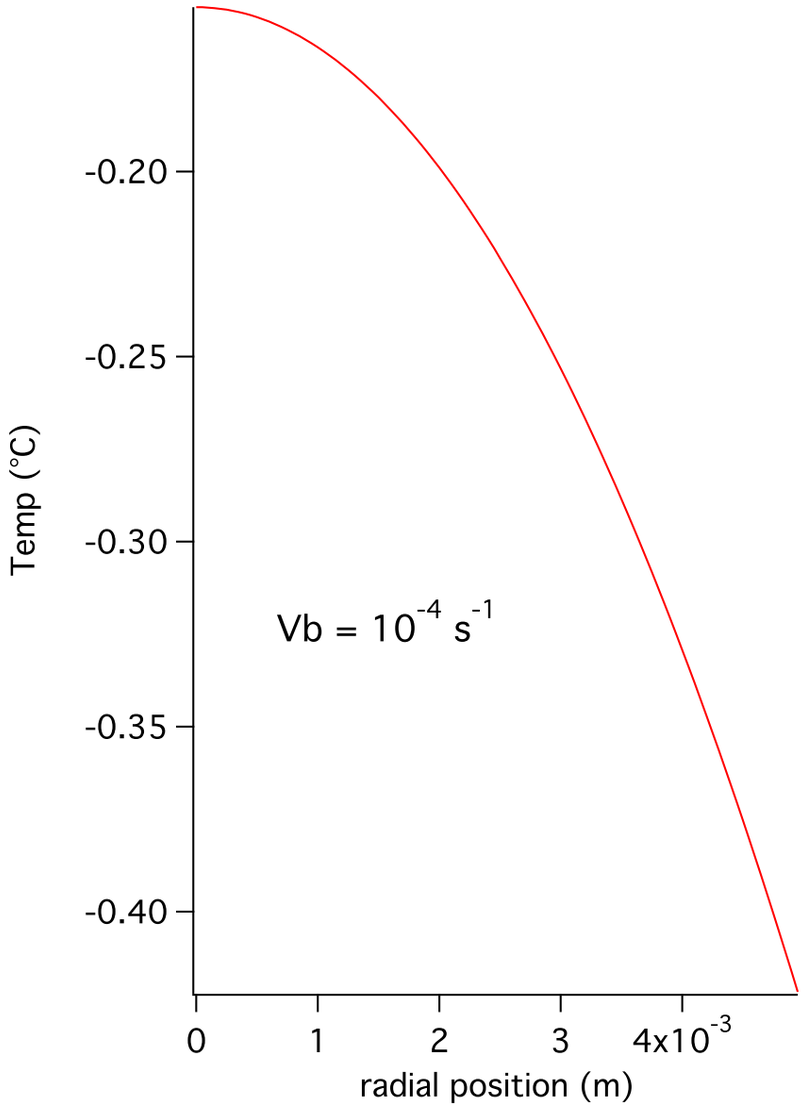
steady state 1D diffusion + source term

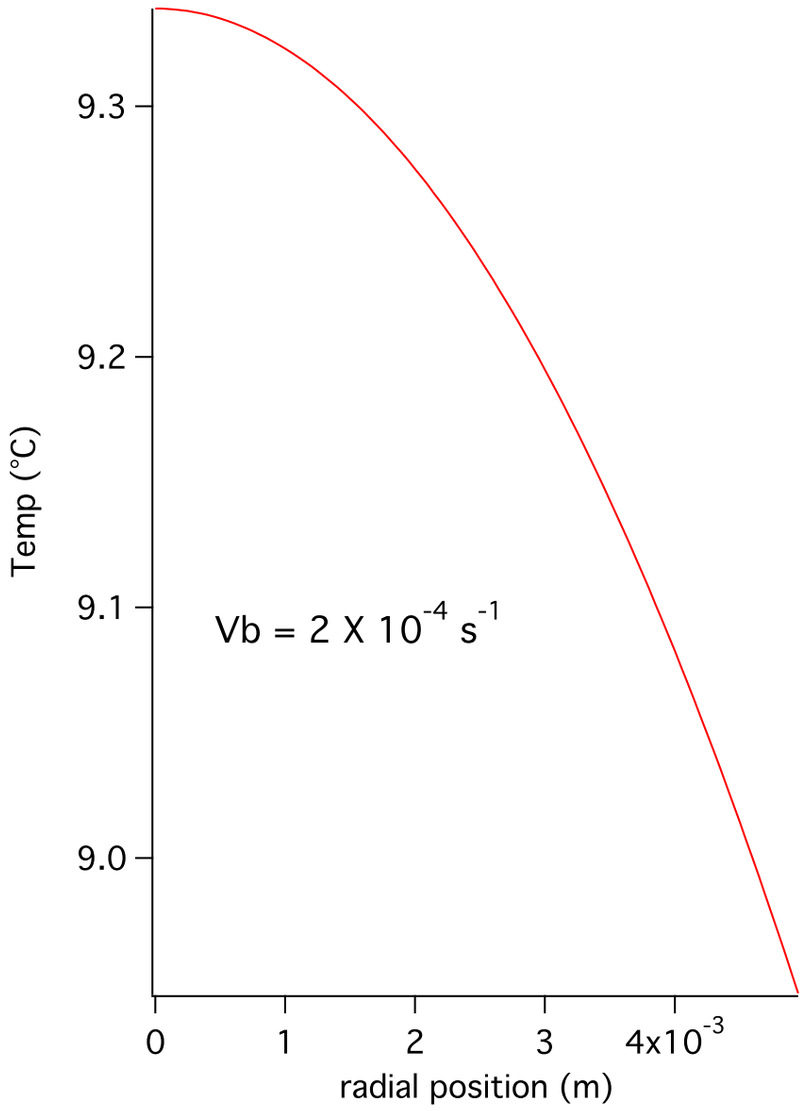
use the 1D bioheat equation to determine the temperature in Maurice's fingers (5mm of tissue between bone and skin) at steady state

what are the characteristic lengths involved in the problem ?

estimate the time required to reach this steady state

blood flow rate per unit volume of tissue: $V_b \sim 10^{-4}\ s^{-1}$ or $2 \times 10^{-4}\ s^{-1}$
blood density: $\rho_b \sim 1000\ kg/m^3$
blood specific heat: $C_b \sim 3600\ J/kg.K$
blood temperature $T_b = 37°C$
thermal conductivity of tissue: $\lambda \sim 0.5\ W/m.K$
metabolic heat generation rate: $q_m \sim 0$ in fingers (no muscles)
outside temperature: $T_a = -20°C$
heat transfer coefficient with air: $h \sim 2\ W/m^2.K$





Steady state solution of $\dfrac{\partial^2\theta}{\partial x^2} - \dfrac{\rho_b C_b V_b}{\lambda}\theta = 0 \qquad \theta = T - T_b$

With symmetry condition (zero gradient) at x=0 and continuity of flux (h (T-Ta) =λ grad T) at $x=x_s$.

$$T = Tb + \frac{T_a - T_b}{\cosh(x_S/l_b) + \lambda/(hl_b)\sinh(x_S/l_b)}\cosh(x/l_b)$$