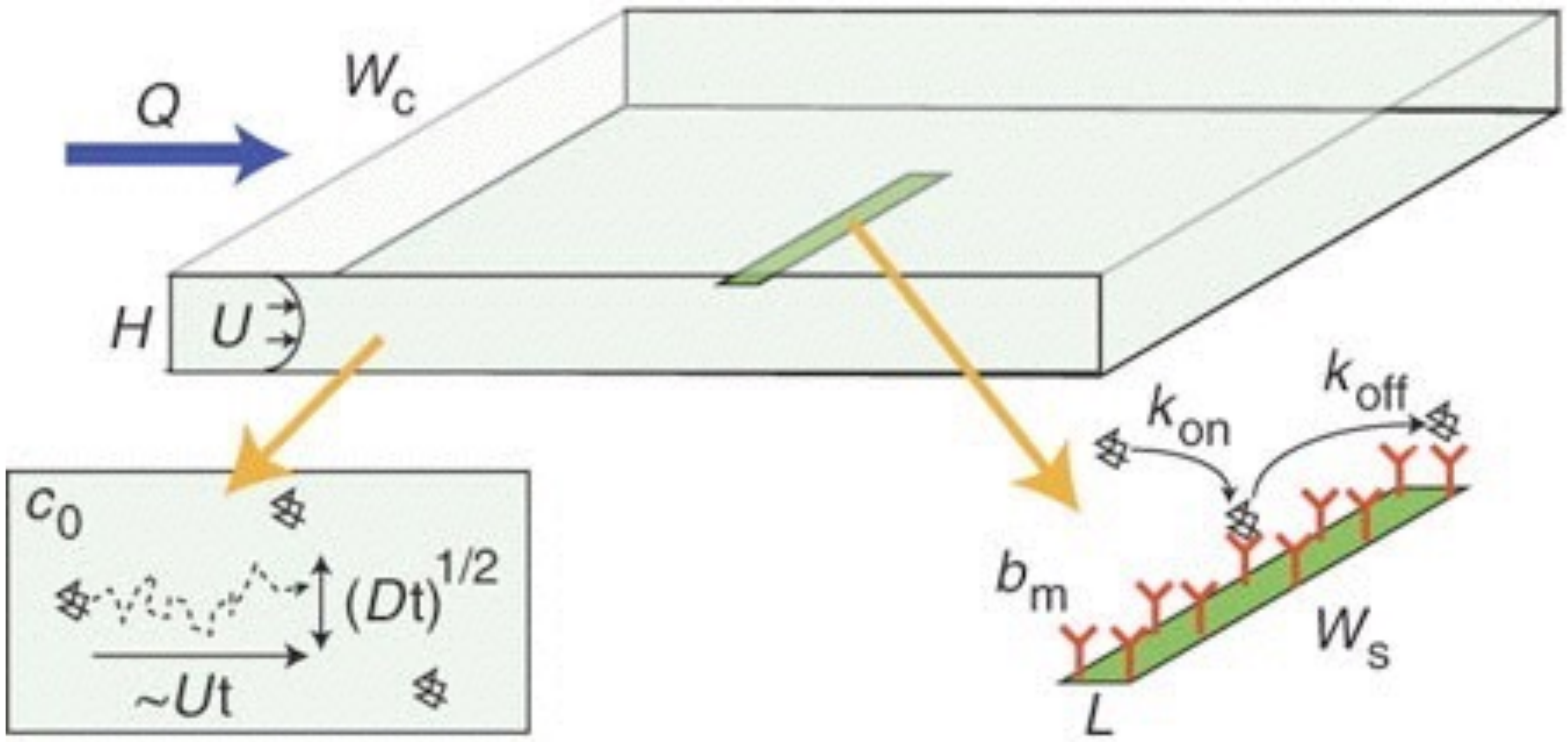
The microchip problem, T Squires et al. Nat. Biotech. 2008

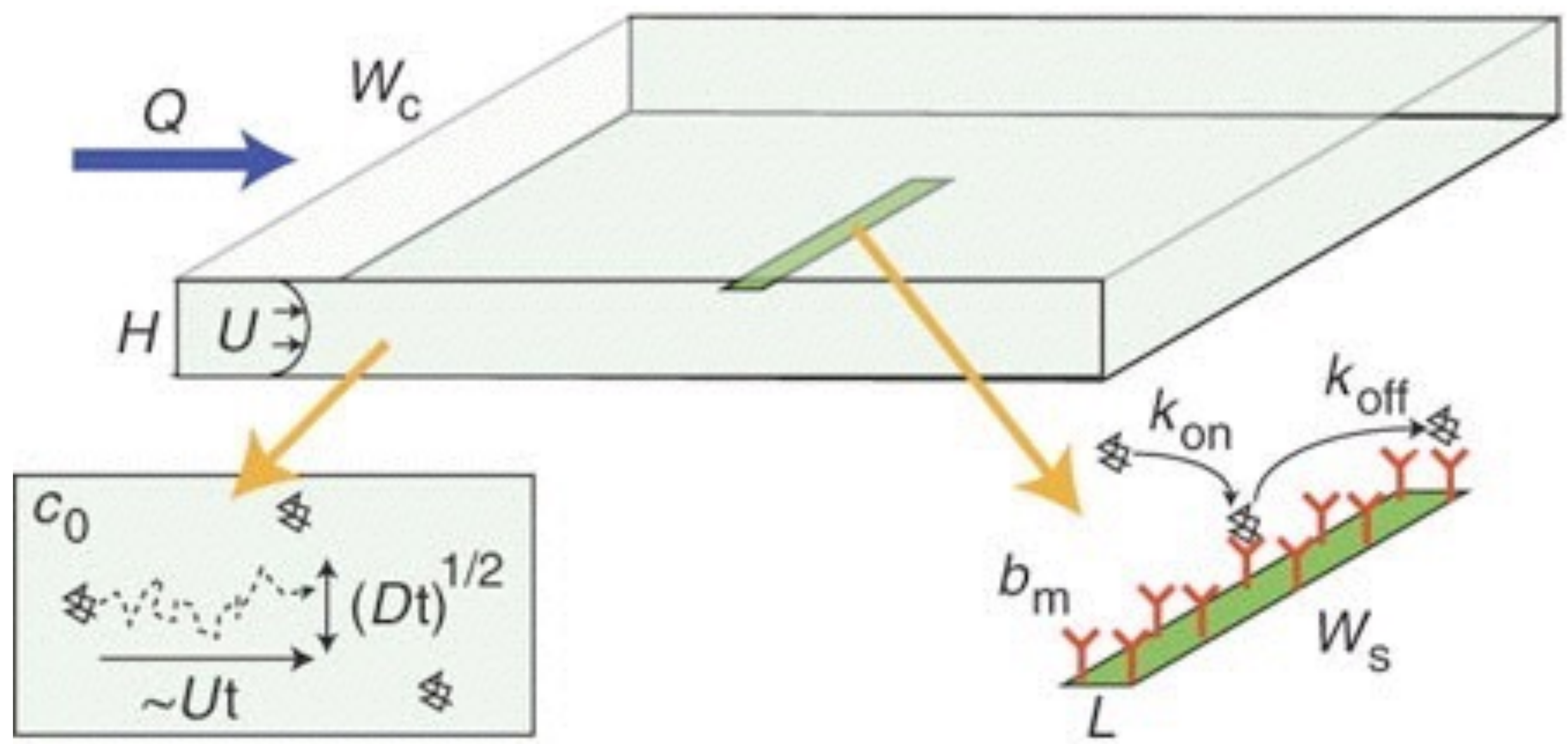


Given the following parameters

- Flow rate $Q = U\,H\,W_c$
- Poiseuille flow profile inside the channel
- Geometry channel height H, width Wc, reactive strip length L, width Ws
- $H << W_c$
- Inlet concentration C_0, diffusion coefficient D of analyte

Compute the total flux of analyte J towards the reactive strip, at steady state (ignore kinetics and assume very fast adsorption)

Compare J to a purely diffusive flux $J_D = DC_0/L \times L\,W_s$

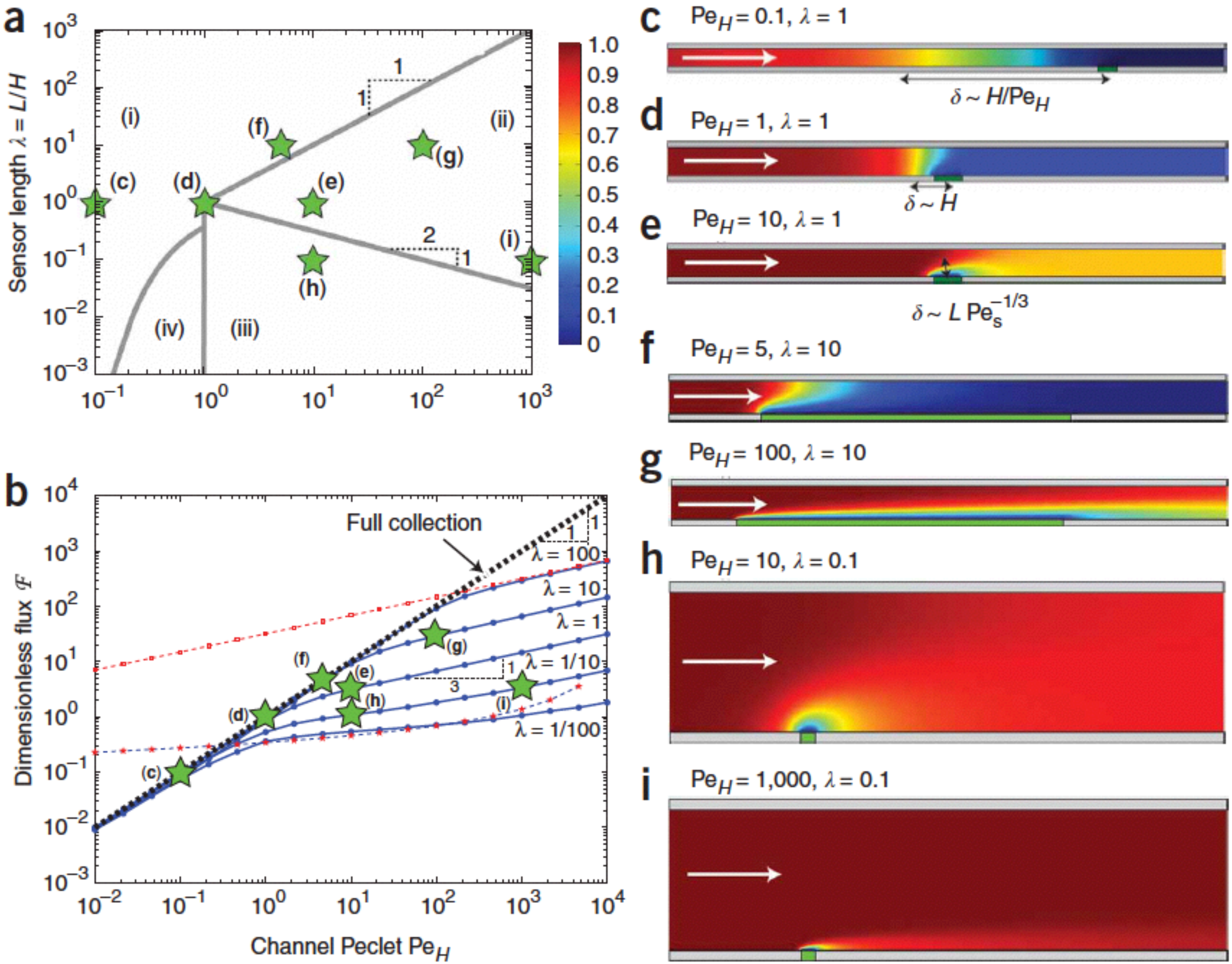


$$J_{total} = -D \int_0^L \frac{\partial C}{\partial y}(y=0)dx$$

$$y(1-y)\frac{\partial C}{\partial x} = \frac{1}{Pe_H}\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right)$$

$$y(1-y)\frac{\partial C}{\partial x} = \frac{1}{Pe_H}\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right)$$

assume δ(x) << L and δ(x) << H,
simplify the equation
derive a scaling law for δ(x)
estimate the mass flux
check the conditions of validity of δ(x) << L and δ(x) << H


a
Sensor length λ = L/H
Channel Peclet Pe_H
(i)
(ii)
(iii)
(iv)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
b
Dimensionless flux ℱ
Full collection
λ = 100
λ = 10
λ = 1
λ = 1/10
λ = 1/100
c
Pe_H = 0.1, λ = 1
δ ~ H/Pe_H
d
Pe_H = 1, λ = 1
δ ~ H
e
Pe_H = 10, λ = 1
δ ~ L Pe_s^-1/3
f
Pe_H = 5, λ = 10
g
Pe_H = 100, λ = 10
h
Pe_H = 10, λ = 0.1
i
Pe_H = 1,000, λ = 0.1