Heat exchangers in marine mammals, an example of countercurrent heat exchangers

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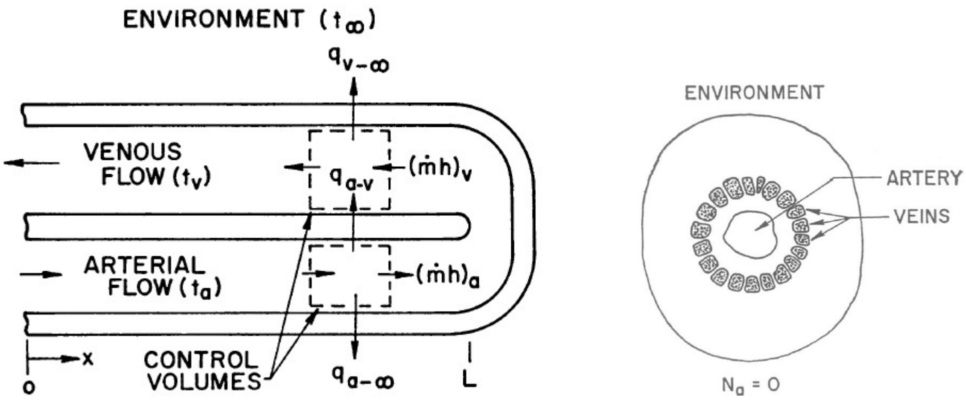

FIGURE 1 – Counter current heat exchanger made or arteries and veins in the fins of a dolphin. Figures reproduced from Mitchell & Myers, Biophys. J. 8, 897 (1968).

On the control volume of length dx in the arterial flow the balance of heat is :

$$\dot{m}CT_a(x) - \dot{m}CT_a(x+dx) + (T_a(x) - T_v(x))hdx = 0 \quad (1)$$

where $\dot{m}$ is the mass flow rate, C is the specific heat per unit mass :

$$\dot{m}C\frac{dT_a}{dx} + (T_a(x) - T_v(x))h = 0 \quad (2)$$

With a similar equation on the venous side, with additional heat transfer to the environment :

$$-\dot{m}C\frac{dT_v}{dx} + (T_v(x) - T_e)k - (T_a(x) - T_v(x))h = 0 \quad (3)$$

We need to solve the coupled system of equations with the following boundary conditions :
- The arterial blood arrives in the heat exchanger ($x = 0$) at temperature : $T_a(0) = T_0$.
- At the end of the exchanger ($x = L$) arterial and venous blood are at the same température : $T_a(L) = T_v(L)$.

To solve the equations, we introduce dimensionless variables :
- $\xi = x/L$
- $t_a = (T_a(x) - T_e)/(T_0 - T_e)$
- $t_v = (T_v(x) - T_e)/(T_0 - T_e)$
- $K = kL/(\dot{m}C)$
- $H = hL/(\dot{m}C)$

The coupled equations are in dimensionless variables :

$$\frac{dt_a}{d\xi} + H(t_a - t_v) = 0 \tag{4}$$

$$\frac{dt_v}{d\xi} - Kt_v + H(t_a - t_v) = 0 \tag{5}$$

with boundary conditions : $t_a(0) = 1$, $t_a(1) = t_v(1)$.

We can express t_v as a function of t_a :

$$t_v = t_a + \frac{1}{H}\frac{dt_a}{d\xi} \tag{6}$$

and :

$$\frac{dt_v}{d\xi} = \frac{dt_a}{d\xi} + \frac{1}{H}\frac{d^2t_a}{d\xi^2} \tag{7}$$

and we get a second order differential equation for t_a :

$$\frac{d^2t_a}{d\xi^2} + (2H - K)\frac{dt_a}{d\xi} - HKt_a = 0 \tag{8}$$

The general solution is $t_a = C_1 \exp(\lambda_1 \xi) + C_2 \exp(\lambda_2 \xi)$ where λ_1 and λ_2 are the roots of $\lambda^2 + (2H - K)\lambda - HK = 0$. The solution with the appropriate boundary conditions is [1] :

$$t_a = \exp(K\xi/2)\left[\frac{B\cosh A(1-\xi) + \sinh A(1-\xi)}{B\cosh A + \sinh A}\right] \tag{9}$$

with the corresponding expression for the venous side t_v :

$$t_v = \exp(K\xi/2)\left[\frac{B\cosh A(1-\xi) - \sinh A(1-\xi)}{B\cosh A + \sinh A}\right] \tag{10}$$

where $A = (K/2)\sqrt{1 + 4H/K}$ et $B = \sqrt{1 + 4H/K}$.

The solutions depend on two dimensionless parameters, on one hand,K comparing the heat flux between the veins and the environment and the heat flux due to the blood flow ; on the other hand H comparing the heat flux between the veins and the arteries and the heat flux due to the blood flow. the solutions are shown on fig. 2 for a few values of H and K. In particular when H is much larger than 1, the temperature of venous blood rises because of the exchange with the arterial flow. So the counter current heat exchanger might be beneficial to the animal by reducing the heat loss to the environment. The macroscopic balance of heat in the exchanger gives : $Q = \dot{m}C(T_a(0) - T_v(0)) = \dot{m}C(T_0 - T_e)[1 - t_v(0)]$, where Q is the loss of heat per unit time. If t_v rises back to a value close to 1, the heat loss decreases.

However, some physiological estimates give values on the order of 0.2 for H and 0.1 for K. In this case, the temperature of the venous blood decreases continuously through the heat exchanger. So it is not yet clear if this arrangement of arteries and veins which is found in many species of marine mammals is really beneficial to reduce heat loss to the surrounding water, or if it serves another purpose.

1. Mitchell & Myers, An analytical model of the counter-current heat exchange phenomena, Biophys. J. 8, 897 (1968)

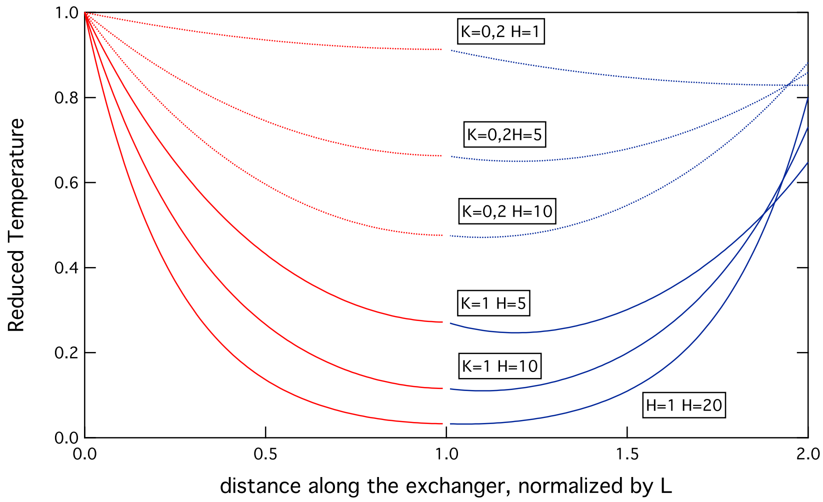

FIGURE 2 – Temperature within the countercurrent heat exchanger for two values of K (Dashed lines, $K = 0, 2$; plain lines $K = 1$) and different values of H. In red the arterial side, in blue the venous side.