Transport phenomena
133è promotion
Thermal regulation of starlings

1. *Justify the range of wavelengths used to measure the surface temperature of birds.*

The electromagnetic emission of a body at temperature T has a maximum at wavelength $\lambda_M(m) = 2.88 \times 10^{-3}/T$ (Wien's displacement law). Around room temperature (300K), the maximum of emission is at a wavelength of 10 μm within the range of sensitivity of the camera. Using this range of wavelengths optimizes the intensity of the signal from a body at 300 K.

2. *The infrared camera measures the electromagnetic flux emitted by a surface. Given the range of wavelengths in which the camera sensor operates, how should one modify Stefan's law to convert the measured flux into a temperature (the explicit calculation is not required, but only its analytical form) ? Is the temperature variation of the measured flux identical to the variation in Stefan's law ?*

The electromagnetic flux measured by the camera J_m is the integral of the flux emitted in the range λ_1, λ_2 (assuming a flat response of the camera sensor in this range, and a constant emissivity ϵ) :

$$J_m = \int_{\lambda_1}^{\lambda_2} \frac{2\pi hc^2}{\lambda^5} \frac{\epsilon d\lambda}{\exp(hc/\lambda k_B T) - 1}$$

If we use the reduced variable $\zeta = \lambda k_B T/hc$, we get :

$$J_m = \frac{2\pi\epsilon(k_B T)^4}{h^3 c^2} \int_{\zeta_1}^{\zeta_2} \frac{d\zeta}{\zeta^5 \exp(1/\zeta) - 1}$$

We get the same dependence on temperature as Stefan's law, but with a smaller numerical factor, since we integrate only on a finite range of wavelengths and not on the whole spectrum. However when the temperature changes, if λ_1 and λ_2 keep the same value, the integration limits ζ_1 and ζ_2 change and the temperature dependence is not strictly the same as Stefan's law. If the temperature variations remain small compared to the absolute temperature, we still have a proportionality between the measured power and T^4.

3. *The temperature of the wind tunnel walls is equal to 20° C and we consider that air is at the same temperature. The emissivity ϵ of birds is estimated to be 0.95. Neglecting the radiation from ambiant air and ignoring any view factors between the surface of birds and the tunnel walls, estimate the radiative heat flux for each of the parts A, B, C and the total flux exchanged on the bird.*

The heat flux emitted by the bird's surface is given by Stefan's law : $\epsilon\sigma T_s^4$. The flux of heat emitted by the walls of the wind tunnel is σT_a^4. Since the absorptivity is equal to the emissivity the flux of heat absorbed by the bird is $\epsilon\sigma T_a^4$. Assuming that the view factor is equal to 1 (the bird is completely enclosed in the wind tunnel) we get the net flux of heat from the bird :

$$j_R = \epsilon\sigma(T_s^4 - T_a^4)$$

Taking $T_a = 293$ K, $\sigma = 5.67 \times 10^{-8}$ W.m^{-2}.K^{-4}, we get the following radiative fluxes (j_{Ri} per unit surface, J_{Ri} integrated on the surface) for the different parts of the bird :

— legs : $j_{RA} = 39$ W.m^{-2}, $J_{RA} = j_{RA} S_A = 0,03$ W

— brachial part of wings : $j_{RB} = 30$ W.m^{-2}, $J_{RB} = j_{RB} S_B = 0,25$ W

— everything else : $j_{RC} = 16,5$ W.m^{-2}, $J_{RC} = j_{RC} S_C = 0,66$ W

And the total amount of heat exchanged by radiation per unit time is $J_R = 0,94$W.

4. *Give an order of magnitude of the dimensionless parameters relevant for the air flow around birds and for the heat transfer. The velocity of the air flow is taken as $U = 10$ m/s.*

The total area of the wings is 330 cm^2 (taking into account the upper and lower sides). We can consider them as a rectangle of area $W \times L = 4L^2$. We have then $4L^2 \approx 160$ cm^2 and $L^2 \approx 40$ cm^2 , hence $L \approx 6,5$ cm the characteristic length for the flow field.

The relevant dimensionless number for the flow is the Reynolds number computed with L, $Re = UL/\nu$. With $U = 10$ m/s and $\nu = 1,5 \times 10^{-5}$ m^2/s, $Re \approx 4 \times 10^4$.

For heat transfer the relevant dimensionless number is the Peclet number $Pe = RePr$. For air, the Prandtl number Pr is equal to 0,7 and the order of magnitude of the Peclet number is 3×10^4.

For the legs, the relevant lengthscale is the average diameter D_P which is 3 mm. The corresponding Reynolds number is $R_P \approx 2000$ and the Peclet number Pe_P is around 1500Th.

5. *Is it necessary to take into account the transfer by pure diffusion ?*

The Peclet number is much larger than 1 for the wings, body and legs as well. Heat transport par convection is then dominant over the transport by pure diffusion. However, in a situation where a transport boundary layer exists, we should take into account the diffusive flux through the boundary layer to compute the heat flux.

6. *Using a dimensional analysis of the heat transport equation, give an expression for the local heat flux on the surface of the bird for the parts B and C, considered as flat plates and for the legs as well.*

Since the Reynolds number is much larger than 1 for all parts of the bird, the flow around the bird is a boundary layer type flow. Assuming that the flow remains laminar in the boundary layer, the thickness of this layer $\delta(x)$ varies as $\sqrt{\nu x/U}$, x being the distance from the leading edge of the wings of the stagnation point on front of the legs. Within the boundary layer, the velocity profile is linear : $u_x(y) \approx Uy/\delta(x)$ where y is the coordinate normal to the surface. In steady state $(\partial_t T = 0)$ and in the boundary layer approximation $(\partial_{xx} T \ll \partial_{yy} T)$ the transport equation for heat is :

$$u_x \frac{\partial T}{\partial x} = U \frac{y}{\delta(x)} \frac{\partial T}{\partial x} = \kappa \frac{\partial^2 T}{\partial y^2}$$

Evaluating both sides of the equation at $y = \delta_T(x)$ where $\delta_T(x)$ is the thickness of the thermal boundary layer, we get :

$$\delta_T^3 \sim \frac{\kappa}{U} x \delta(x) = \frac{\kappa}{\nu} \left(\frac{\nu x}{U}\right)^{3/2}$$

and :

$$\delta_T \sim Pr^{-1/3} \delta(x).$$

The local heat flux at position x on the bird surface $(y = 0)$ is :

$$J(x) = \lambda \left(\frac{\partial T}{\partial y}\right)_{y=0} \approx \lambda \frac{\delta T}{\delta_T(x)} = \lambda \frac{\delta T Pr^{1/3}}{\delta(x)}.$$

7. *Give an expression in scaling law for the Nusselt number on parts B and C and for the legs.*

The heat flux integrated over the length L in the streamwise direction is :

$$J_{total} = \lambda \delta T Pr^{1/3} \int_0^L \delta(x)^{-1} dx = \lambda \delta T Pr^{1/3} \left(\frac{UL}{\nu}\right)^{1/2}$$

With pure diffusion, the heat flux integrated over the length L would be $J_{dif} \sim \lambda \delta T / L \times L = \lambda \delta T$. The Nusselt number Nu is the ratio J_{total}/J_{dif} and we have :

$$Nu = Pr^{1/3} Re_L^{1/2}$$

For the legs, the derivation is exactly similar but the relevant characteristic lengthscale is the diameter D_P, instead of L and we have $Nu = Pr^{1/3} Re_{D_P}^{1/2}$.

8. *Estimate the total heat flux due to the air flow around the bird. Physiologists have noted that birds increase their heat transfer by moving the legs away from the body and dragging them into air. What is the specific contribution of legs to the heat flux ?*

For air, $Pr = 0,7$ and $Pr^{1/3} = 0,9$.

For the different parts of the bird we have :

— legs : $Re_P \approx 2000$ and the Nusselt number Nu_A is on the order 40.The total heat flux is then :

$$J_{CA} = S_A \ Nu_A \ J_{difA} \sim S_A \ Nu_A \ \frac{\lambda \delta T_A}{D_P} \ \approx 1,8\text{W}$$

— brachial part of wings : surface 80 cm^2 = 2 ×40 cm^2. If we keep the same aspect ratio of 4, we have $L^2 = 10$ cm^2 and $L \approx 3$ cm. $Re_L \approx 2 \times 10^4$ and $Nu_B \approx 130$. The total heat flux is :

$$J_{CB} = S_B \ Nu_B \frac{\lambda \ \delta T_B}{L} \approx 4,8\text{W}$$

— everything else : surface 400 cm^2, we have $L^2 = 100$ cm^2 and$L \approx 10$ cm. $Re_L \approx 6 \times 10^4$ and $Nu_C \approx 220$. The total heat flux is :

$$J_{CC} = S_C \ Nu_C \frac{\lambda \ \delta T_C}{L} \approx 6,6\text{W}$$

Adding the contributions from the three parts we get an amount of heat exchanged per unit time on the order of 13 W, the legs contributing for 15%.

9. *What is the dominant mechanism of heat transfer : radiation or convection by the flow ?*

The heat transfer by convection is an order of magnitude larger than the transfer by radiation.

10. *S. Ward and colleagues estimated, by an independent method, the metabolic power of starlings in flight. This power increases linearly with flight speed U from 8 W at 6 m/s to 13 W at 14 m/s. How does this power compare with the heat flux estimated in the simple model developed above ?*

At 10 m/s, the metabolic power would be $8 + 5 \times 4/8 = 10,5$ W. We find a total amount of heat exchange on the order of 14W (convection+radiation) which is larger than this value. The heat flux has to be smaller than the metabolic power, since this power accounts also for the mechanical energy used for propulsion. Our simple model overestimates the heat flux but we find the correct order of magnitude.