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Gallery of Fluid Motion

Drop Medusa: Direct numerical simulations of high-frequency Faraday waves on spherical drops

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The dynamics of a drop under vibration is a classical and long-standing fluid dynamics problem. Lamb [1] studied the free oscillations of a viscous drop, and Lundgren *et al.* [2] carried out the first numerical study of the flow. Although several numerical techniques were then used to study oscillatory phenomena [3–5], these studies were limited to axisymmetric, moderate amplitude, low capillary mode oscillations, and were unable to attain long-time, finite-amplitude motion. Recently, three-dimensional numerical simulations of drops forced by moderate amplitude and low-frequency radial vibration were carried out by Ebo-Adou *et al.* [6], covering spherical harmonics ranging from one to six. In this spherical analog of Faraday waves, the authors observed regular and simple patterns. At higher amplitudes and frequencies, it is inevitable that many spherical harmonics will be excited and interact nonlinearly.

Here, we extend the numerical study of Ebo-Adou *et al.* [6] to high frequency and amplitude. Unlike our previous Gallery of Fluid Motion entries [7], where complexity like contact line motion was present, here, we have chosen a levitated drop under radial acceleration. As a continuation to the study of nonlinear waves on drops, we show the atomization process under such conditions.

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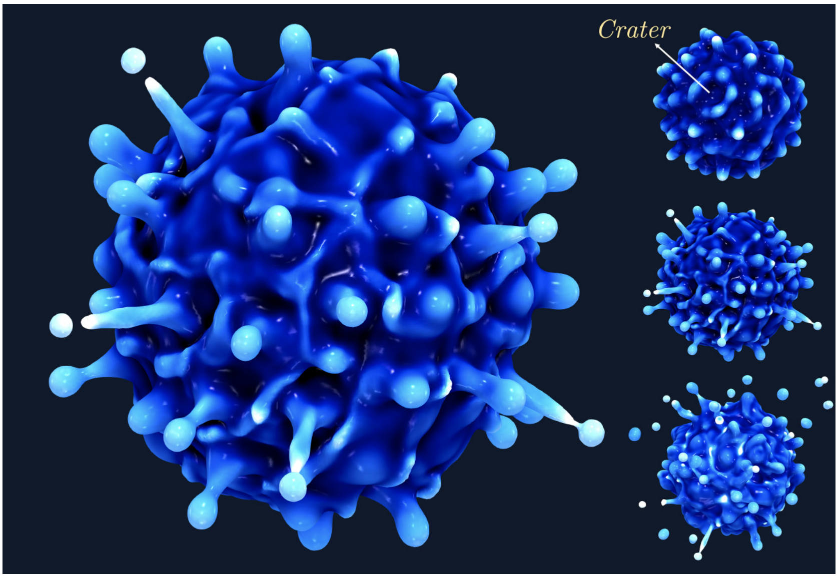

FIG. 1. Drop Medusa: (left) prominent threads of jets forming on the interface that resemble the living snakelike hair of the Greek mythological figure, Medusa; (right) interfacial evolution at $t = 380T$, $385T$, and $387T$, top to bottom, respectively [12].

We performed three-dimensional direct numerical simulations of a drop of volume $\mathcal{V} = 100\,\mu\text{L}$ using our in-house Navier-Stokes multiphase solver, BLUE [8]. A triply periodic cubic computational domain, encompassing water and air, is decomposed into $8 \times 8 \times 8$ subdomains each of resolution 64^3. The corresponding global mesh structure is 512^3 grid cells, each of linear dimension 22.5 μm. The density of the liquid drop and its surroundings are set to those of water and air, $998\ \text{kg/m}^3$ and $1.205\ \text{kg/m}^3$, and their dynamic viscosities to 10^{-3} kg/m s and 1.82×10^{-5} kg/m s, respectively, while the surface tension is set equal to 0.0714 N/m. The volumetric force on a sphere is analogous to the modulated gravitational force on a planar interface [6] and the frequency is set to $f = 1040$ Hz, as per Vukasinovic *et al.* [9] with an initially slightly perturbed interface (of amplitude $\epsilon = 0.005R$) and proportional to the 20th axisymmetric spherical harmonic. The choice of these parameters ensured that the radius of the drop is larger than the capillary wavelength $2\pi/k$, related to the vibration frequency by $\omega^2 = \sigma k^3/\rho$, such that we obtain capillary waves on the drop [10]. Unlike the icosahedral and cubic patterns that are excited at lower frequencies [6], high frequency driven vibration excites higher modes, as we will discuss in this paper. Neither a linear stability analysis nor experiments have been found on which to base our choice of acceleration amplitude. Therefore, we ran numerical experiments by varying the acceleration amplitude until we observed the appearance of waves. As in Panda *et al.* [7], we ramped up the acceleration to $A = 300g$ at a rate of $50g$ per 60 forcing time periods $T(= 1/f)$ up to $t = 360T$ to set up an irregularly perturbed interface. The kinetic energy decreases significantly before increasing again, suggesting that the initial perturbation has been damped and reorganized, so that the subsequent appearance and dynamics are independent of the initialization.

Figure 1 shows the behavior of the Faraday waves on the drop at $t = 380T$, $385T$, and $387T$, colored according to the magnitude of the interfacial pressure. The craters (shown at $t = 380T$) collapse to form jets that eject droplets [9,11]. In order to better understand this phenomenon, we compute the spectral coefficients obtained from the spherical harmonic decomposition $\zeta(\theta, \phi, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} \hat{\zeta}_l^m(t) Y_l^m(\theta, \phi)$, where Y_l^m is the spherical harmonic function of degree $l \in [0, \infty)$ and order $m \in [-l, l]$. In Fig. 2(a), we plot the square modulus $|\hat{\zeta}_l^m(t)|$ of the complex coefficients at

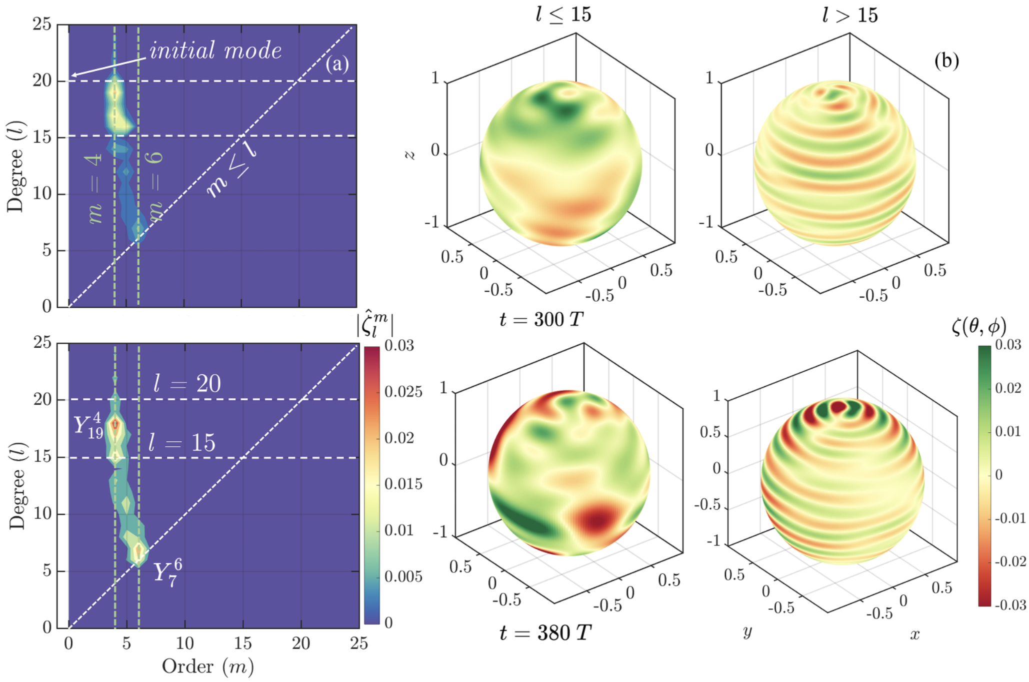

FIG. 2. (a) Spherical harmonic spectrum: modulus of the spherical harmonic coefficients $|\hat{\zeta}_{l,m}|$ at $t = 300T$ (top) and $380T$ (bottom); (b) interface reconstruction at $t = 300T$ (top) and $380T$ (bottom) from only the low degree (left, $l < 15$) or high degree (right, $l > 15$) spherical harmonics.

$t = 300T$ and $380T$. Initially at $t = 0$, the only nonzero coefficient is that of Y_{20}^0. At $t = 300T$, the excited spectral coefficients are mainly in the narrow band $m \in [4, 6]$ and $l \in [15, 20]$; for this reason, we use $l = 15$ as the boundary between high- and low-degree modes. We then reconstruct the surface to examine the role of these modes. For $l > 15$, we observe low-m modes that are centered with respect to the z axis that is distinguished at $t = 0$ via the axisymmetric initial condition Y_{20}^0. It is somewhat surprising that the orientation of the initial condition persists, despite the damping out of the initial surface perturbation and the emergence of new modes. By $t = 380T$, the amplitudes of lower-l modes have greatly increased in the system, as shown in the lower part of Fig. 2(a). More specifically, a range of $l \in [6, 20]$ is excited with $m \in [4, 6]$. The dominant modes are found to be Y_{19}^4 and Y_7^6. The reconstructed interface for $l \leqslant 15$ shows large patches of increased ζ with no apparent order. The amplitude of $|\hat{\zeta}_l^m(t)|$ for $l > 15$ also increases, and remains oriented around the z axis.

The superposition of all of these modes results in a highly deformed interface, as shown in Fig. 1, James *et al.* [11] described the way in which an erratic surface ζ leads to negative curvature zones on the drop surface, called craters. We associate these craters with the prominent red patches in Fig. 2(b) and with spherical wavenumbers of order $l \in [5, 15]$, $m \in [4, 6]$ in Fig. 2(a). The craters undergo capillary-driven collapse, which in turn leads to jet formation at $t = 385T$. Following their pinch off and ejection, the jets retract and fall back into the vibrating drop. These violent jet-formation events across the drop surface create a resemblance with the Greek mythological figure, Medusa, captured by the title of this paper.

In this work, we discussed the dynamics of vibration-induced drop atomization. A snapshot of the transient process was shown as a poster in Gallery Fluid Motion 2023 [12] and in Fig. 1. We have shown that the excitation of the higher modes ($l > 15$) lead to the superposition of nonlinear waves that form negative-curvature craters that give rise to jet formation. These jets undergo capillary pinch off, forming smaller droplets that result in atomization. The phenomenon of atomization has broad applications in nasal sprays, agrochemicals, food processing, disease spreading, and ink-jet printing [13].

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