Turing instability

Some researchers denote any steady bifurcation with a finite wavenumber by the name of Turing instability, but many other researchers believe that this name is reserved for bifurcations which arise in reaction-diffusion equations from sufficiently different diffusion constants for different species. Stationary finite-wavenumber instabilities were studied in 1916 by Rayleigh for thermal convection and in 1923 by Taylor for Taylor-Couette flow, long before Turing's 1952 article, and by many other researchers before and since.

Here are some references emphasizing the requirement of different diffusion coefficients.

A well-known and standard textbook on pattern formation:

• Rebecca Hoyle, *Pattern Formation: An Introduction to Methods*, Cambridge Univ. Press, 2006.
"Diffusion can also create patterns more direction: in a famous 1952 paper, Turing predicted that two reacting and diffusing chemicals, an activator and an inhibitor, can produce a pattern if the inhibitor diffuses much faster than the activator."

The leading textbook in mathematical biology

• J.D. Murray, Mathematical Biology: II: Spatial Models and Biomedical Applications, p. 76 (in Third Edition)
"Turing's (1952) idea is a simple but profound one. He said that, if in the absence of diffusion (effectively $D_A = D_B = 0$), A and B tend to a linearly stable uniform steady state then, under certain conditions, which we shall derive, spatially inhomogeneous patterns can evolve by diffusion driven instability if $D_A \neq D_B$."

The first observation of Turing patterns in chemical reactions:

• V. Castets, E. Dulos, J. Boissonade, and P. De Kepper, *Experimental evidence of a sustained standing Turing-type nonequilibrium chemical pattern*, Phys. Rev. Lett. **64**, 2953-2956, (1990).
"The necessary conditions for Turing patterns are very stringent. ... Moreover, the inhibitor should diffuse much faster than the activator."

The second observation of Turing patterns in chemical reactions:

• R. Dennis Vigil, Q. Ouyang and Harry L. Swinney, *Turing patterns in a simple gel reactor*, Physica A **188**, 17-25 (1992)
"It is believed that these complexes are immobile and result in a rescaling of the effective diffusivity of some of the species [12], which is a necessary condition for a Turing instability to occur."

Turing's original paper:

• A.M. Turing, *The Chemical Basis of Morphogenesis*, Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences **237**, 37-72 (1952).
Although the paper does not mention explicitly that the patterns are caused by differing diffusion coefficients, one finds the following concerning diffusion coefficients μ and ν or μ' and ν':
p. 52: $\nu = 0, \mu = 1$ and $\mu' = 1, \nu' = 1/2$
p. 55: Equation 9.5 contains $\mu' - \nu'$ as denominator.
p. 62: $\mu = 1/2, \nu = 1/4$
p. 71: $\mu' = 2, \nu' = 1$

Wikipedia:

• `https://en.wikipedia.org/wiki/Turing_pattern`
"The pattern arises due to Turing instability which in turn arises due to the interplay between differential diffusion (i.e., different values of diffusion coefficients) of chemical species and chemical reaction."

Scholarpedia:

• `http://www.scholarpedia.org/article/Oregonator`
"Turing patterns appear in the Oregonator when $D_z > D_x$ (Becker and Field, 1985)"