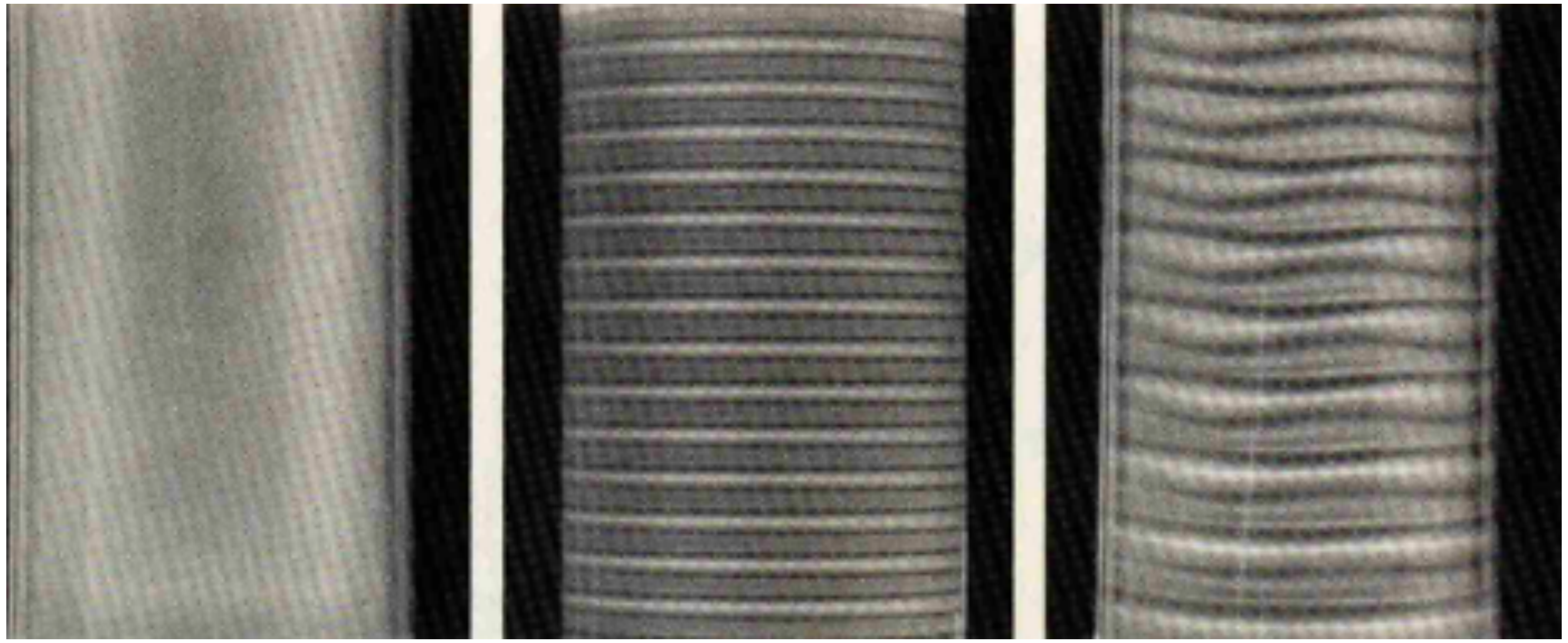
PHYSICS TODAY NOVEMBER 1991

# TAYLOR–COUETTE FLOW: THE EARLY DAYS

Fluid caught between rotating cylinders has been intriguing physicists for over 300 years with its remarkably varied patterns and its chaotic and turbulent behavior.

Russell J. Donnelly



(Courtesy of Harry Swinney and Randall Tagg, University of Texas, Austin.)

Taylor's paper, published in the *Philosophical Transactions of the Royal Society of London*, can fairly be called one of the most influential investigations of 20th-century physics. The correspondence that Taylor obtained between theory and experiment for the stability rested in an important way on the no-slip boundary condition for the flow at the solid surfaces. This success was taken by many as perhaps the most convincing proof of the correctness of the Navier–Stokes equations and of the no-slip boundary condition for the fluid at the cylinder walls. Such use of Taylor–Couette flow to confirm fundamental ideas in fluid dynamics has become a tradition.

**Donnelly, Physics Today, 1991**

The Couette-Taylor system has served as a paradigm for testing ideas on stability in systems described by nonlinear partial differential equations since the landmark work of Taylor[1] on flow between concentric rotating cylinders. He measured the critical Reynolds number for the primary instability and showed that it agreed within a few percent with the predictions of a linear stability analysis. This was the first quantitative agreement of theory and experiment for *any* flow instability. However, *linear* stability analyses do not, in general, completely determine the final pattern of secondary flow.

**Tagg, Edwards, Swinney, Marcus, Phys Rev A, 1989**

# TAYLOR–COUETTE FLOW: THE EARLY DAYS

Fluid caught between rotating cylinders has been intriguing physicists for over 300 years with its remarkably varied patterns and its chaotic and turbulent behavior.

Russell J. Donnelly

The striking flow shown in figure 1 is produced in a simple apparatus: A fluid is confined between two concentric cylinders, with the inner and perhaps the outer cylinder able to rotate. The cellular motion that develops with rotation was discovered and described mathematically by Geoffrey I. Taylor in 1923. A similar apparatus, with the inner cylinder suspended from a torsion fiber and the outer cylinder rotating, was used even earlier as a viscometer. Maurice Couette described this arrangement in his thesis, which he presented in Paris in 1890. For this reason, modern investigators refer to flow between rotat-

# TAYLOR-COUETTE FLOW: THE LATER DAYS

VIII. *Stability of a Viscous Liquid contained between Two Rotating Cylinders.*

Cambridge
1923

By G. I. Taylor, F.R.S.

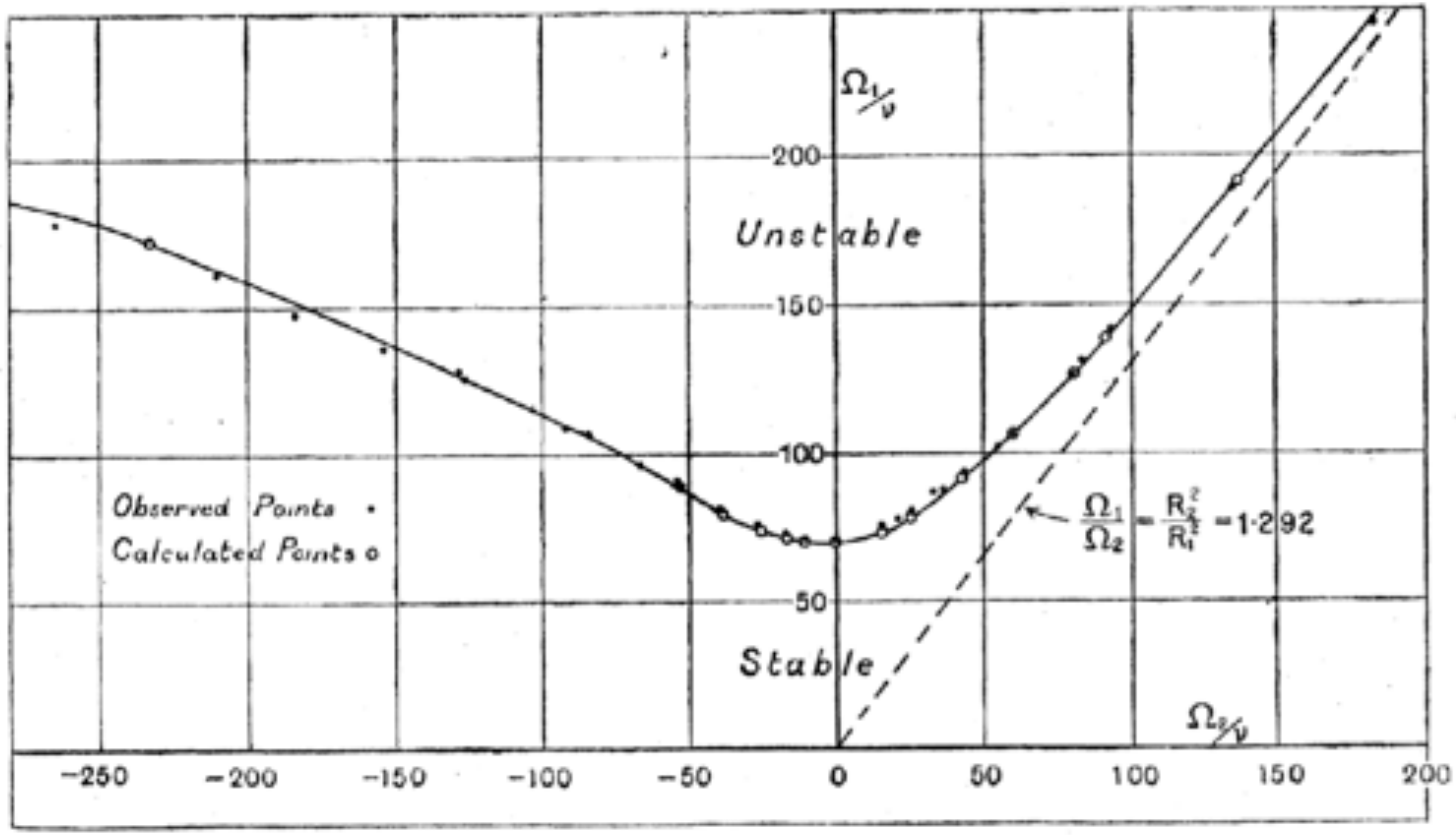
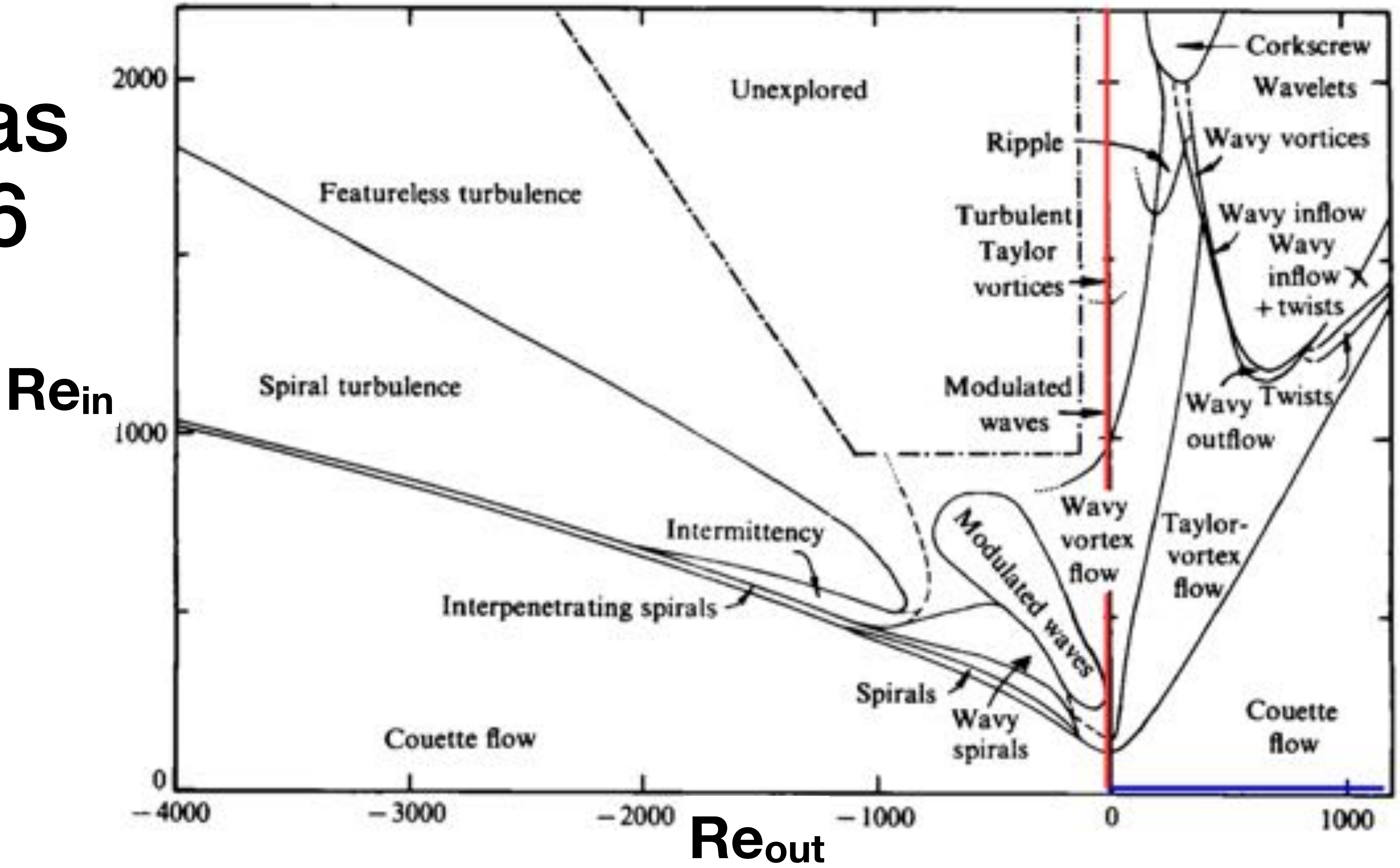


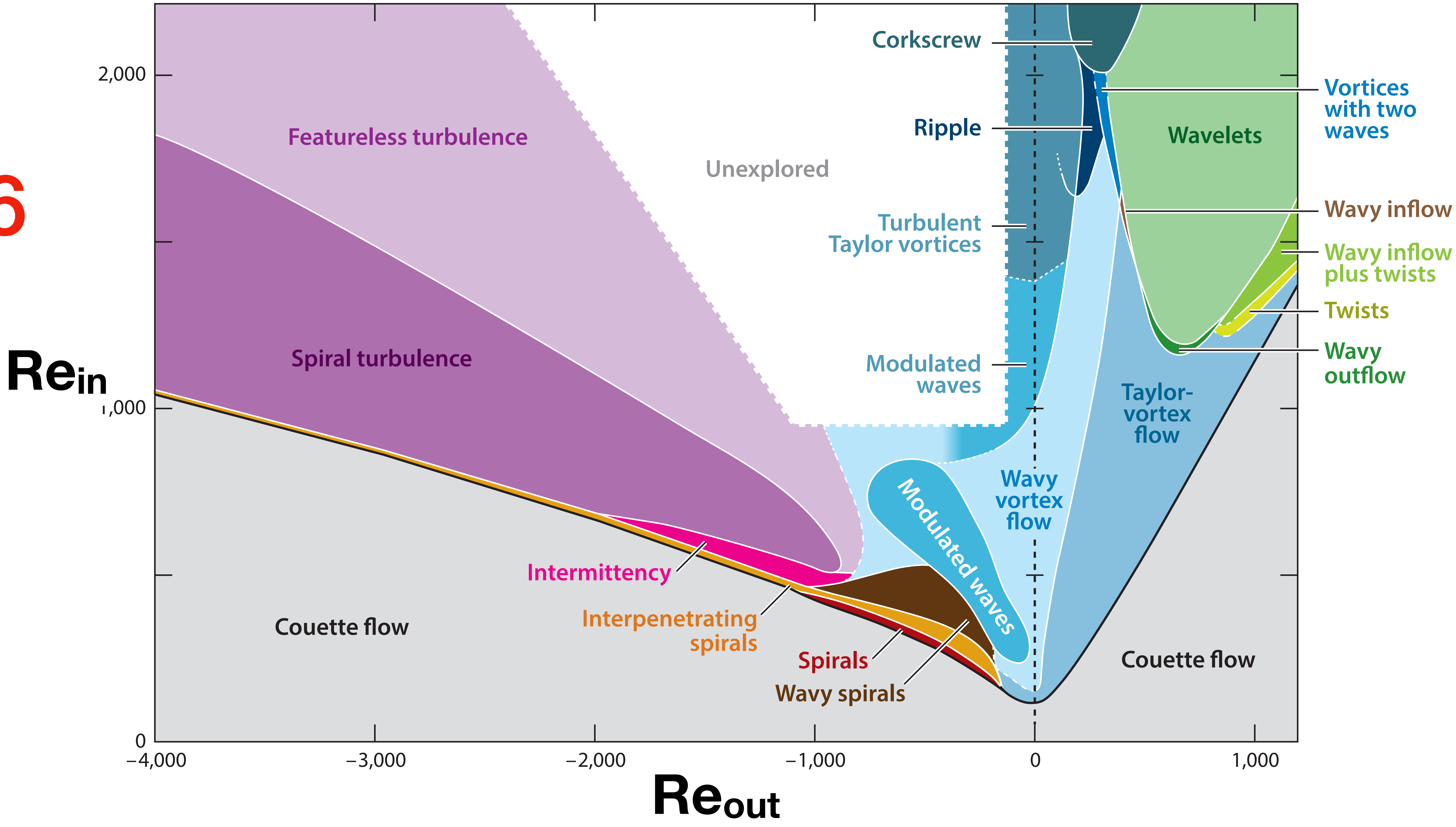

Fig. 18. Comparison between observed and calculated speeds at which instability first appears; case when $R_1 = 3{\cdot}55$ cm., $R_2 = 4{\cdot}035$ cm.

# Texas 1986



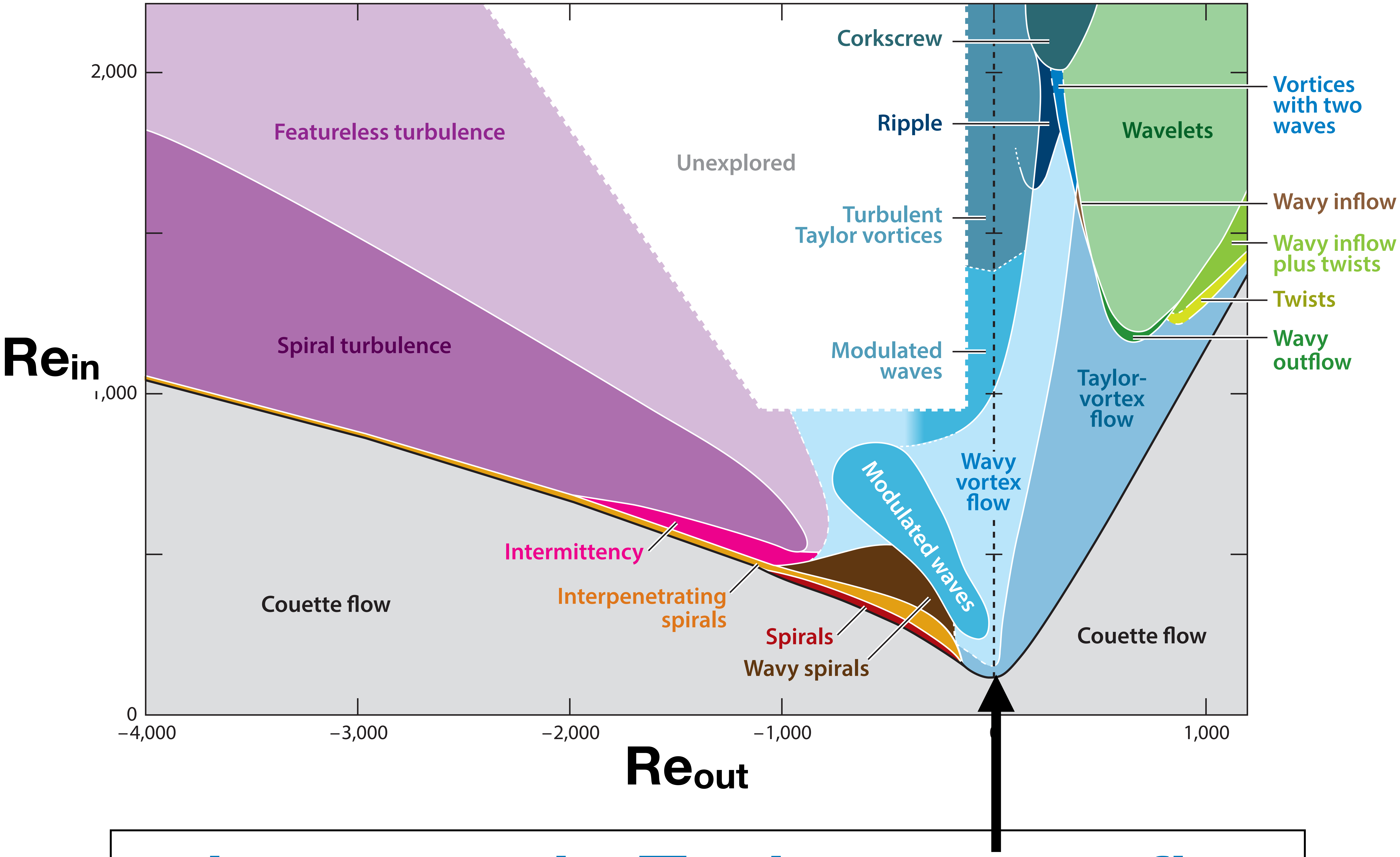

Liu, Andereck, Swinney, J Fluid Mech 1986

Oz
2016




Liu, Andereck, Swinney, J Fluid Mech 1986

Colorized version from Grossmann, Lohse, Sun, Annu Rev Fluid Mech, 2016




# axisymmetric Taylor vortex flow

# T. Brooke Benjamin, Proc R. Soc Lond A, 1978

*Proc. R. Soc. Lond.* A. **359**, 1–26 (1978)
*Printed in Great Britain*

## Bifurcation phenomena in steady flows of a viscous fluid

### I. Theory

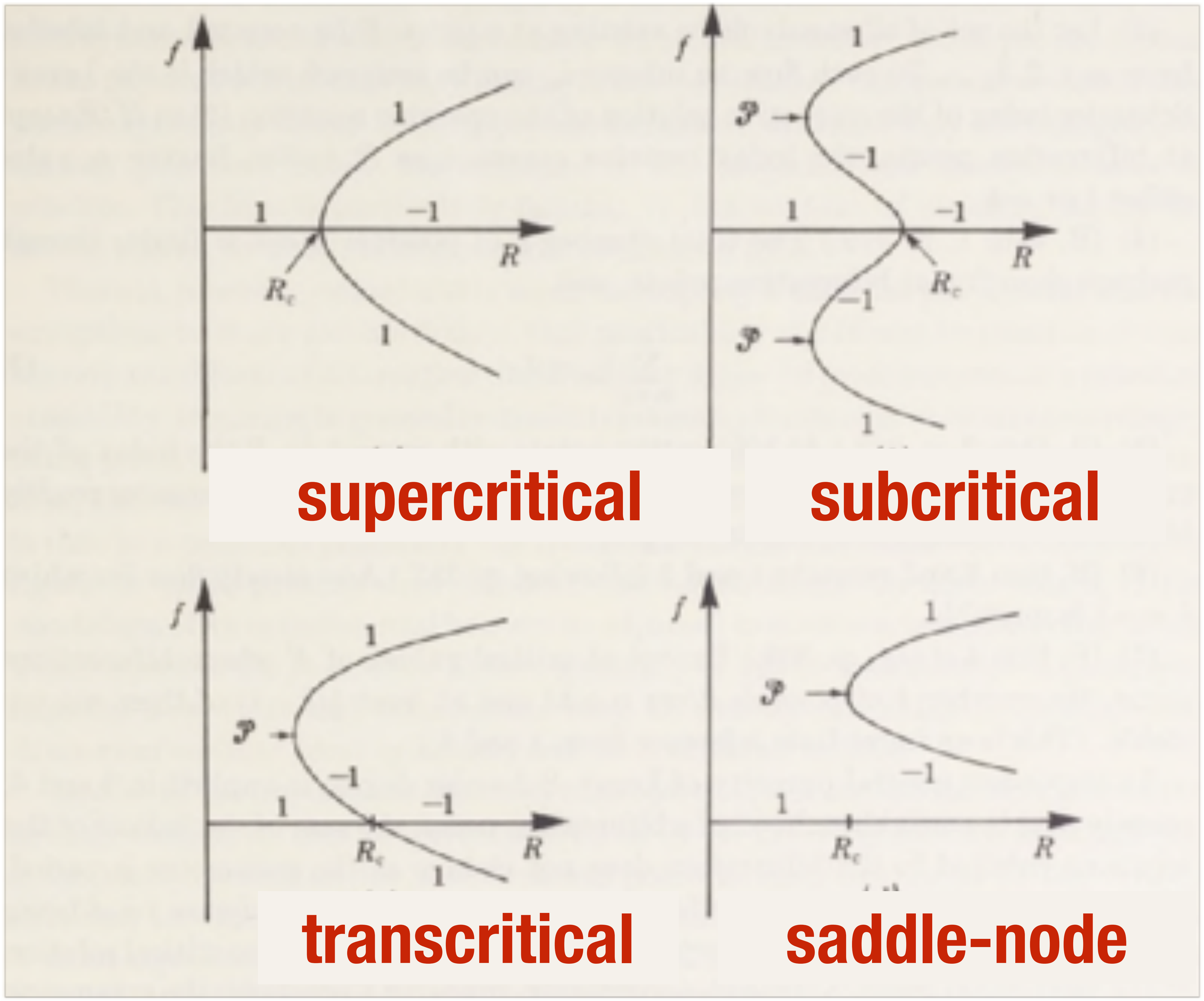
BY T. B. BENJAMIN, F.R.S.
*Fluid Mechanics Research Institute,*
*University of Essex, Colchester CO4 3SQ, U.K.*

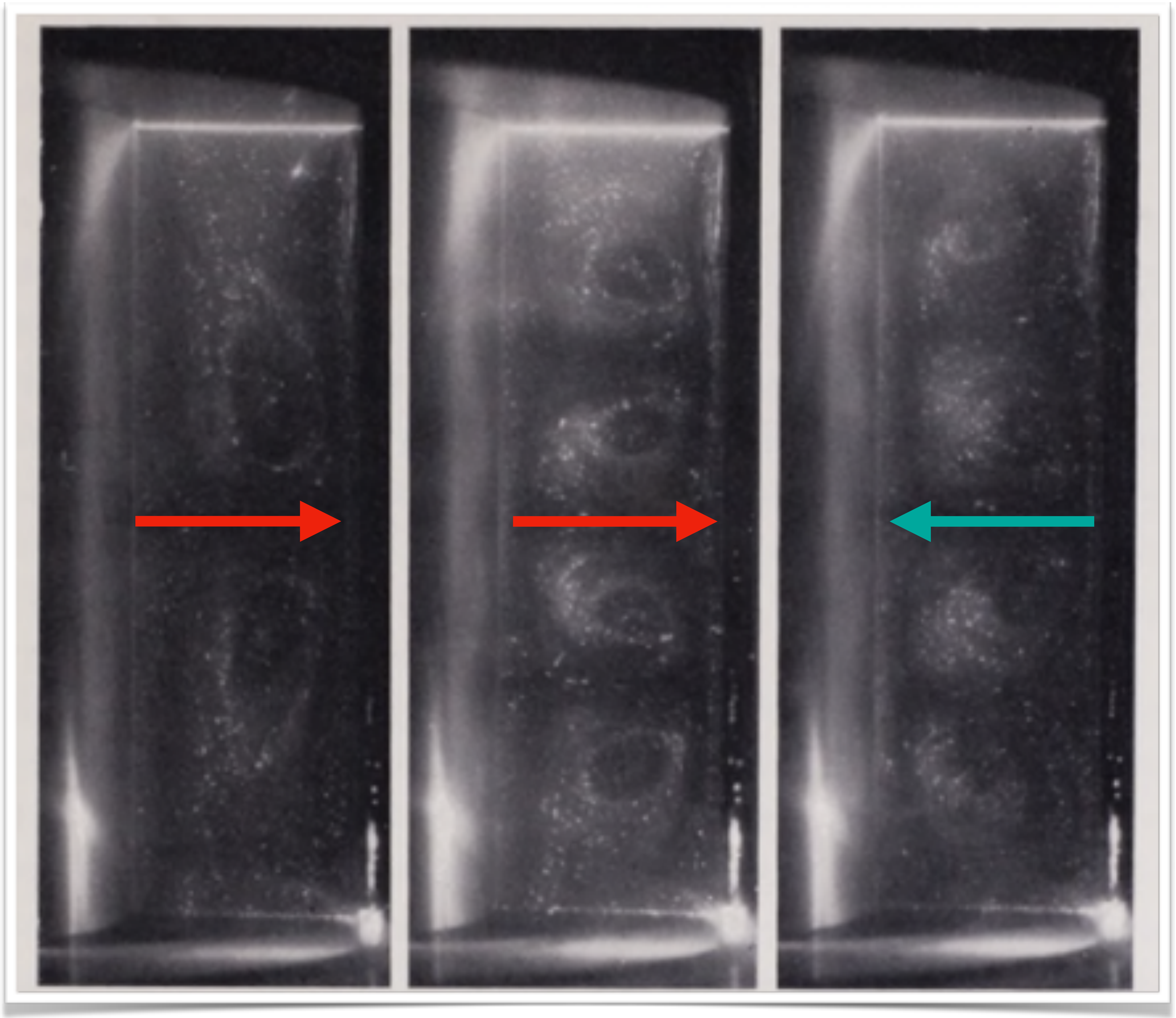
*Proc. R. Soc. Lond.* A. **359**, 27–43 (1978)
*Printed in Great Britain*

## Bifurcation phenomena in steady flows of a viscous fluid

### II. Experiments

BY T. B. BENJAMIN, F.R.S.
*Fluid Mechanics Research Institute,*
*University of Essex, Colchester CO4 3SQ, U.K.*





primary flow
two large cells
due to end effects
(Ekman pumping)

“normal”
four-cell

“abnormal”
four-cell

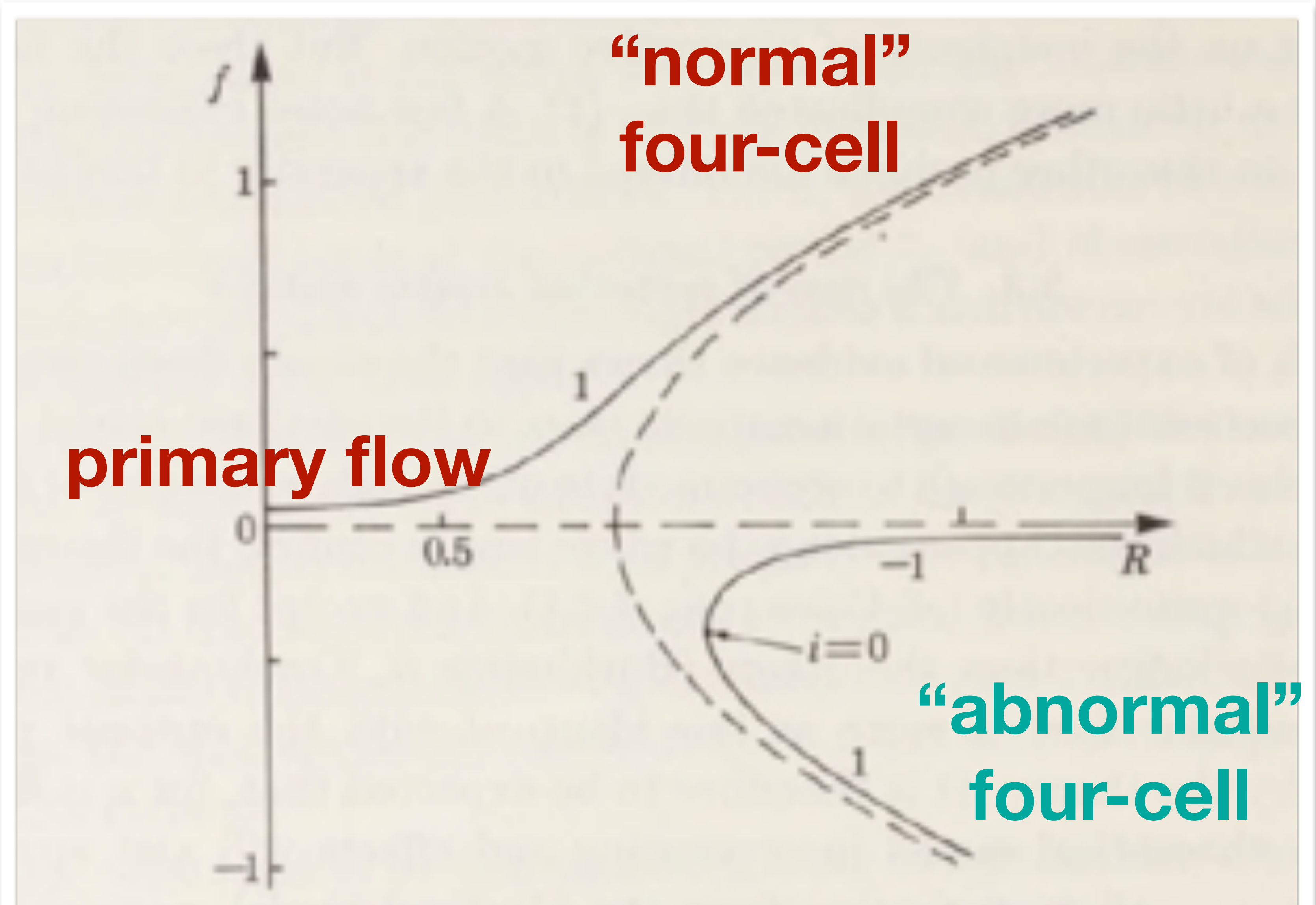



FIGURE 2. Decoupling of supercritical bifurcation.

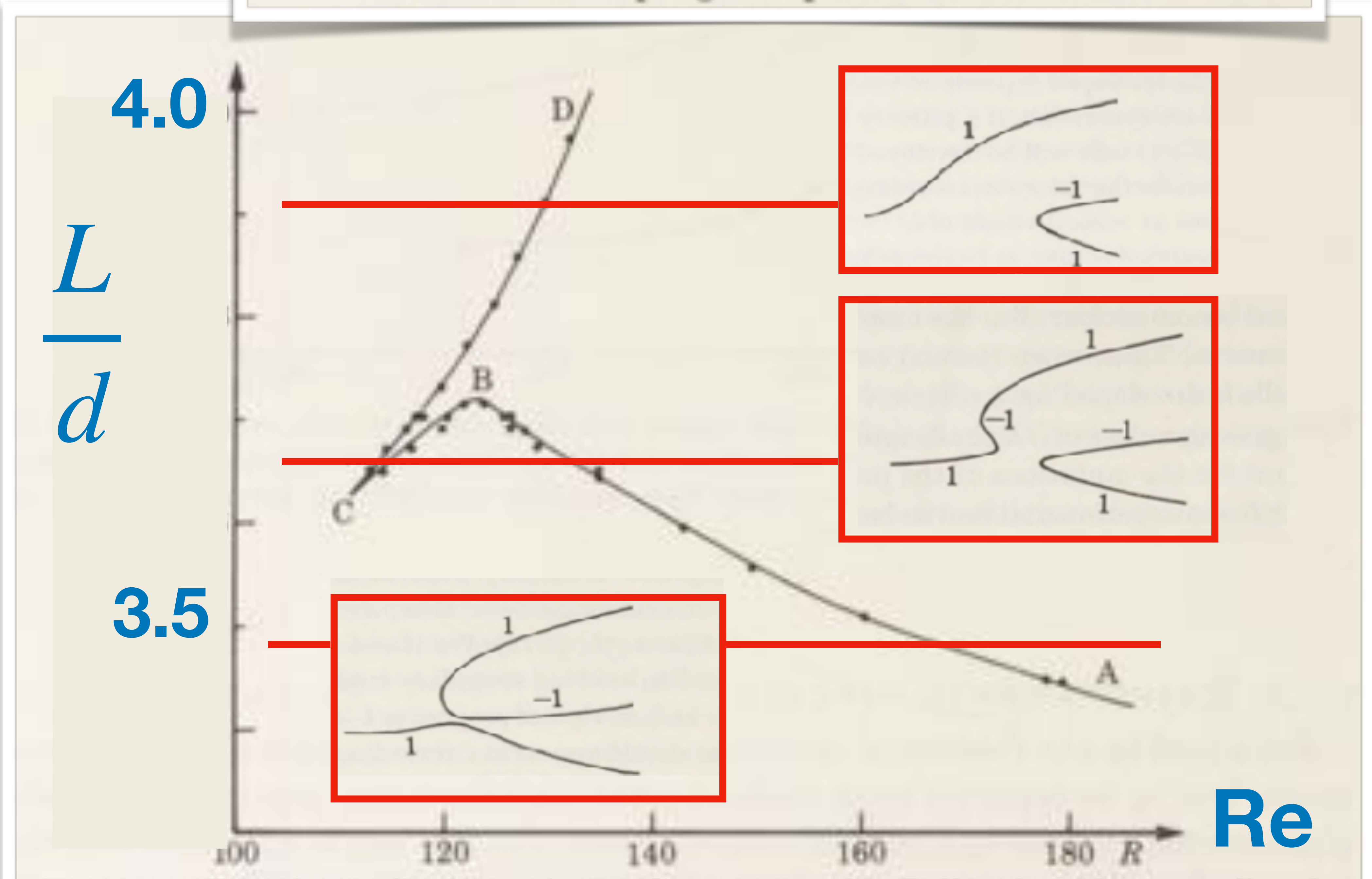
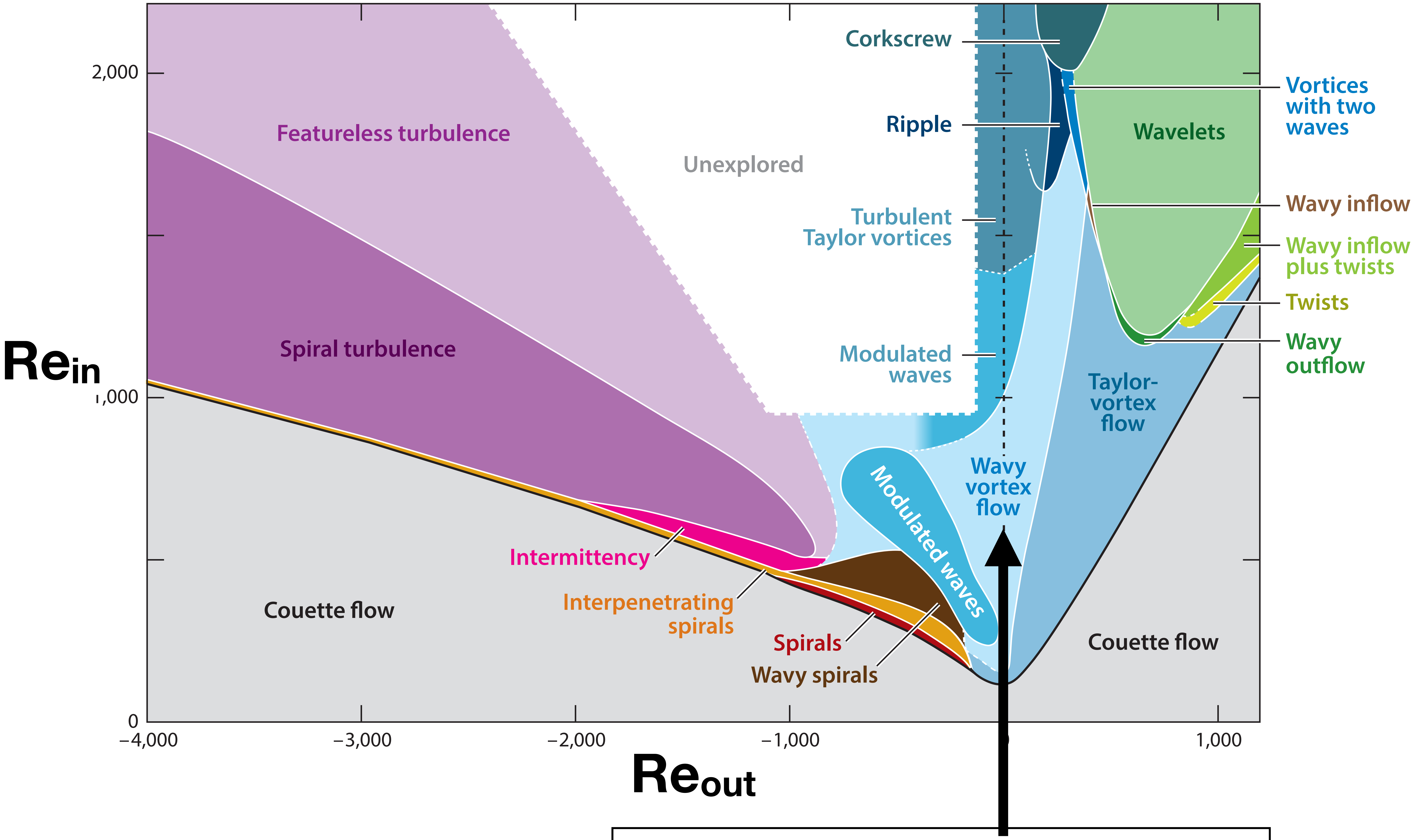
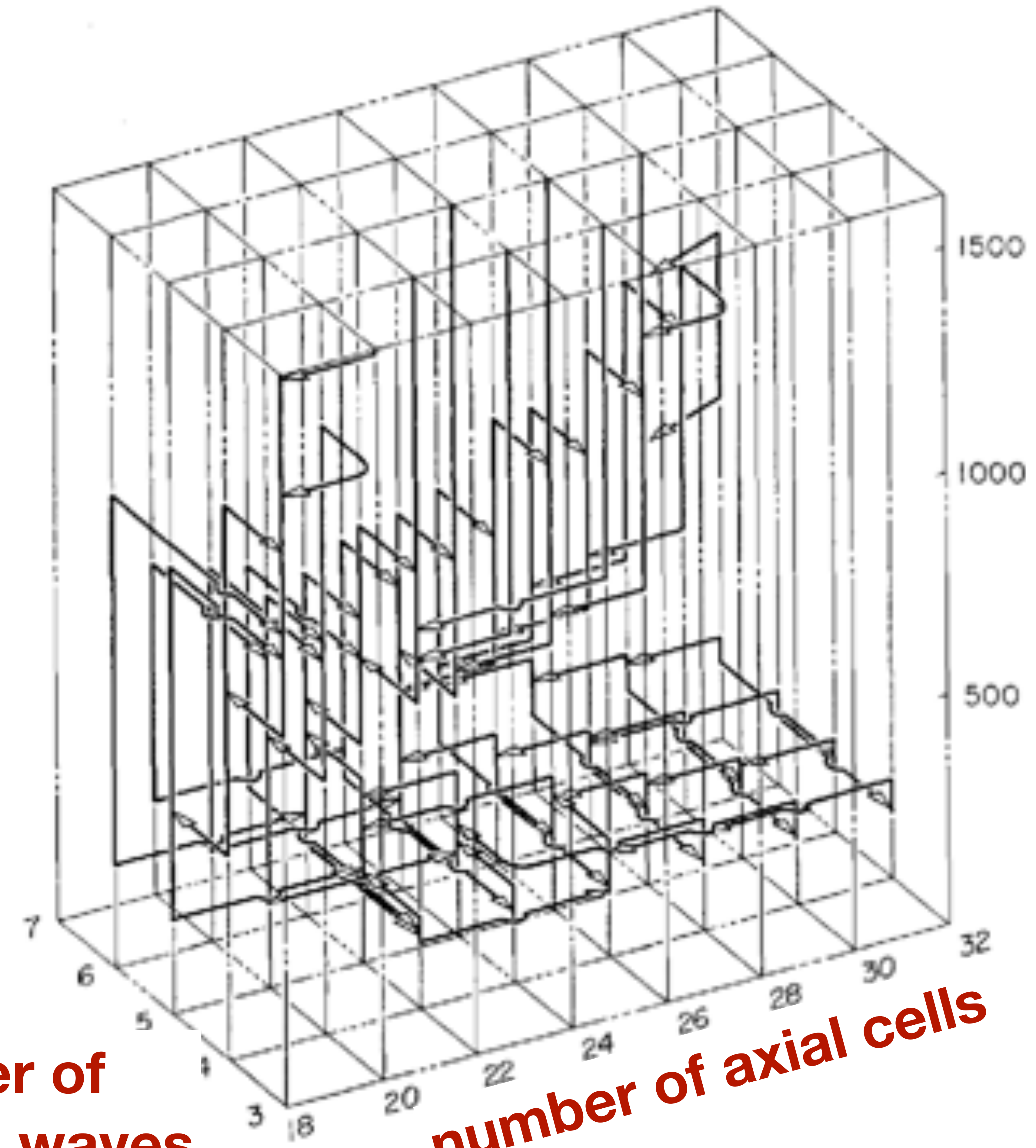



FIGURE 5. Experimentally determined bifurcation set in (R, l/d)-plane for two-cell and four-cell flows.




# Wavy vortex flow

# Inner-cylinder rotation: wavy vortices




**Coles, JFM 1965**

$$\eta \equiv \frac{r_{\text{in}}}{r_{\text{out}}} = 0.874$$

# Mechanism for wavy vortices

**Axial dependence (shear) of azimuthal velocity of Taylor vortices:**

**Kelvin-Helmholtz-like mechanism (explains wide range of azimuthal wavenumbers)**

AIP | Physics of Fluids

**Mechanisms for the transition to waviness for Taylor vortices**

Denis Martinand, Eric Serre, and Richard M. Lueptow

Citation: Physics of Fluids **26**, 094102 (2014); doi: 10.1063/1.4895400

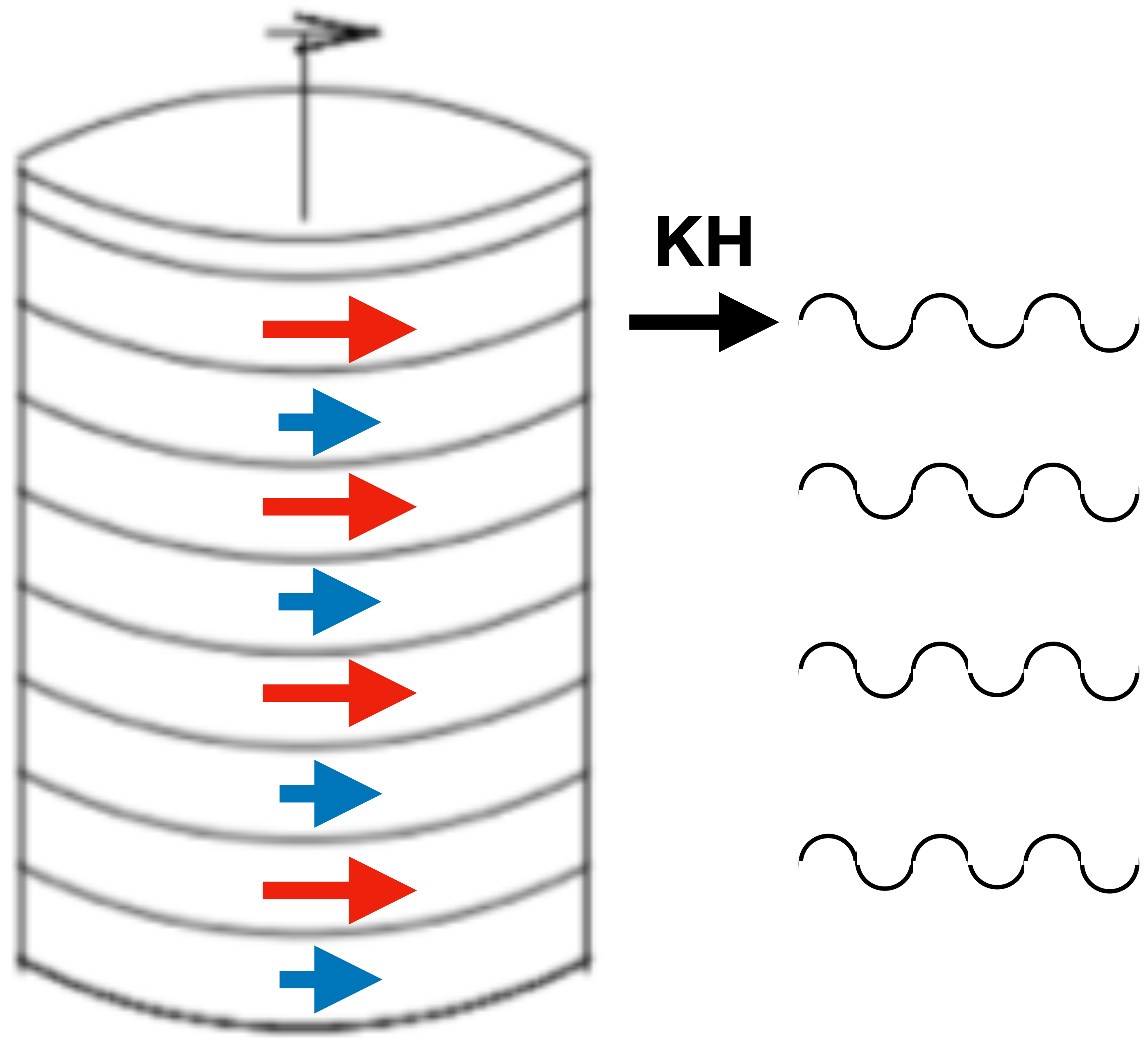
PHYSICAL REVIEW FLUIDS 3, 123902 (2018)

**Self-sustaining process in Taylor-Couette flow**

Tommy Dessup, Laurette S. Tuckerman, and José Eduardo Wesfreid

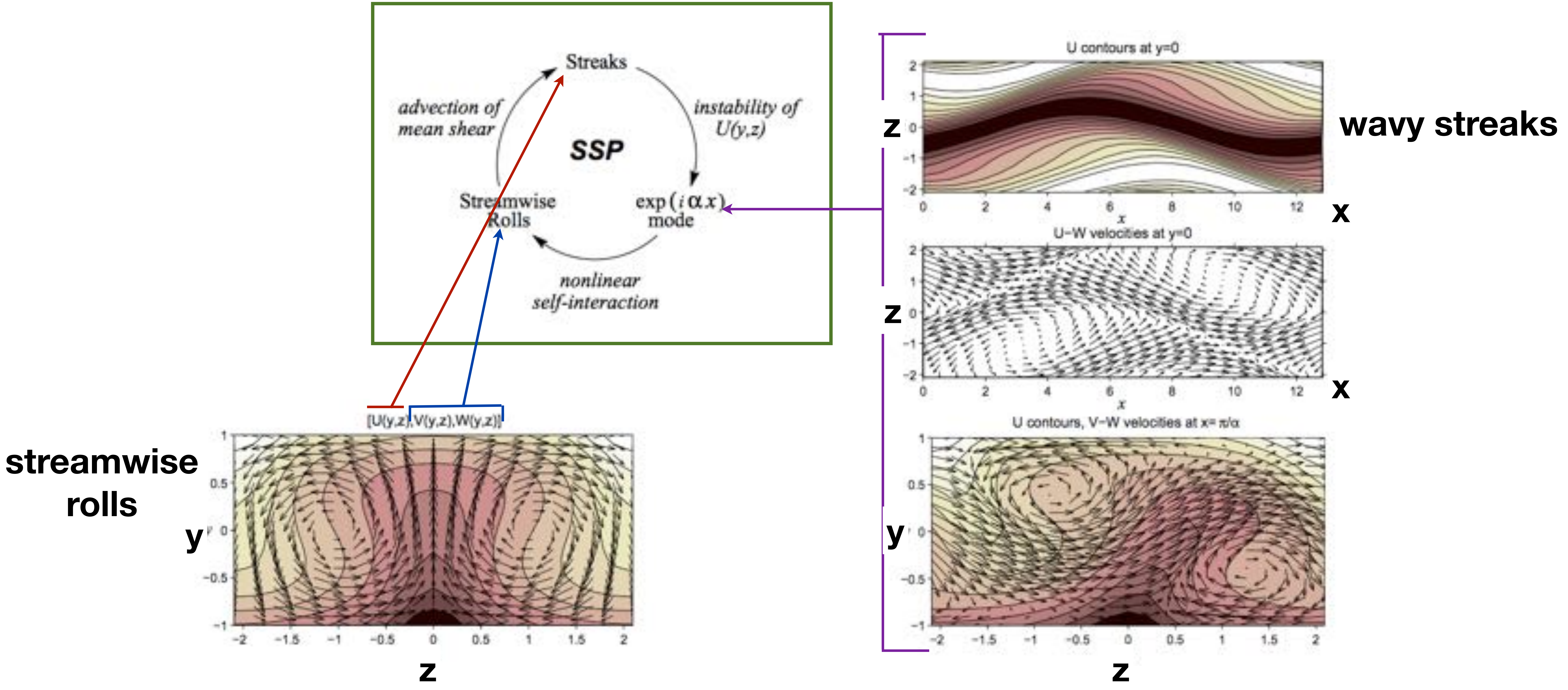
Dwight Barkley

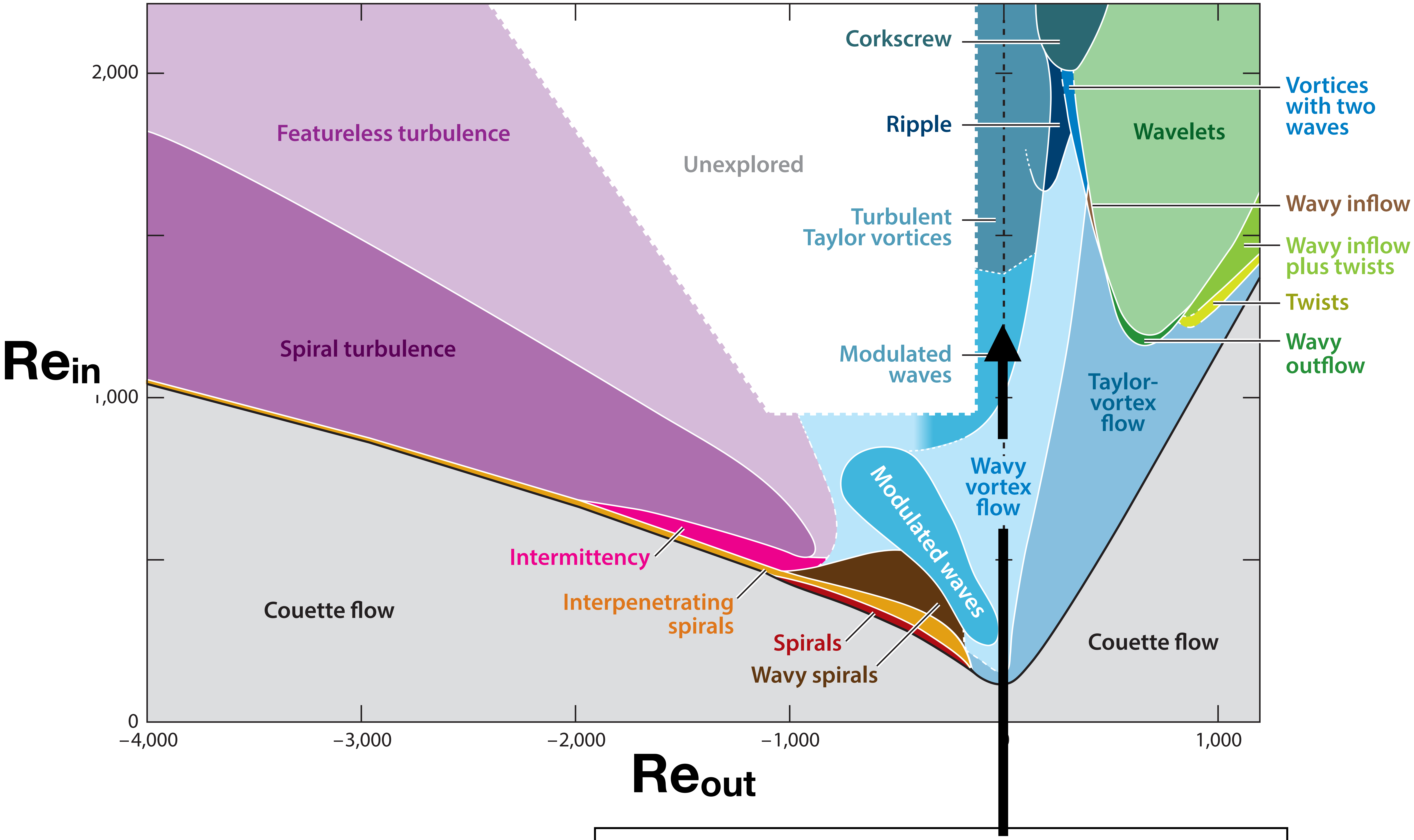
Ashley P. Willis



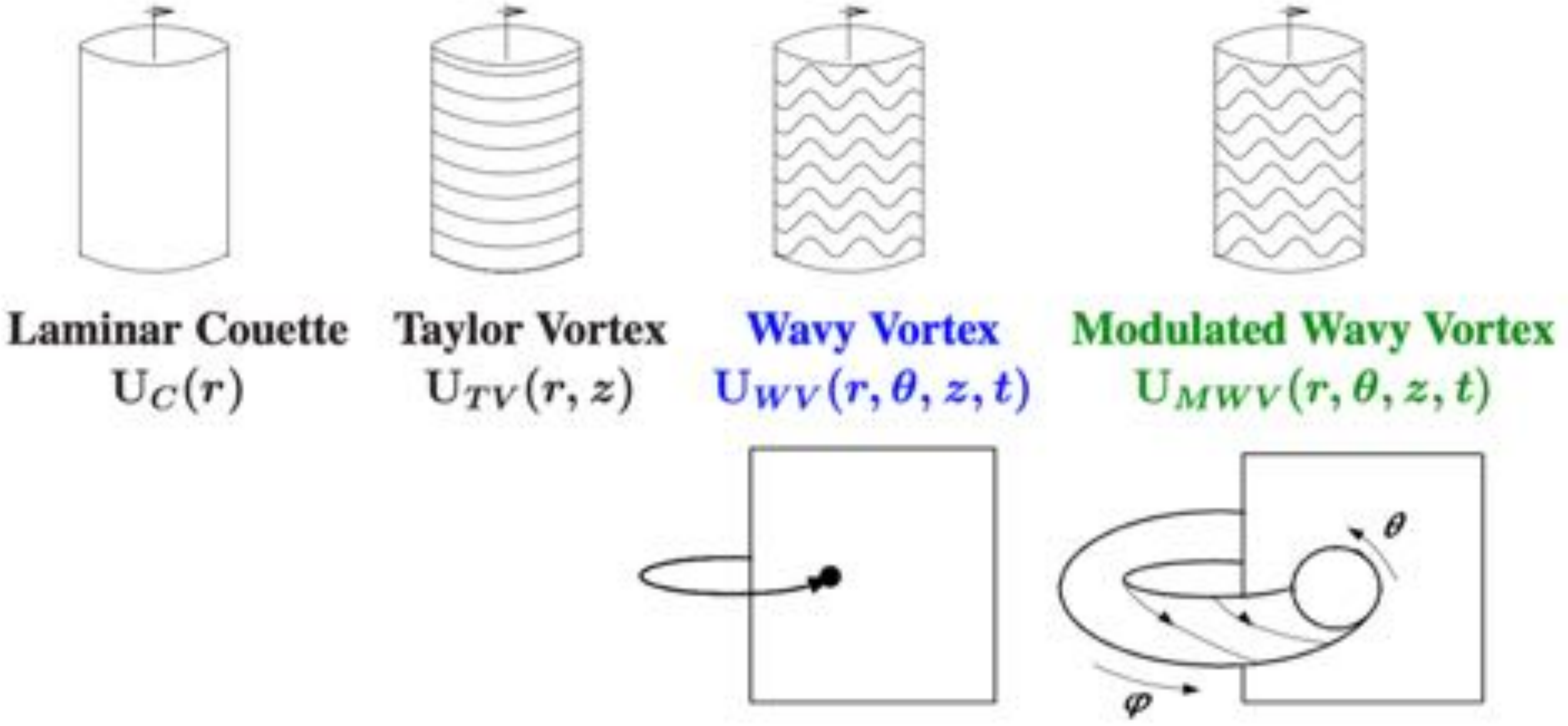
# Waleffe: self-sustaining process (SSP)

*F. Waleffe & J. Kim, How streamwise rolls and streaks sustain in a shear flow: Part 2, AIAA paper 98-2997 (Albuquerque, June 1998)*
*F. Waleffe, On a self-sustaining process in shear flows, Phys. Fluids 9, 883-900 (1997)*






# Modulated Waves

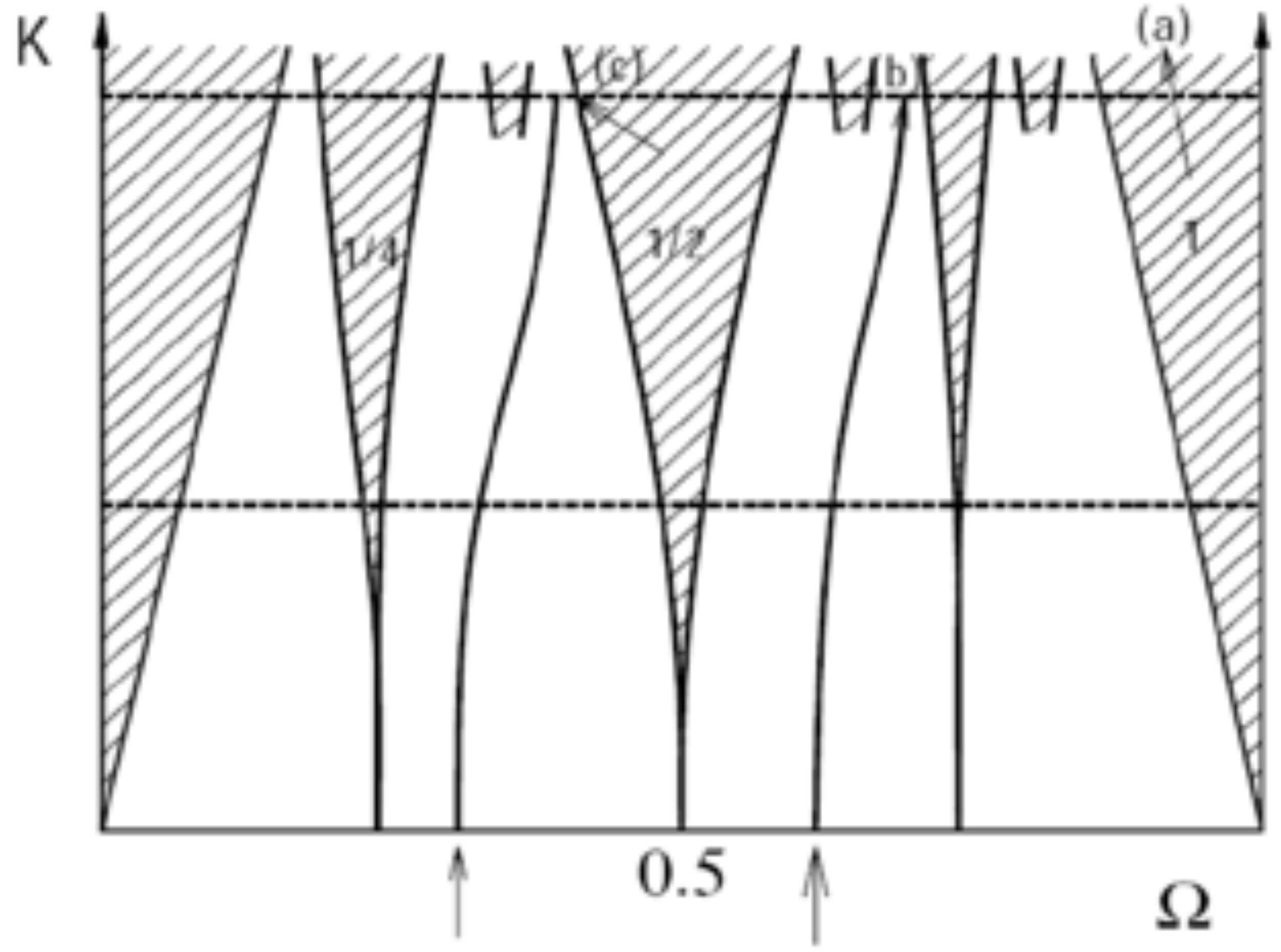


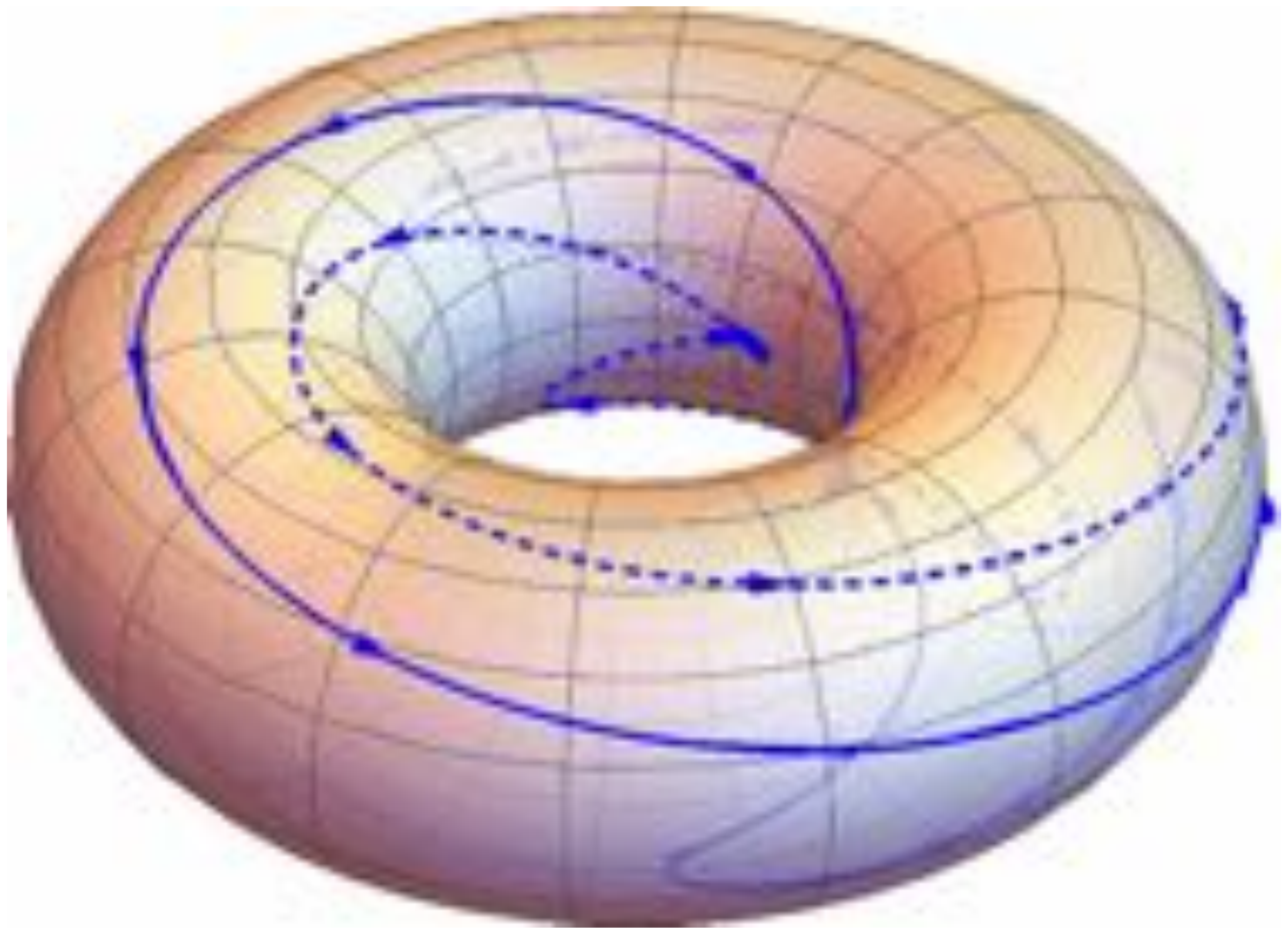

Hopf bifurcation

Secondary Hopf or
Neimark-Sacker bifurcation

# Nonlinear dynamics on a torus → frequency locking

Schematic representation of frequency-locking tongues



saddle-node bifurcations create finite-period limit cycles on the torus

**No frequency-locking in modulated wavy vortex flow! Why not?**

**Rand (1981): Symmetry! In rotating frame,**

**wavy vortex flow is steady and modulated wavy vortex flow is periodic.**

**Points on circle (phases in $\theta$) dynamically equivalent $\Longrightarrow$ no saddle-nodes.**

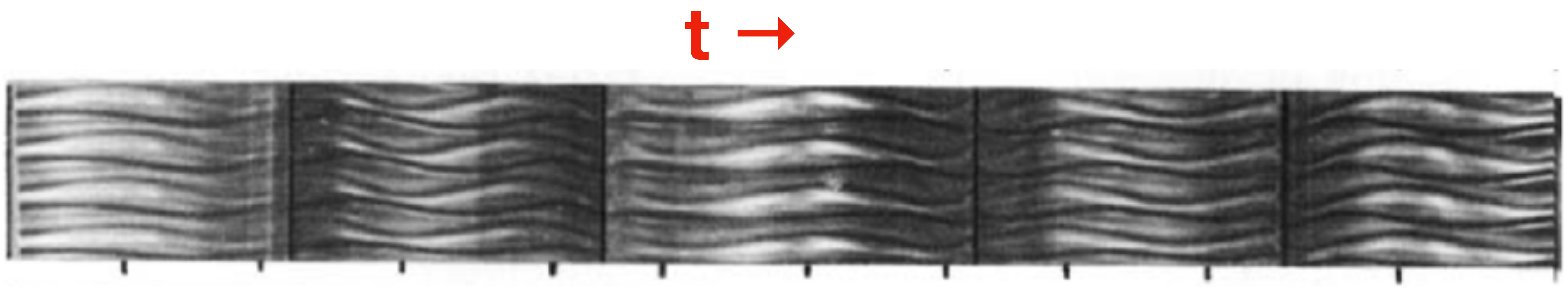
VOLUME 46, NUMBER 15 PHYSICAL REVIEW LETTERS 13 APRIL 1981

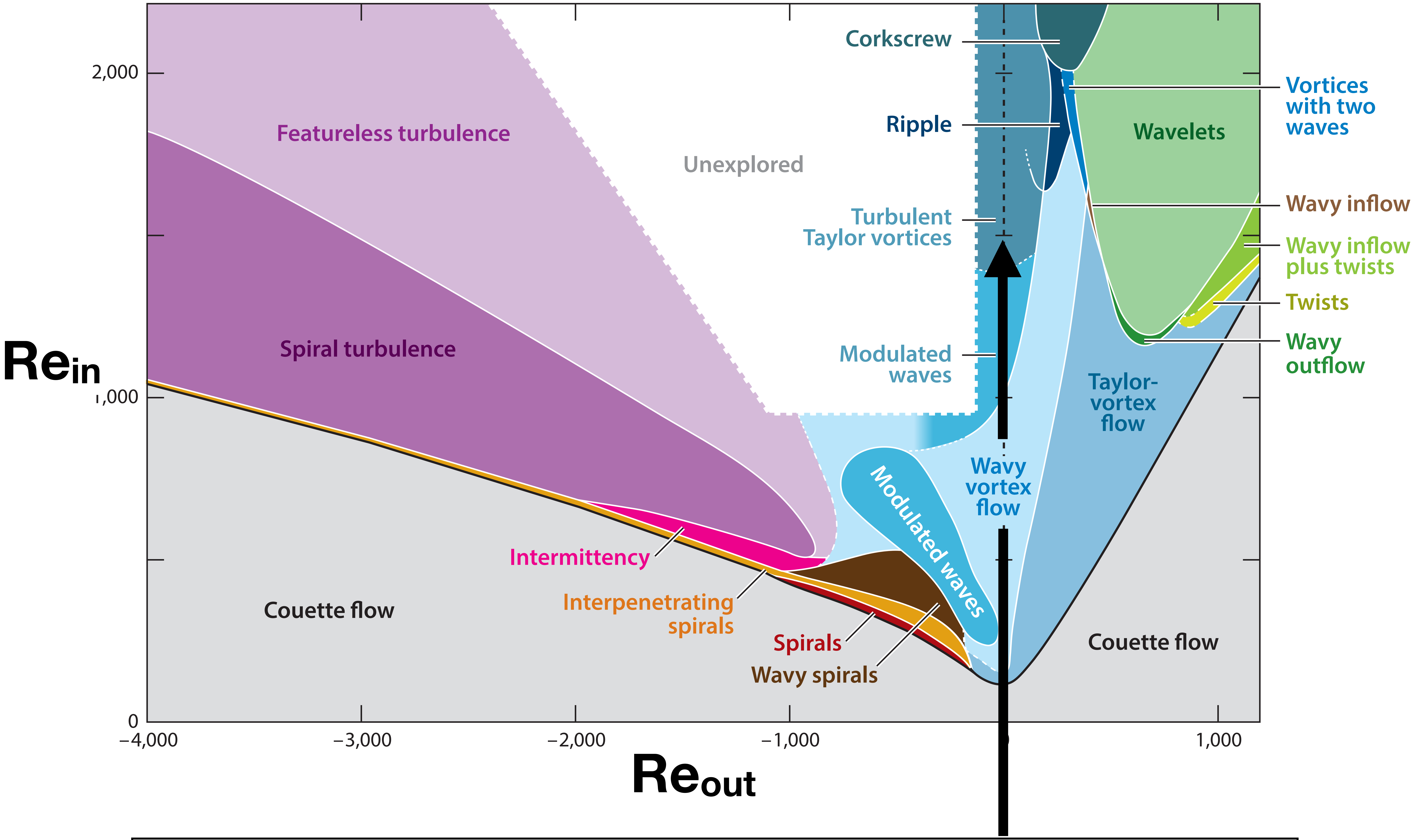
**Doubly Periodic Circular Couette Flow: Experiments Compared with Predictions from Dynamics and Symmetry**

M. Gorman[a] and Harry L. Swinney
*Department of Physics, University of Texas, Austin, Texas 78712*

and

David A. Rand
*Mathematics Institute, University of Warwick, Coventry CV4 7AL, United Kingdom*






Chaotic flow: strange attractor

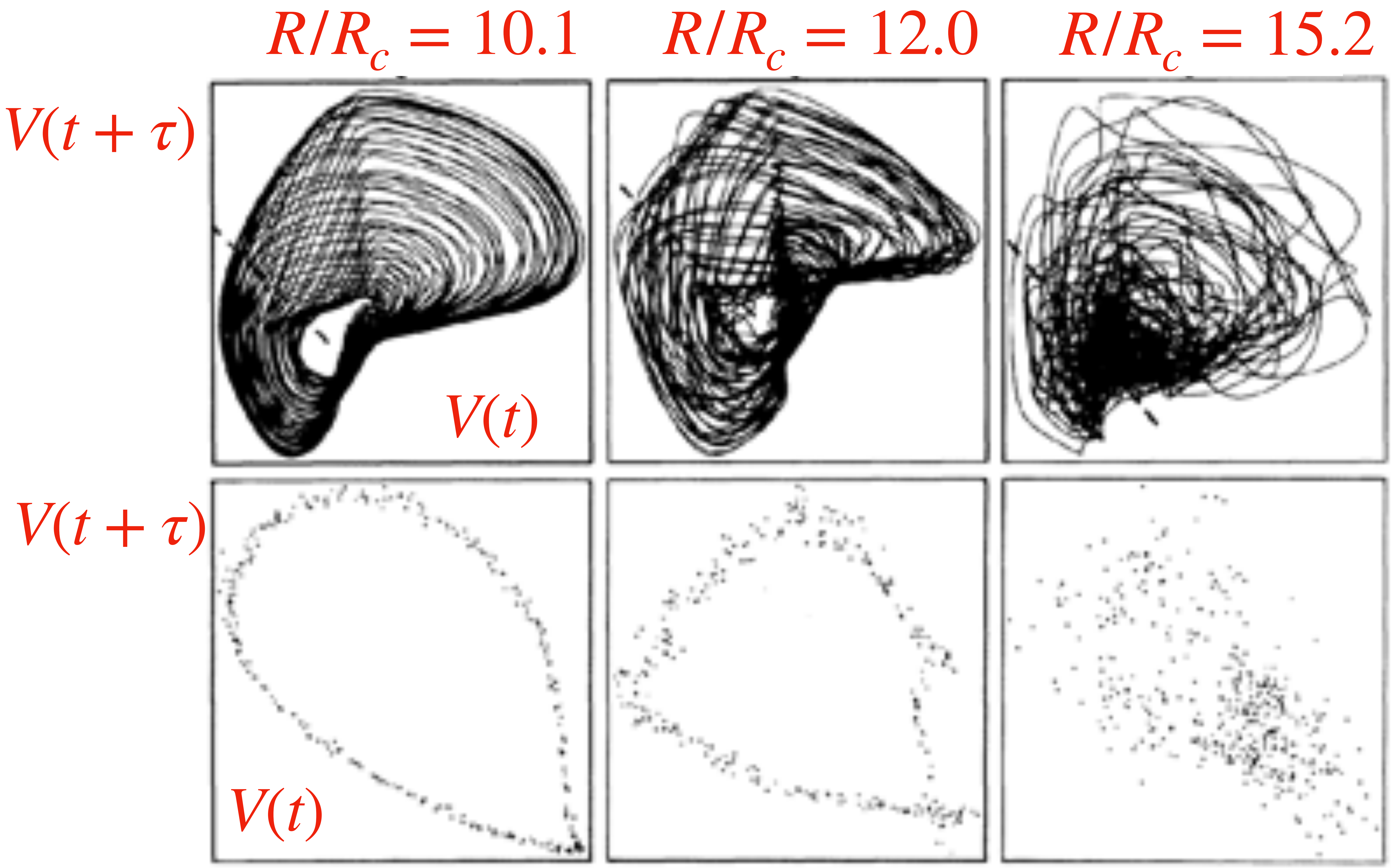
VOLUME 51, NUMBER 16 PHYSICAL REVIEW LETTERS 17 OCTOBER 1983

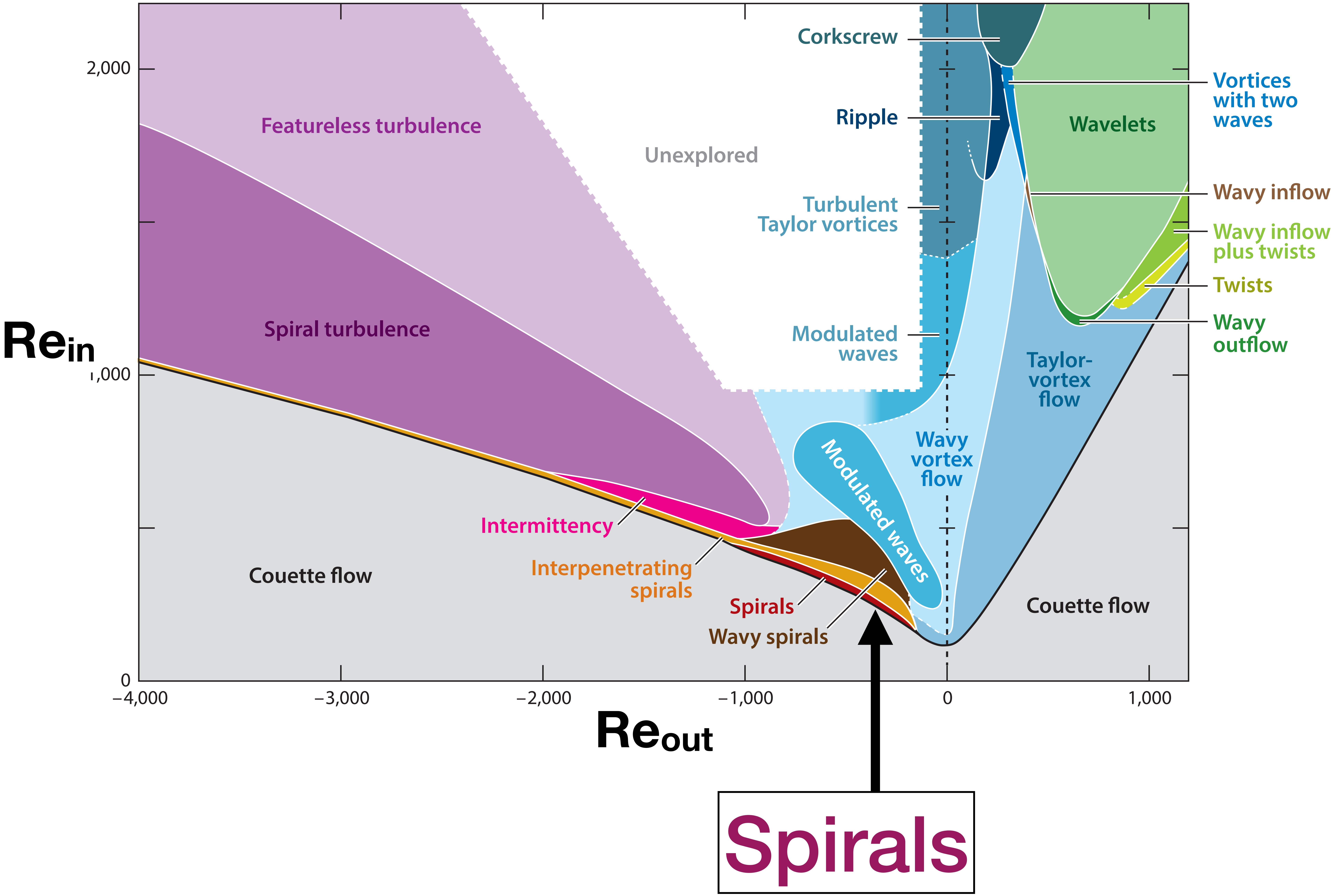
**Low-Dimensional Chaos in a Hydrodynamic System**

A. Brandstäter, J. Swift, Harry L. Swinney, and A. Wolf
*Department of Physics, University of Texas, Austin, Texas 78712*
J. Doyne Farmer and Erica Jen
*Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
P. J. Crutchfield
*Physics Department, University of California, Berkeley, Berkeley, California 94720*

Phase portraits

Poincaré sections defined by $V(t+2\tau) = V^*$





Featureless turbulence
Unexplored
Spiral turbulence
Corkscrew
Ripple
Turbulent
Taylor vortices
Modulated
waves
Vortices
with two
waves
Wavelets
Wavy inflow
Wavy inflow
plus twists
Twists
Wavy
outflow
Taylor-
vortex
flow
Wavy
vortex
flow
Modulated waves
Intermittency
Interpenetrating
spirals
Spirals
Wavy spirals
Couette flow
Couette flow
Re in
Re out
2,000
1,000
0
−4,000
−3,000
−2,000
−1,000
0
1,000
Spirals

# VIII. *Stability of a Viscous Liquid contained between Two Rotating Cylinders.*

By G. I. Taylor, *F.R.S.*

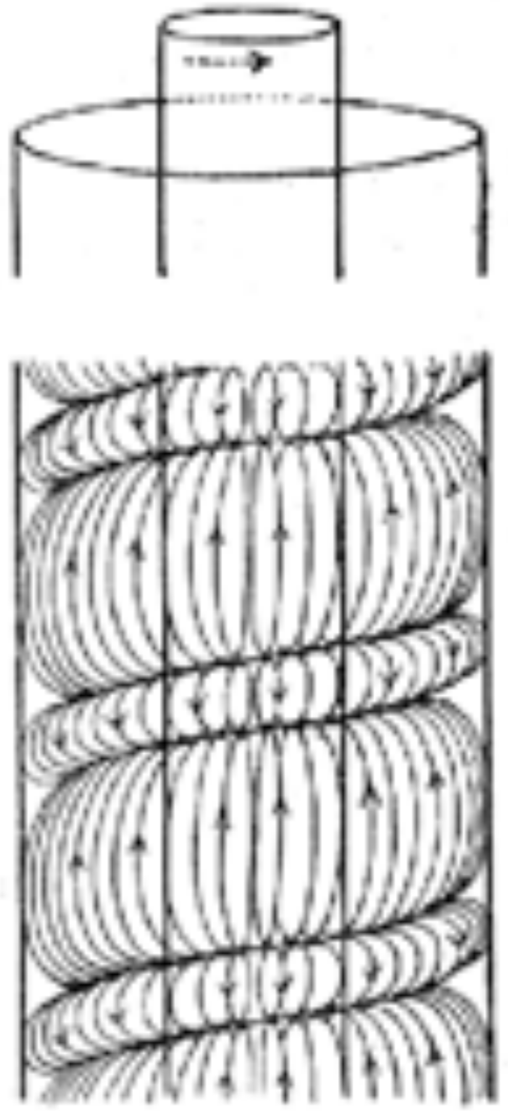


Fig. 19. Spiral form of instability which appears when steady motion is not strictly limited to two dimensions before instability sets in.

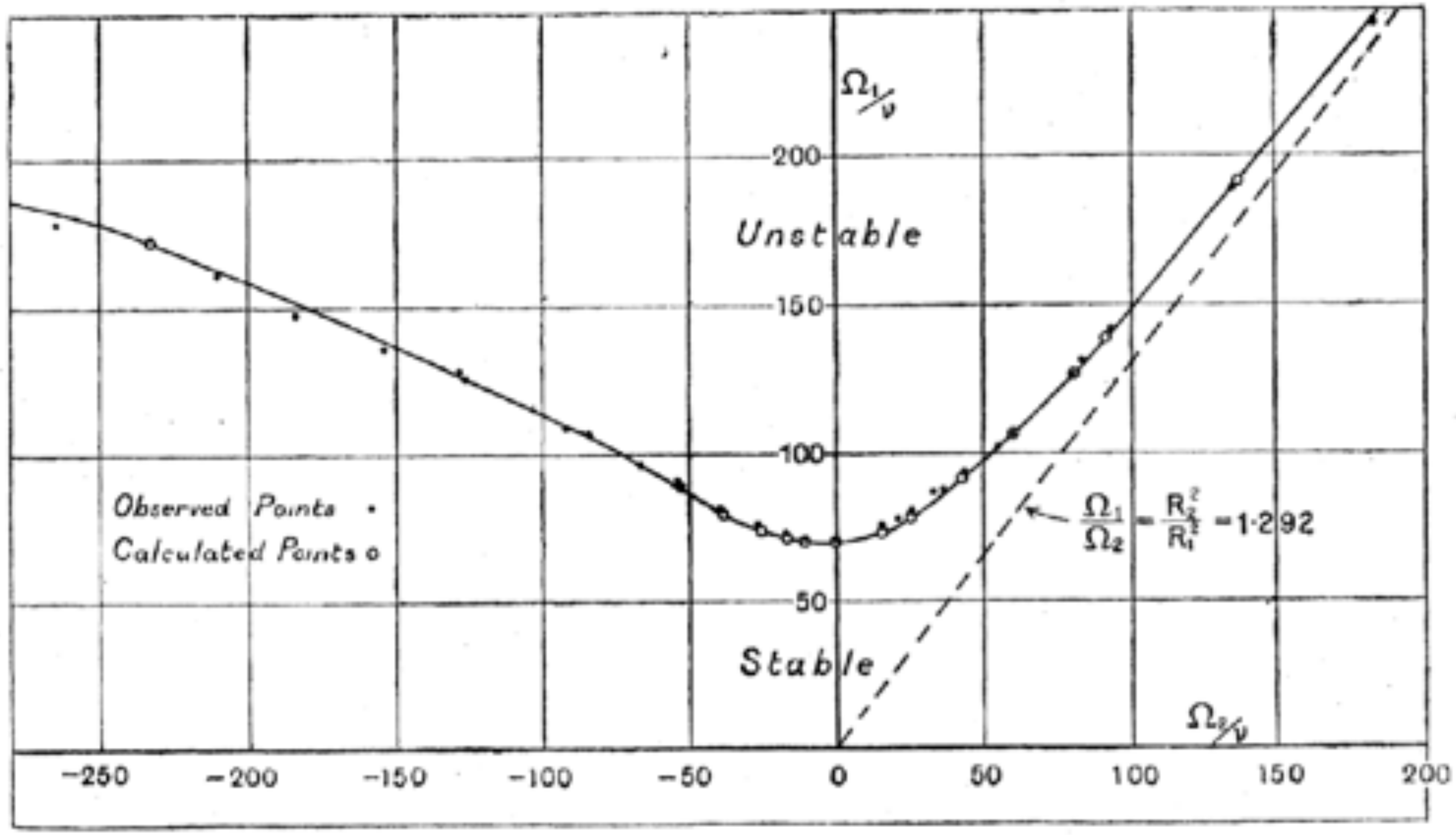



Fig. 18. Comparison between observed and calculated speeds at which instability first appears; case when $R_1 = 3\cdot55$ cm., $R_2 = 4\cdot035$ cm.

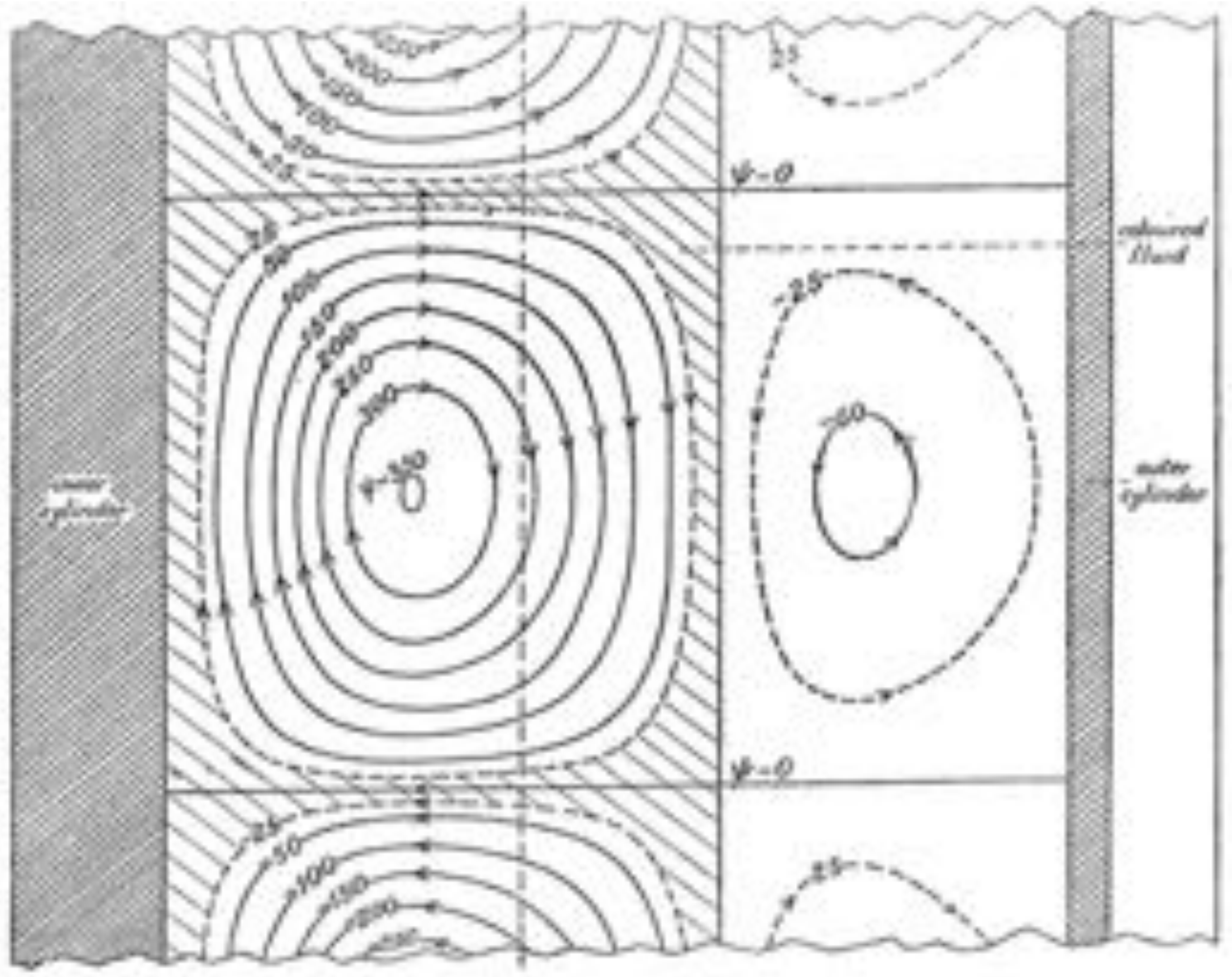
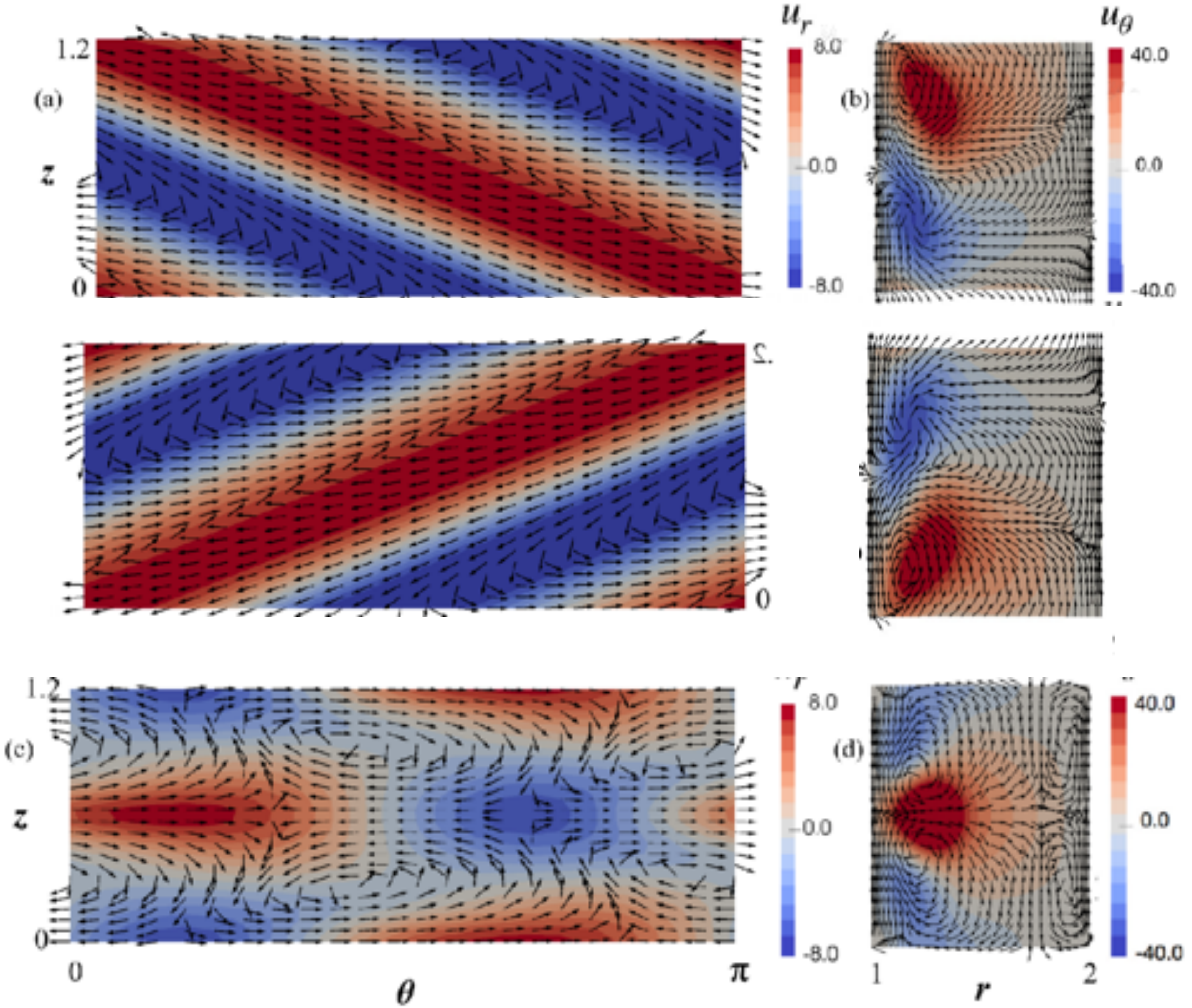



Fig. 7. Stream lines of motion after instability has set in, $\mu = -1\cdot5$.

# Spirals and Ribbons




$S_+$ spirals

↓ reflect

$S_-$ spirals

$\frac{S_+ + S_-}{2}$ ribbons

## Normal Form

$$\frac{dS_+}{dt} = \left[\mu - a|S_+|^2 - b|S_-|^2\right] S_+$$
$$\frac{dS_-}{dt} = \left[\mu - a|S_-|^2 - b|S_+|^2\right] S_-$$

## Solutions

**Trivial**

$$S_+ = S_- = 0$$

**Spirals**

$$S_+ = \sqrt{\mu/a}, S_- = 0 \qquad S_- = \sqrt{\mu/a}, S_+ = 0$$

**Ribbons**

$$S_+ = S_- = \sqrt{\mu/(a+b)}$$

This normal form occurs in many cases.

$D_4$ (symmetry of a square)

Hopf bifurcation in O(2) leads to traveling or standing waves

Steady bifurcation on square lattice leads to stripes vs square patterns

Knobloch, Swift, Golubitsky, …

**theoretical prediction of ribbons by Demay & Iooss (1984)**

**experimental observation by Tagg, Edwards, Swinney, Marcus (1989)**

PHYSICAL REVIEW A VOLUME 39, NUMBER 7 APRIL 1, 1989

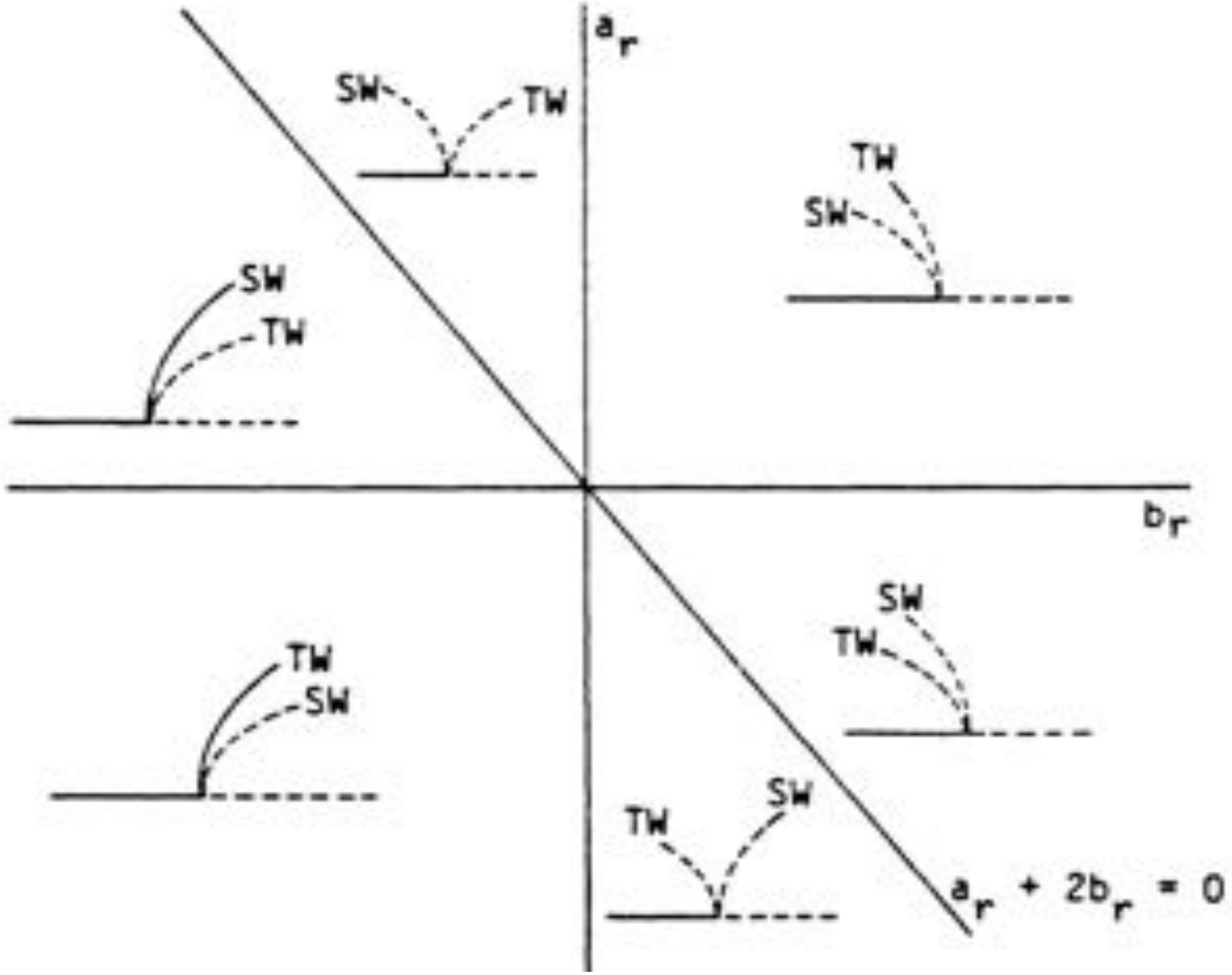
**Nonlinear standing waves in Couette-Taylor flow**

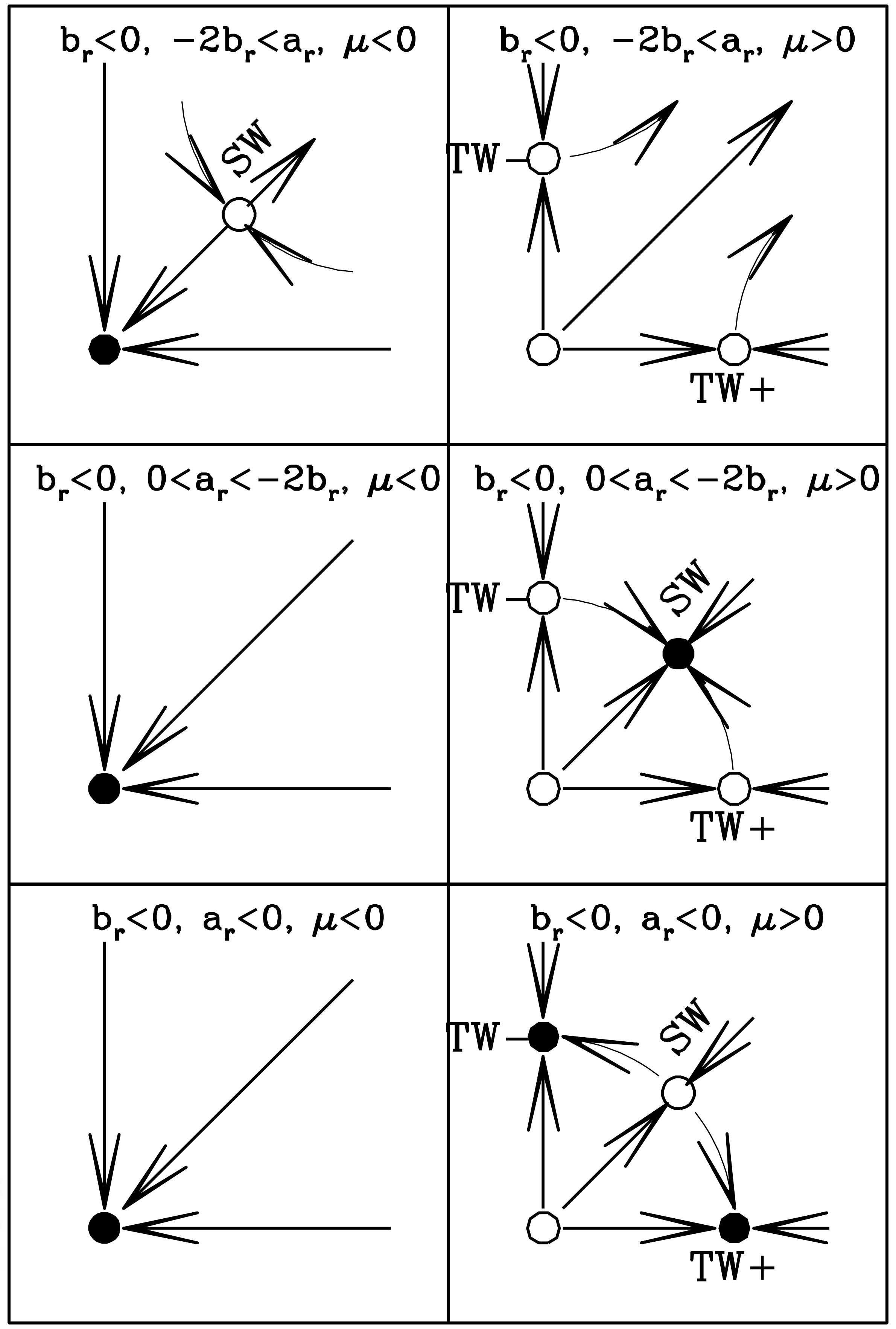
Randall Tagg, W. Stuart Edwards, and Harry L. Swinney

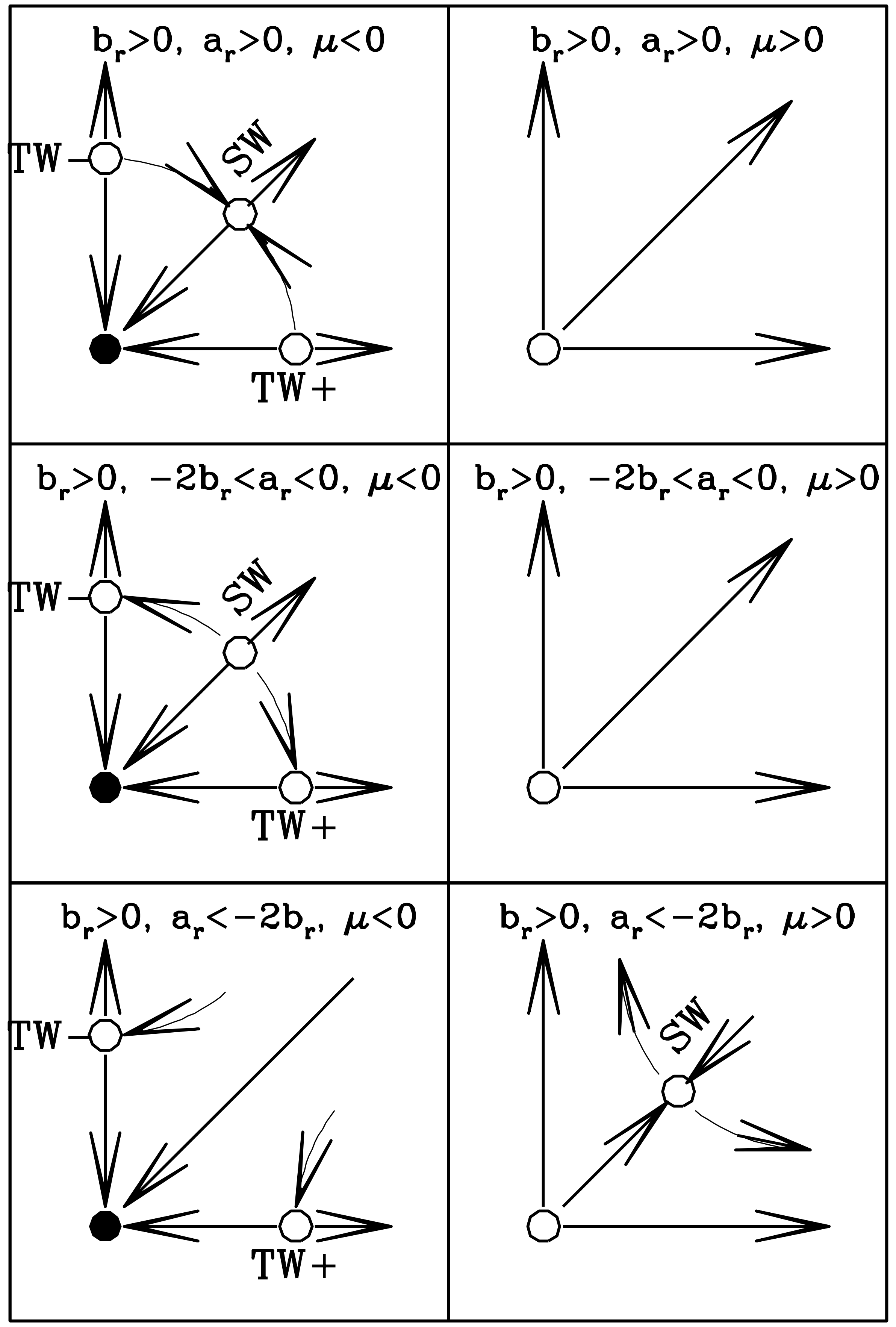
*Center for Nonlinear Dynamics and the Department of Physics, University of Texas, Austin, Texas 78712*

Philip S. Marcus

*Department of Mechanical Engineering, University of California, Berkeley, California 94720*



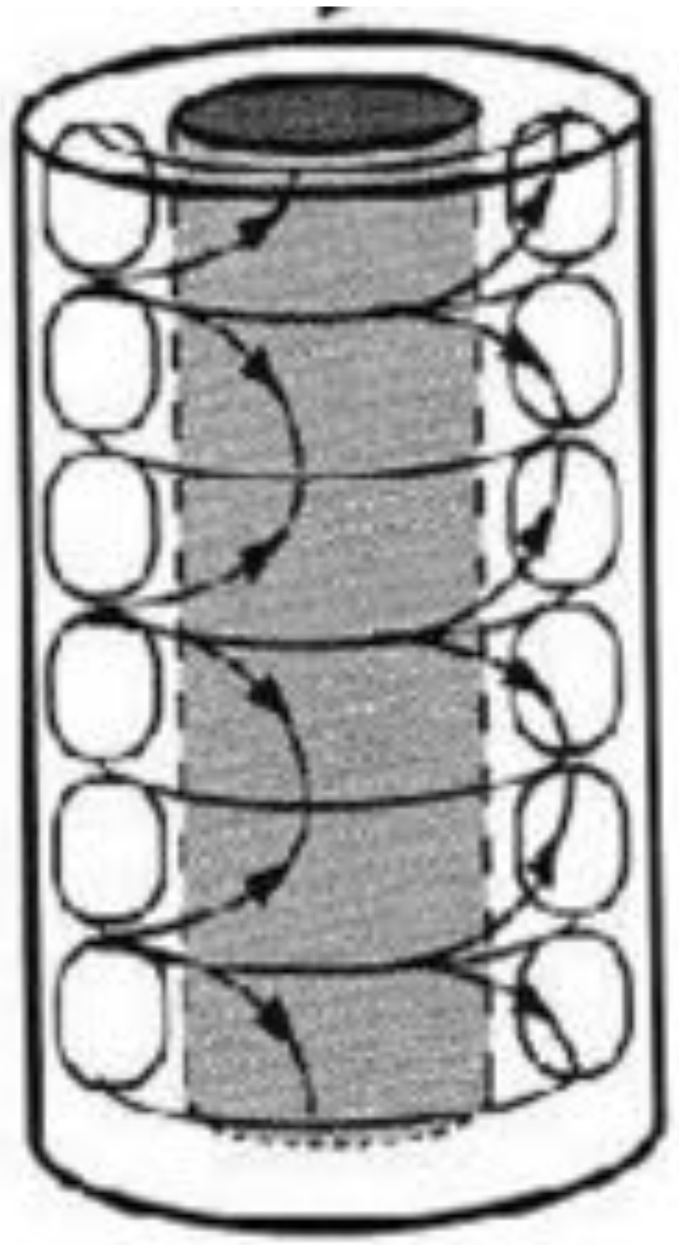




*Knobloch, Phys Rev A 1986*

**TW → spirals**
**SW → ribbons**

# Periodicity: geometric vs assumed
# Taylor-Couette: azimuthal vs axial



$$p(\theta) = p_{\text{per}}(\theta) + \cancel{a\theta} \qquad p(z) = p_{\text{per}}(z) + az$$

$$\frac{dp}{d\theta} = p'_{\text{per}}(\theta) + \cancel{a} \qquad \frac{dp}{dz} = p'_{\text{per}}(z) + a$$

**p must be single-valued function of θ**

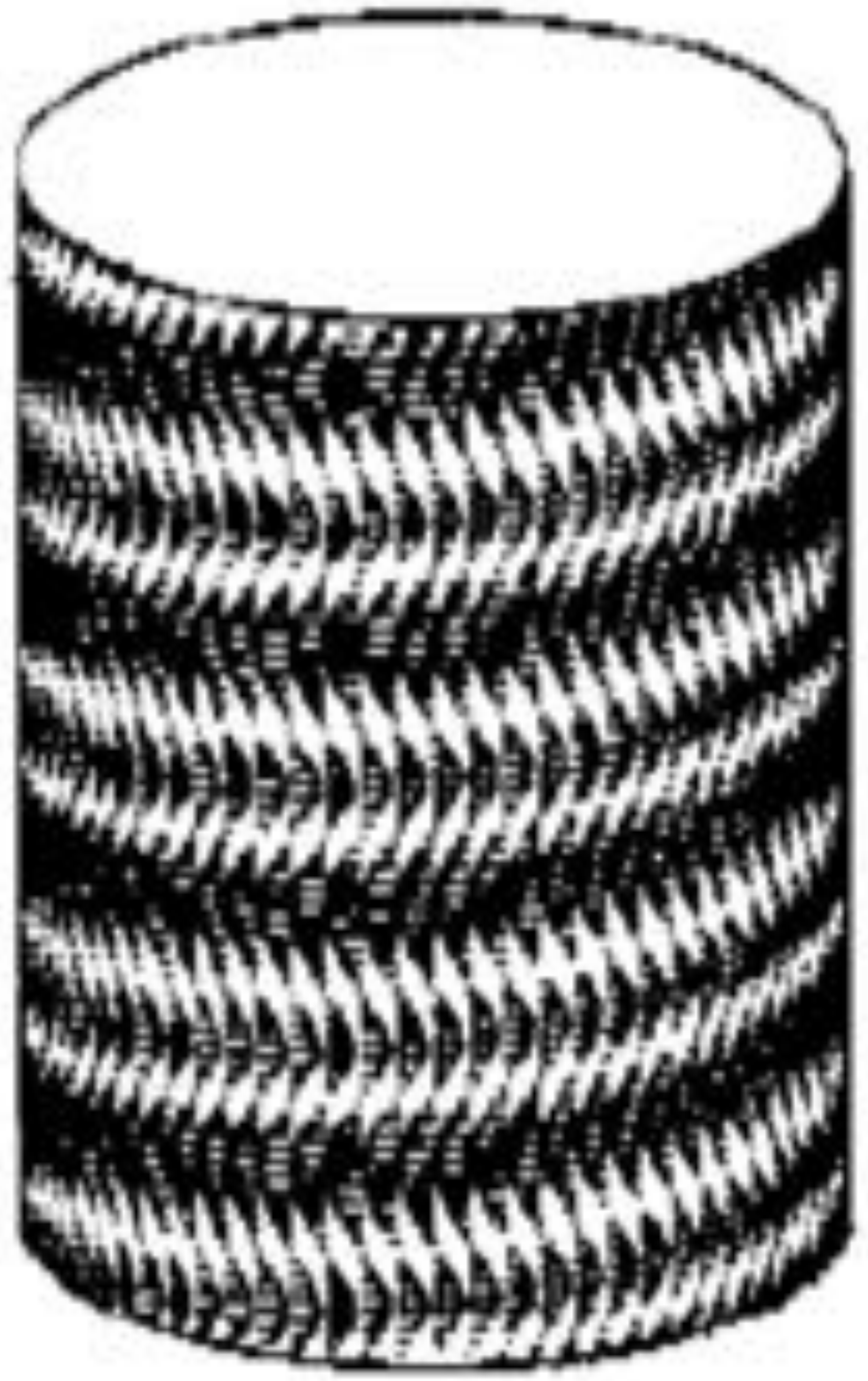
Periodic directions **ALWAYS** require additional condition,
e.g. flux or pressure gradient or combination

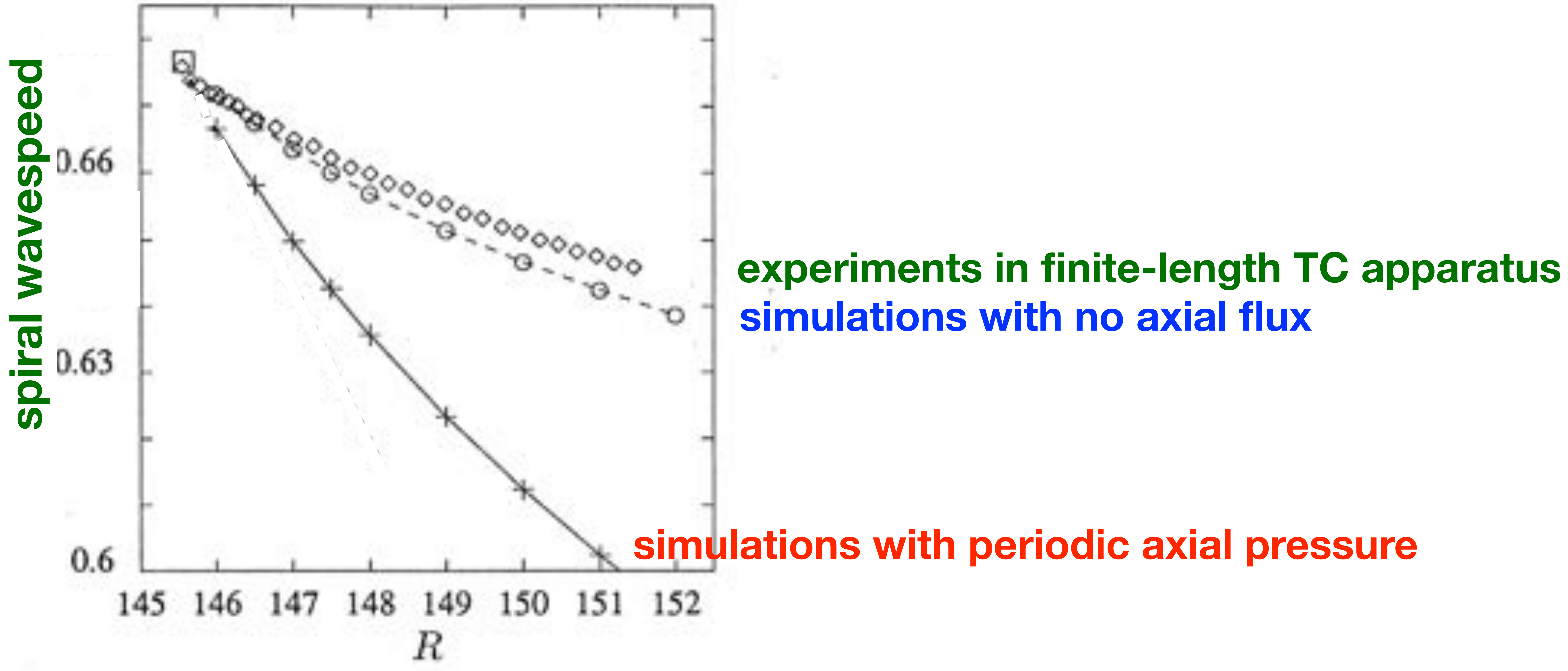
Geometric/azimuthal: **no pressure gradient.** Assumed/axial: **choice.**
Finite boundaries: **no flux**

Periodic traveling waves with nonperiodic pressure

Eur. J. Mech., B/Fluids, 10, no. 2 - Suppl., pp. 205-210, (1991).

W.S. Edwards, R. Tagg, H.L. Swinney,
Center for Nonlinear Dynamics, Univ of Texas at Austin





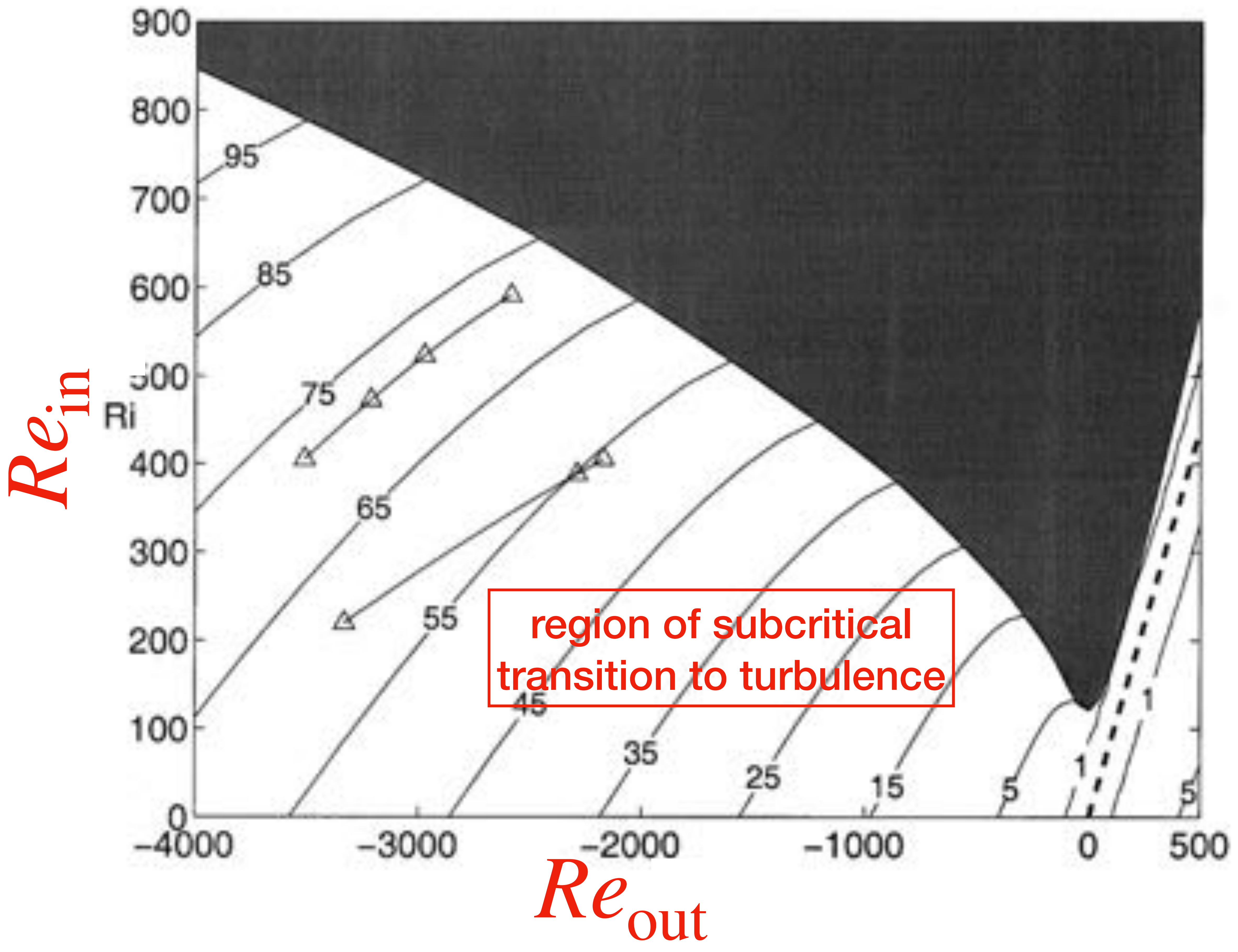

Difference only appears at **nonlinear level** and when **axial reflection symmetry is broken**

# Transient growth

PHYSICS OF FLUIDS MAY 2002

**Energy transient growth in the Taylor–Couette problem**

Álvaro Meseguer[b)]



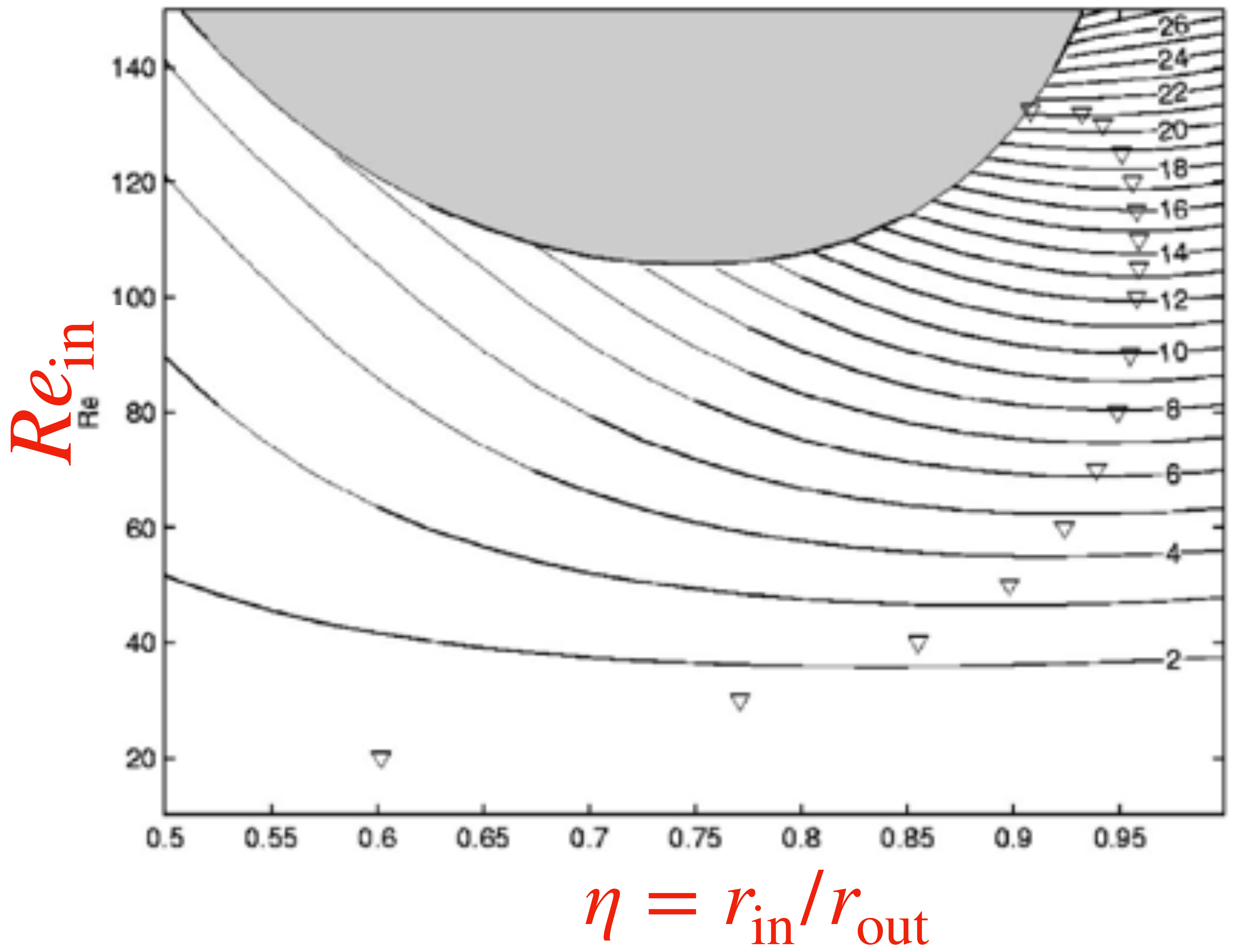

PHYSICS OF FLUIDS OCTOBER 2002

**Transient growth in Taylor–Couette flow**

Hristina Hristova and Sébastien Roch

Peter J. Schmid

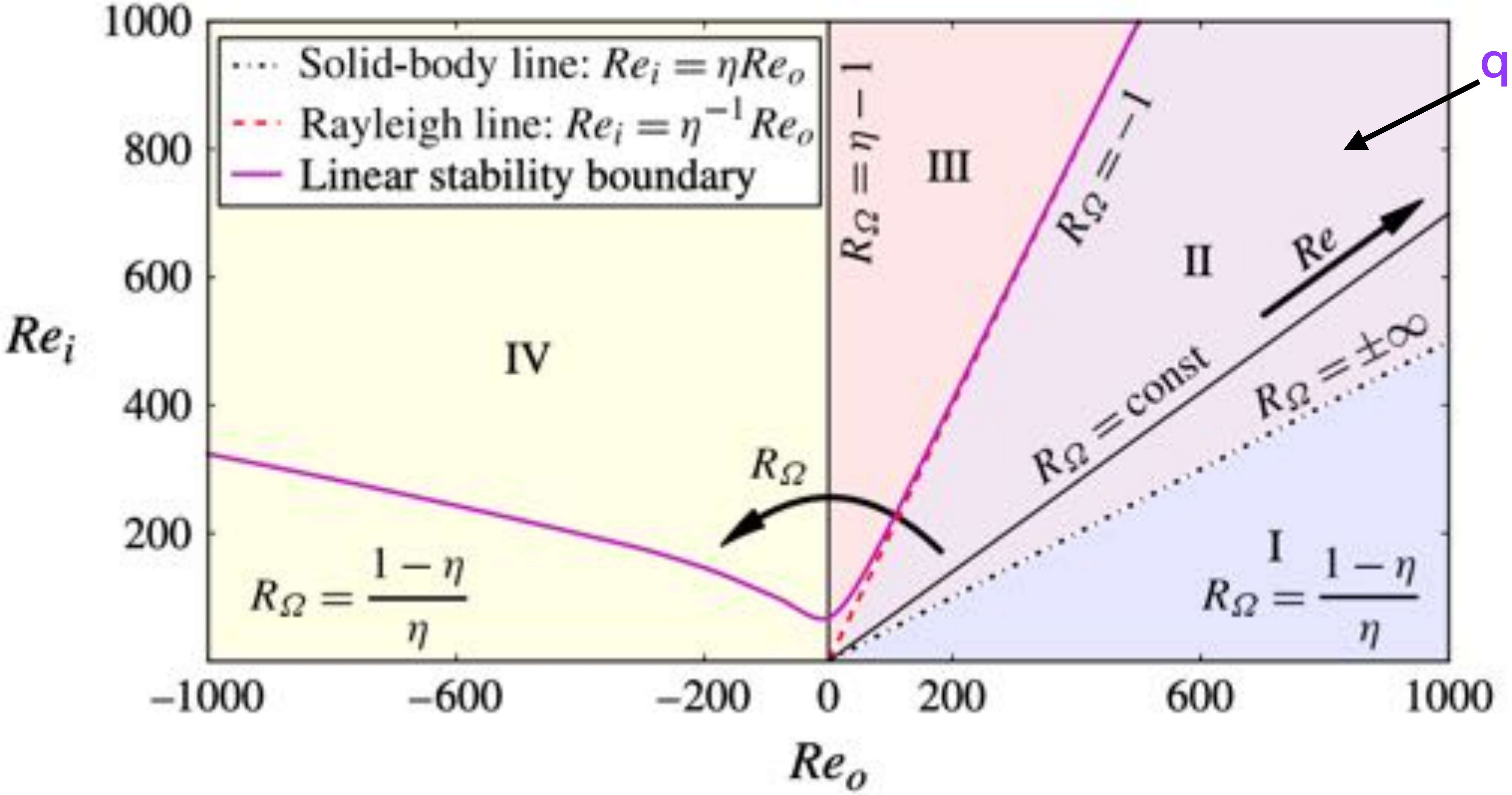
Laurette S. Tuckerman



J. Fluid Mech. (2014), vol. 742, pp 254 290. © Cambridge University Press 2014
doi:10.1017/jfm.2014.12

# Transient growth in linearly stable Taylor–Couette flows

Simon Maretzke[1] †, Björn Hof[2] and Marc Avila[1]




quasi-Keplerian regime

**shear**

$$Re := \frac{2|\eta Re_o - Re_i|}{1+\eta}$$

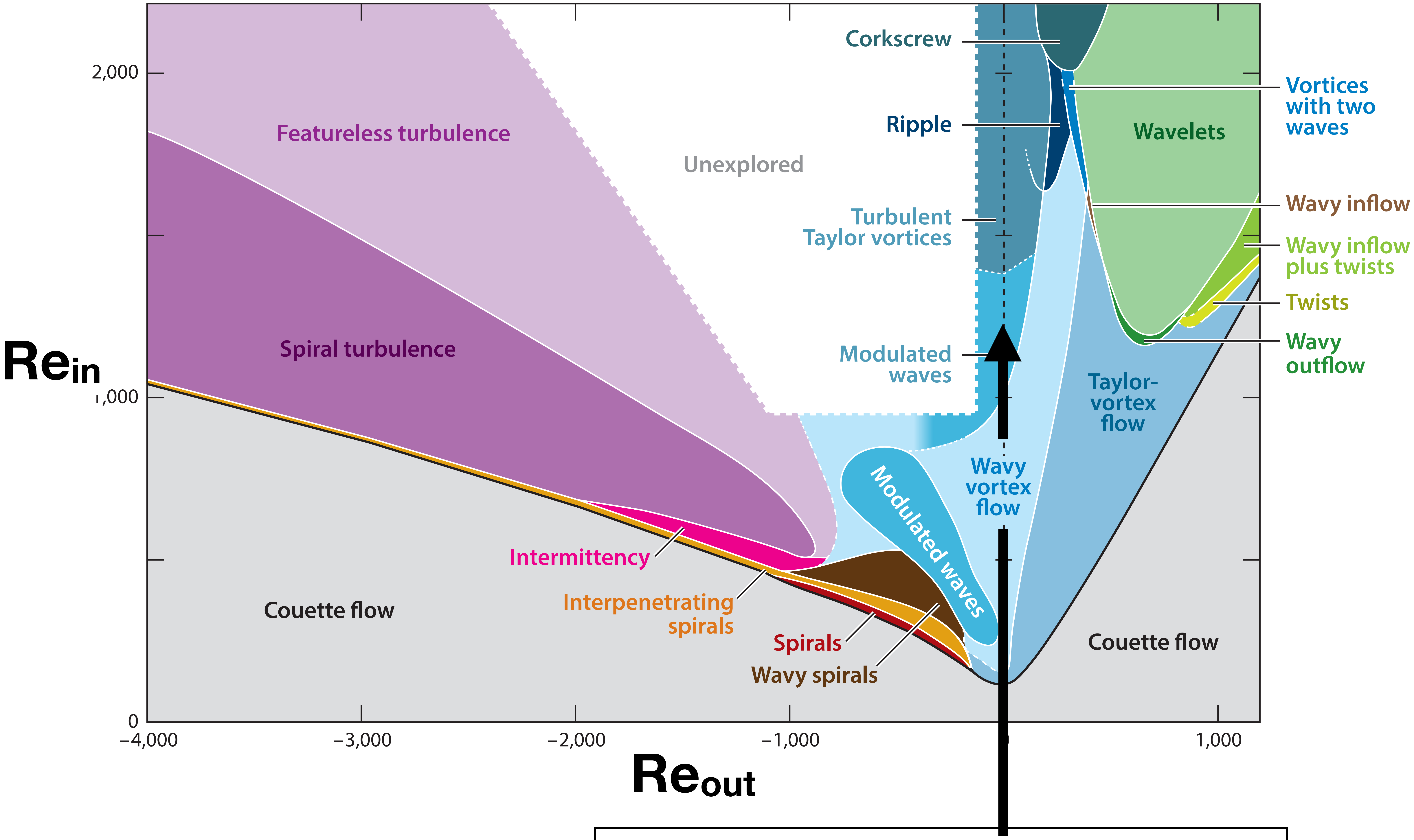
**rotation**

$$R_\Omega := \frac{(1-\eta)(Re_i + Re_o)}{\eta Re_o - Re_i}$$

*Dubrulle et al, Phys. Fluids 2005*

In quasi-Keplerian regime, maximum transient growth is achieved for axially independent perturbations, i.e. Taylor columns.

Transient growth of axially independent perturbations is independent of $R_\Omega$




# Modulated Waves

PRL **94,** 074501 (2005) PHYSICAL REVIEW LETTERS week ending 25 FEBRUARY 2005

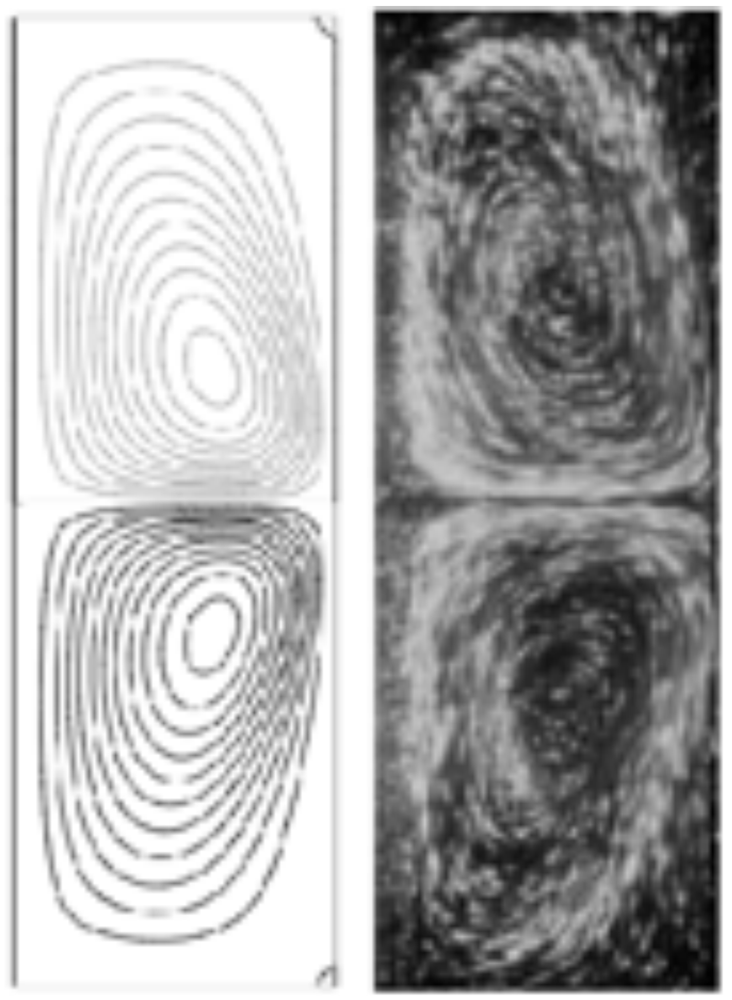
**Symmetry Breaking Via Global Bifurcations of Modulated Rotating Waves in Hydrodynamics**

Jan Abshagen,[1] Juan M. Lopez,[2] Francisco Marques,[3] and Gerd Pfister[1]

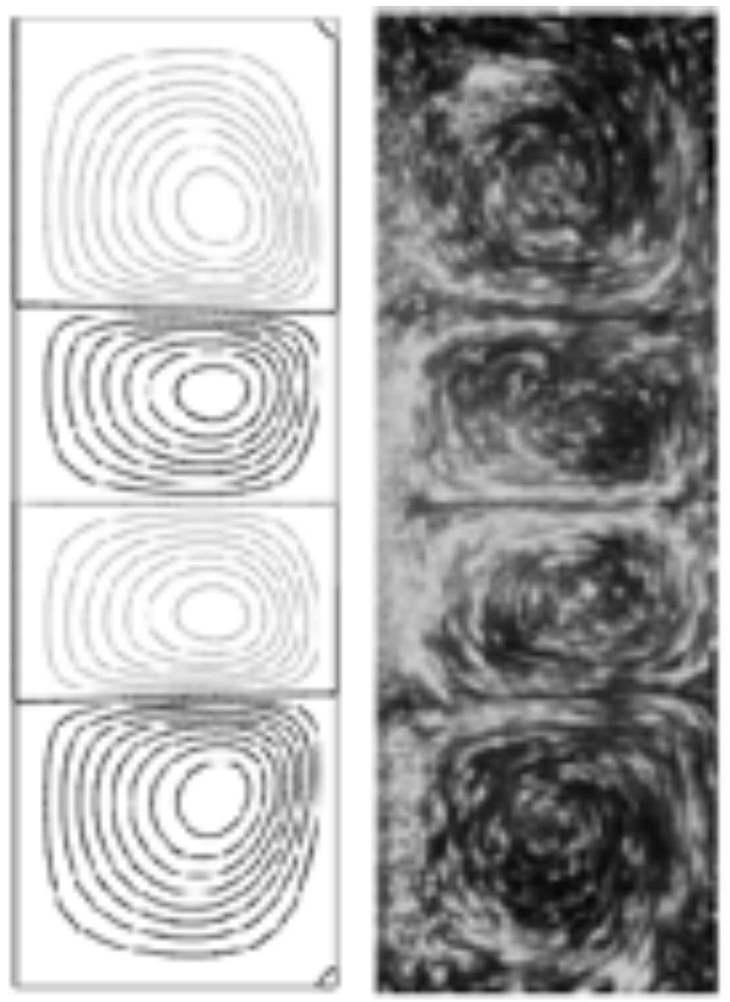
*J. Fluid Mech.* (2005), *vol.* 540, *pp.* 269–299. © 2005 Cambridge University Press 269
doi:10.1017/S0022112005005811 Printed in the United Kingdom

**Mode competition of rotating waves in reflection-symmetric Taylor–Couette flow**

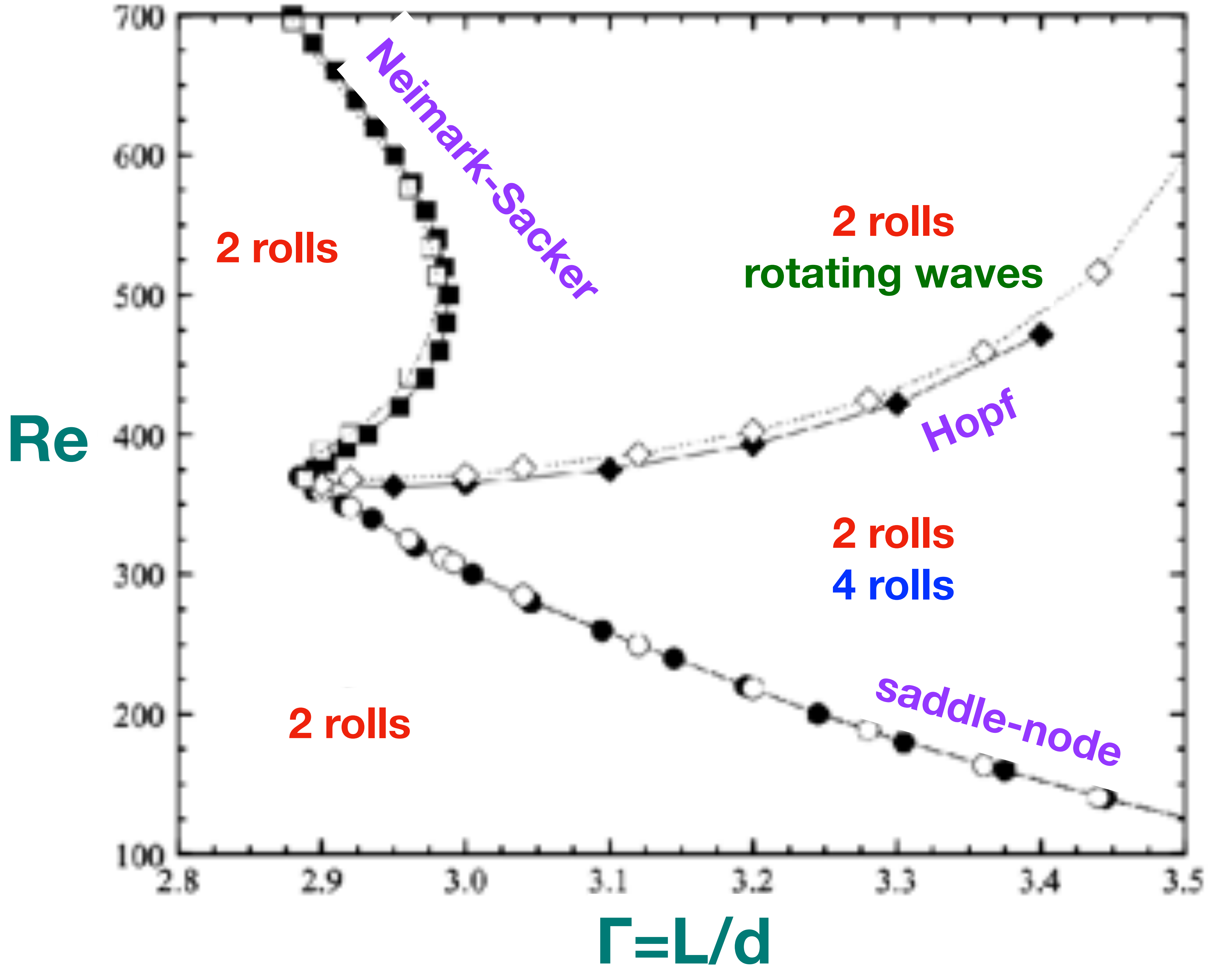
By J. ABSHAGEN[1], J. M. LOPEZ[2], F. MARQUES[3] AND G. PFISTER[1]

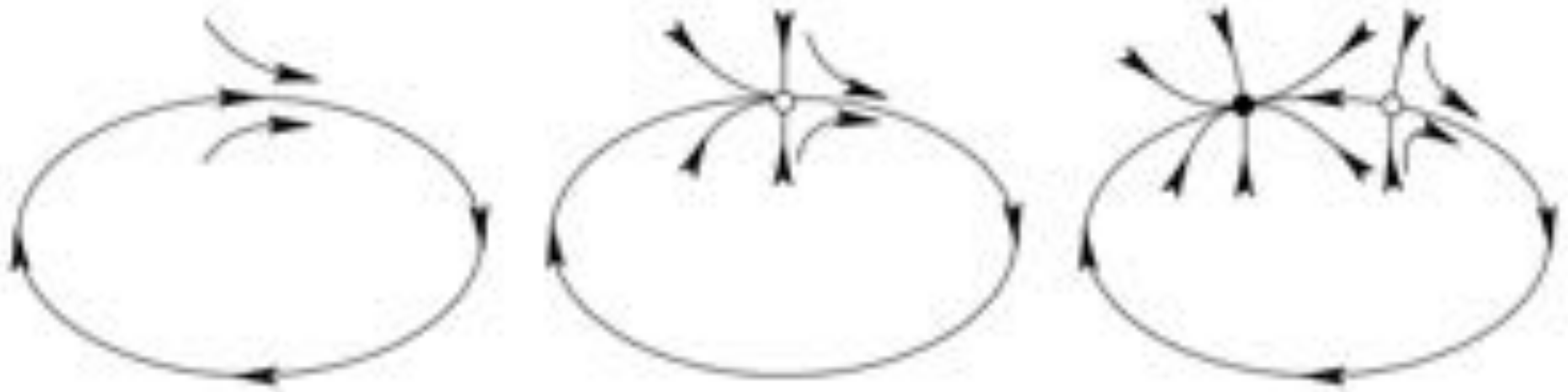


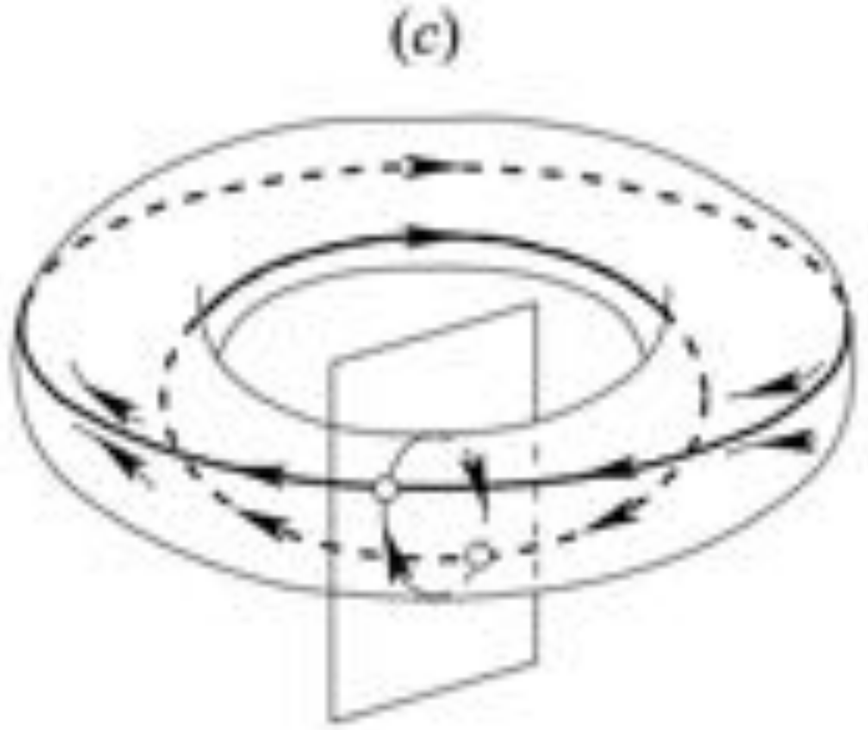
2 rolls



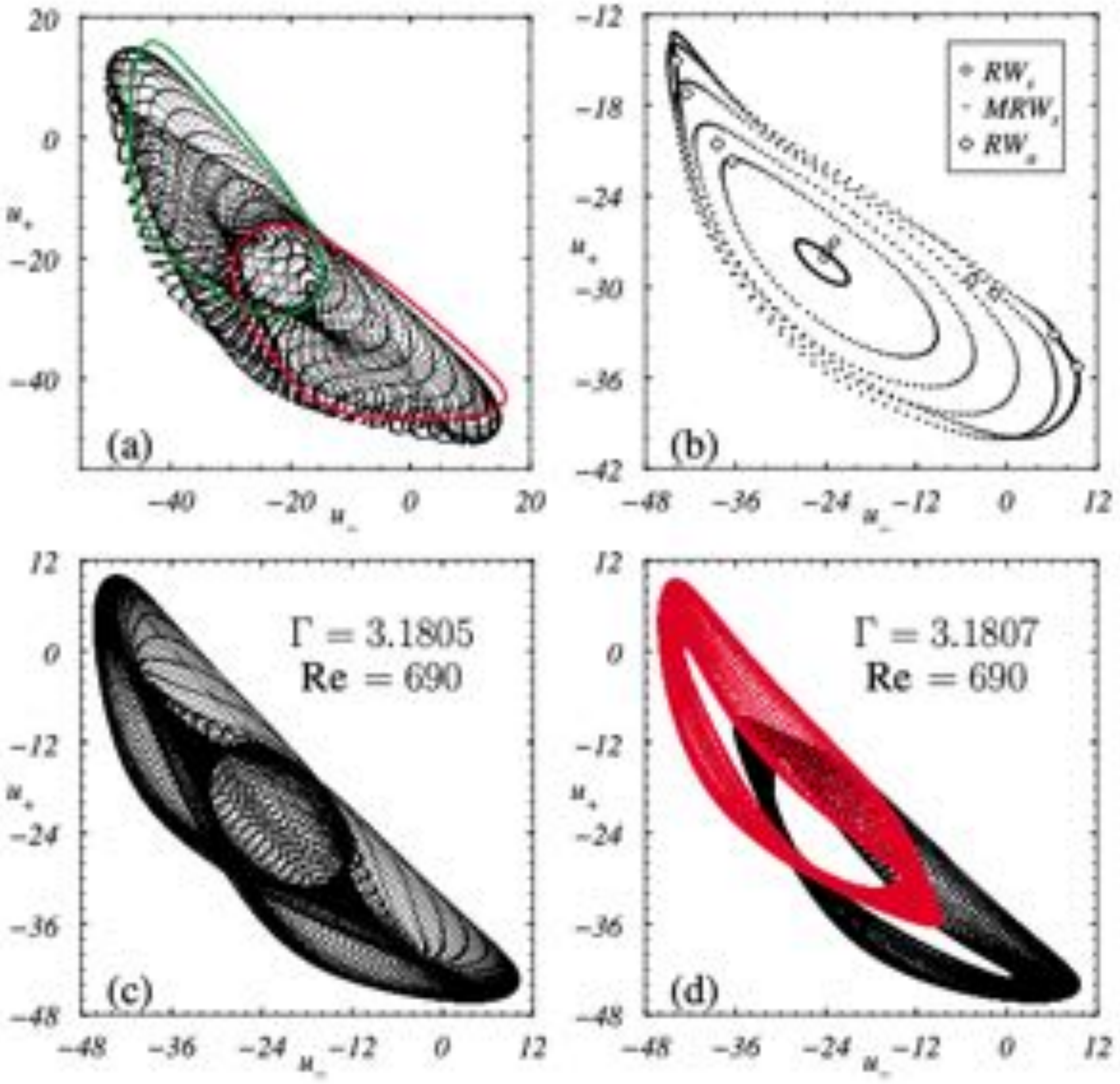
4 rolls

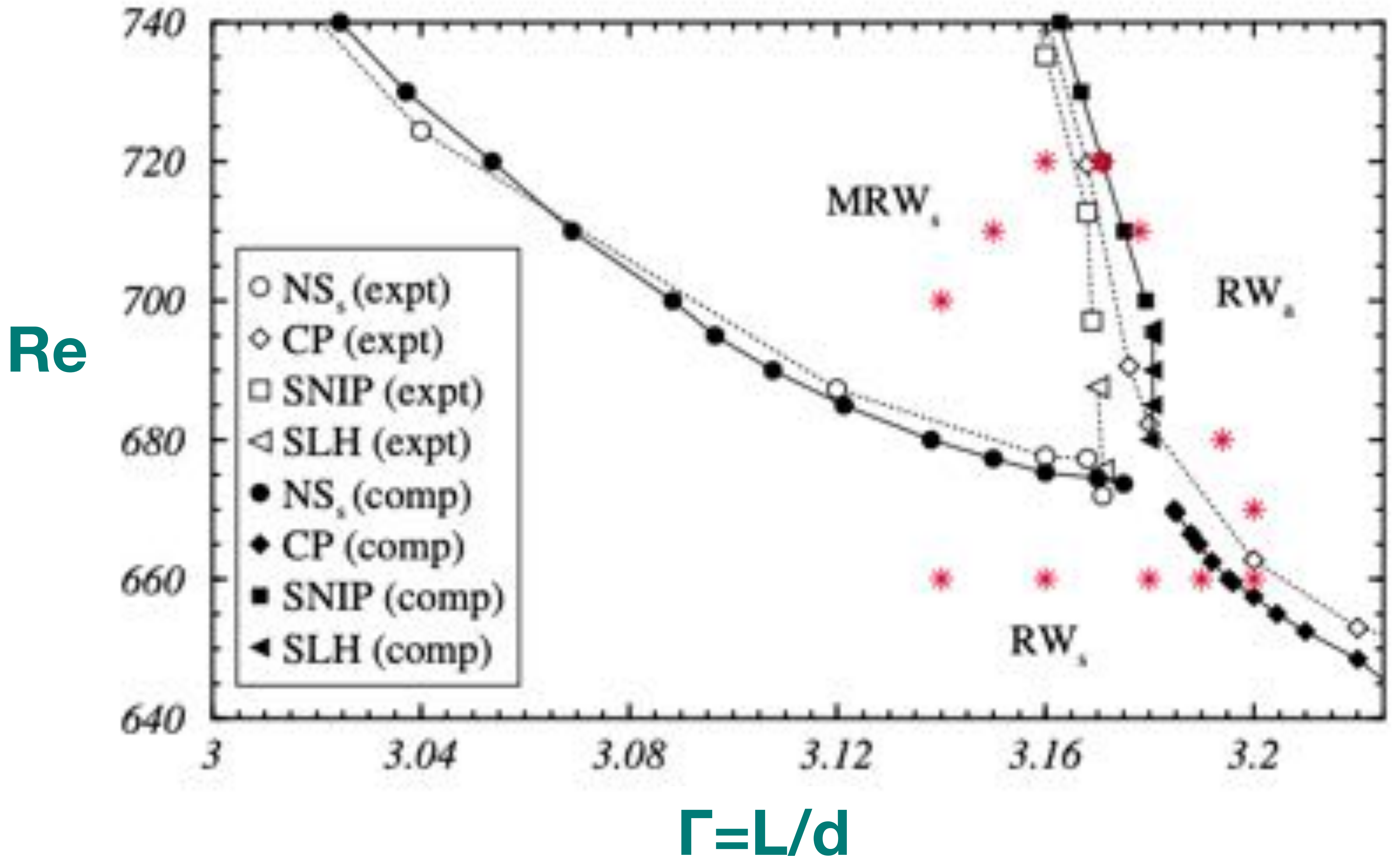







# SNIP (saddle-node infinite period) bifurcation of limit cycles



# Unfolding of codimension-two fold-Hopf bifurcation

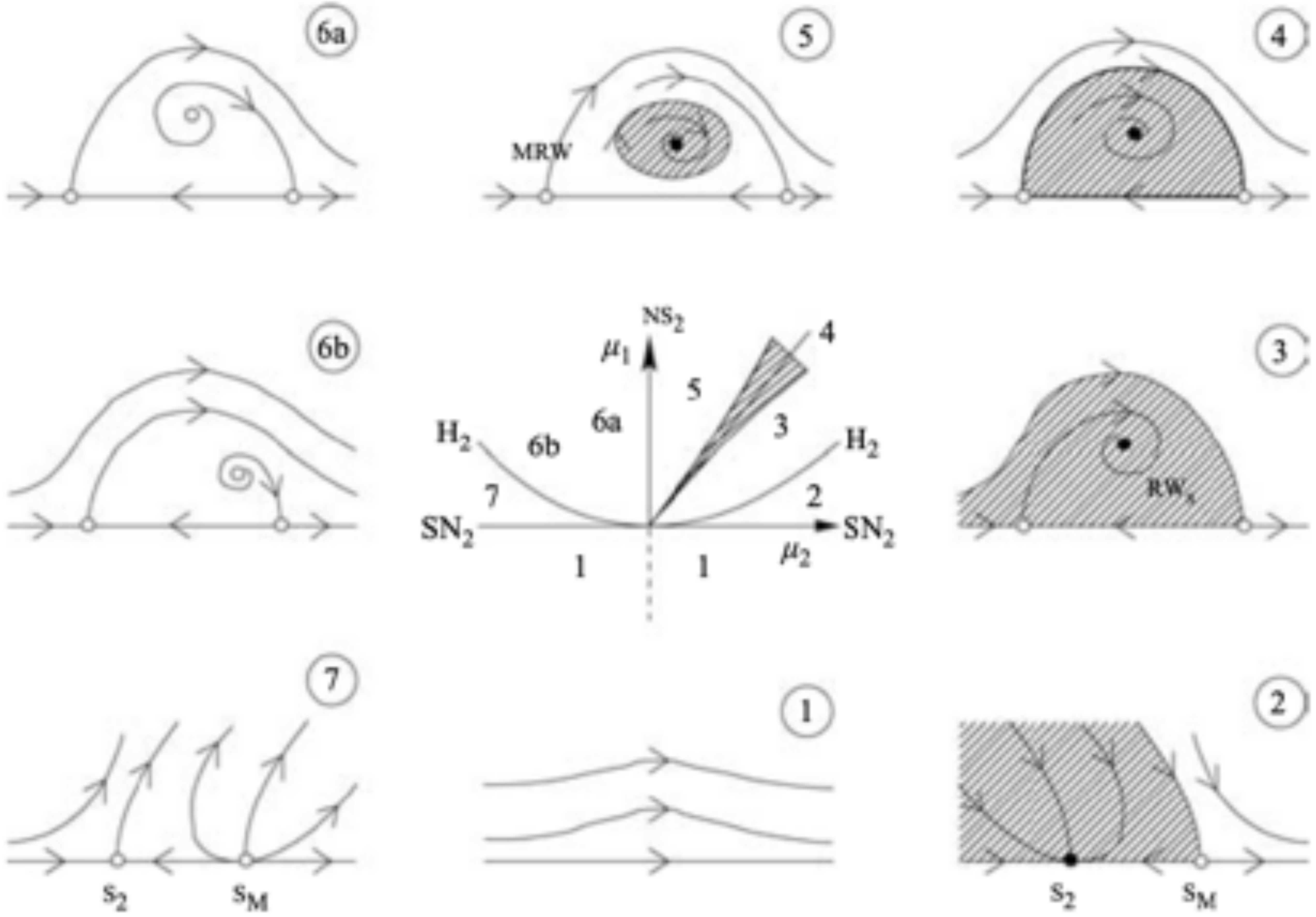
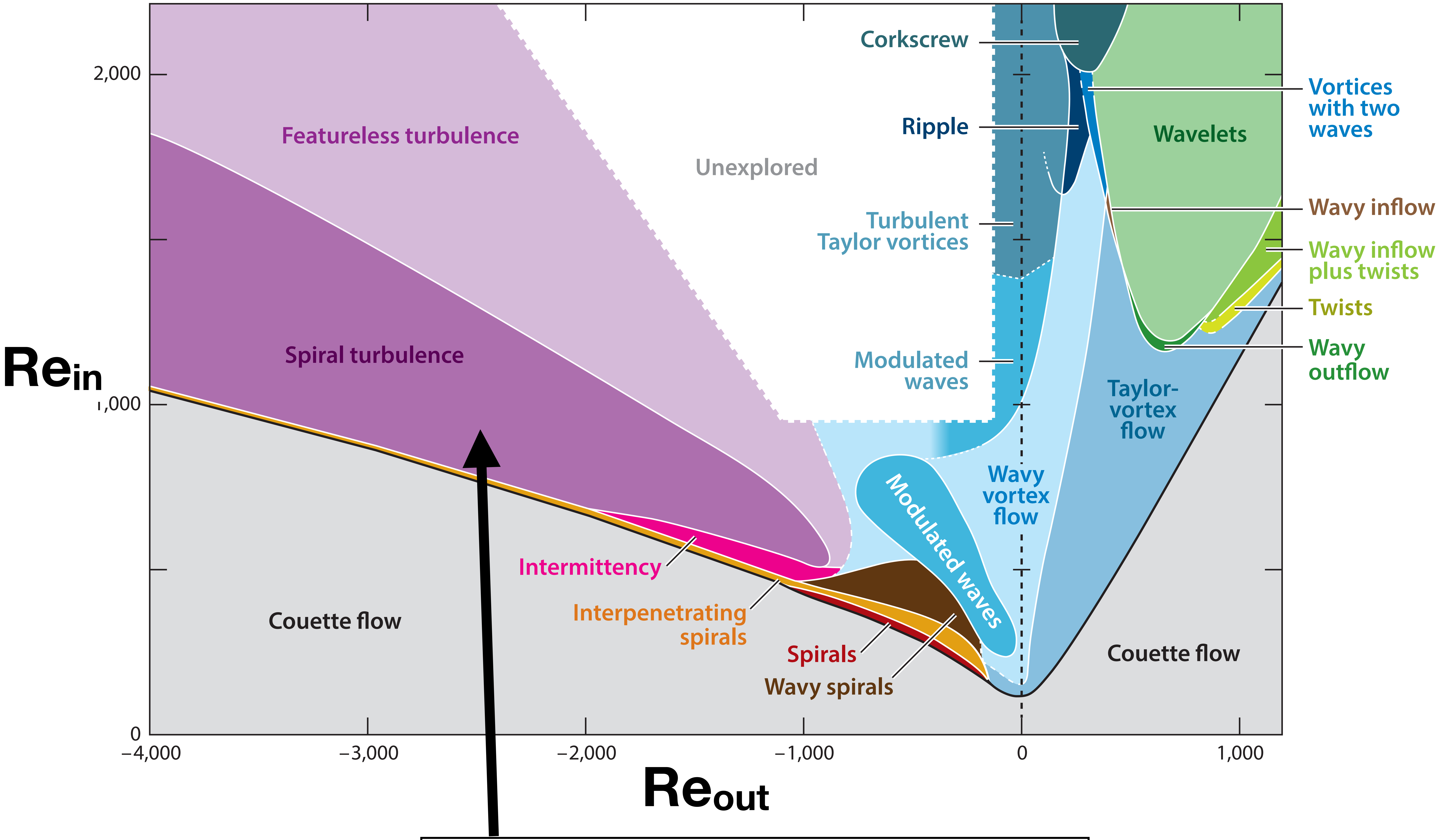



FIGURE 11. Bifurcation diagram of the fold-Hopf bifurcation, in normal form variables, corresponding to the present flow. Filled (•) and open (○) dots correspond to stable and unstable solutions respectively, $\mu_1$ and $\mu_2$ are the two bifurcation parameters, SN$_2$ is a saddle-node bifurcation curve, H$_2$ is the Hopf bifurcation curve, NS$_2$ is the Neimark–Sacker bifurcation curve, and 4 is the horn region of complex dynamics; the straight line inside region 4 is the heteroclinic connection predicted by the formal normal form (6.1) and shown in panel 4 as a thick line.

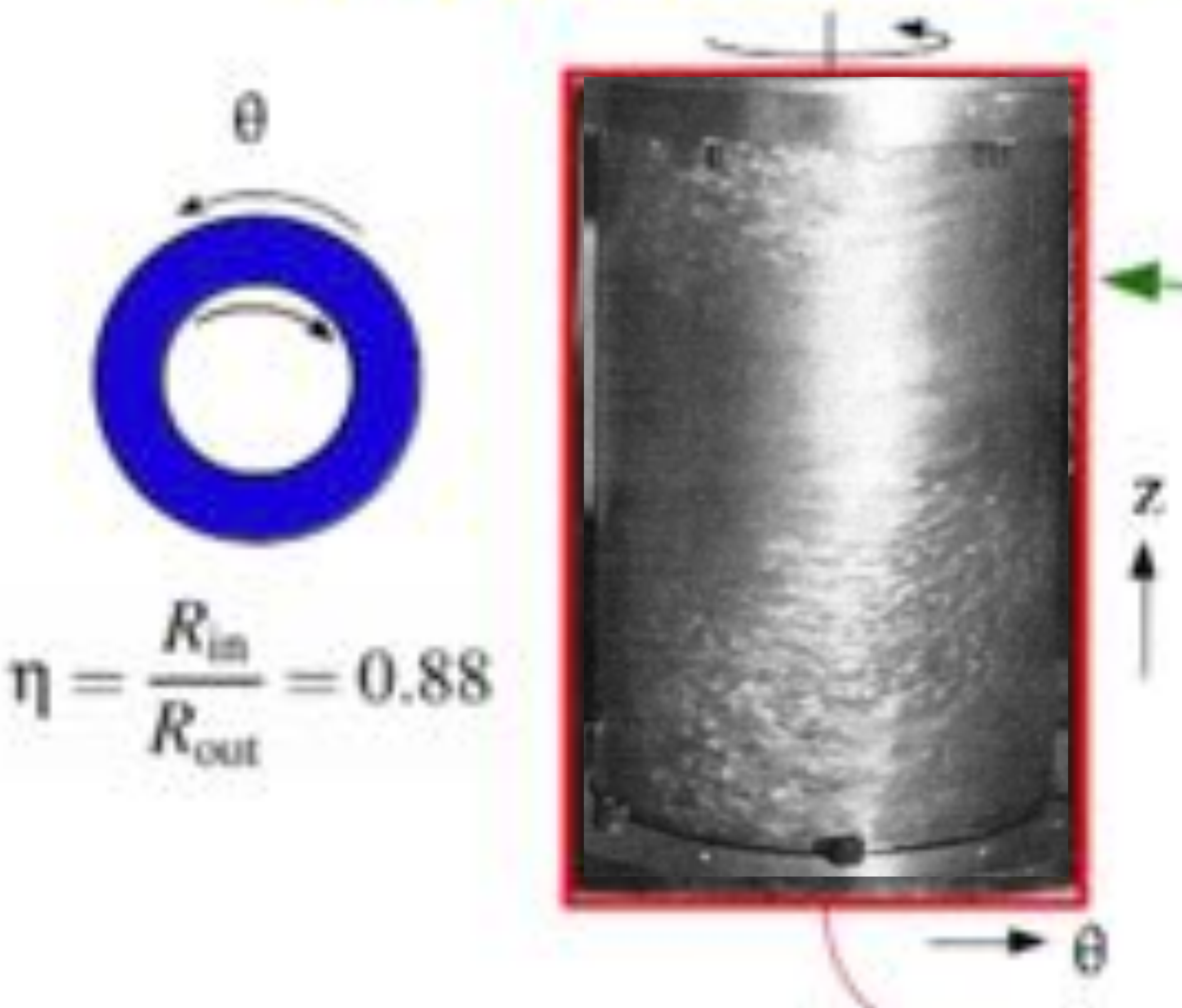
(Kuznetsov 1998)

$$\left.\begin{aligned} \dot{x} &= -\mu_1 + x^2 + \sigma\rho^2, \\ \dot{\rho} &= \rho(-\mu_2 + \chi x + x^2), \\ \dot{\phi} &= \omega, \end{aligned}\right\}$$



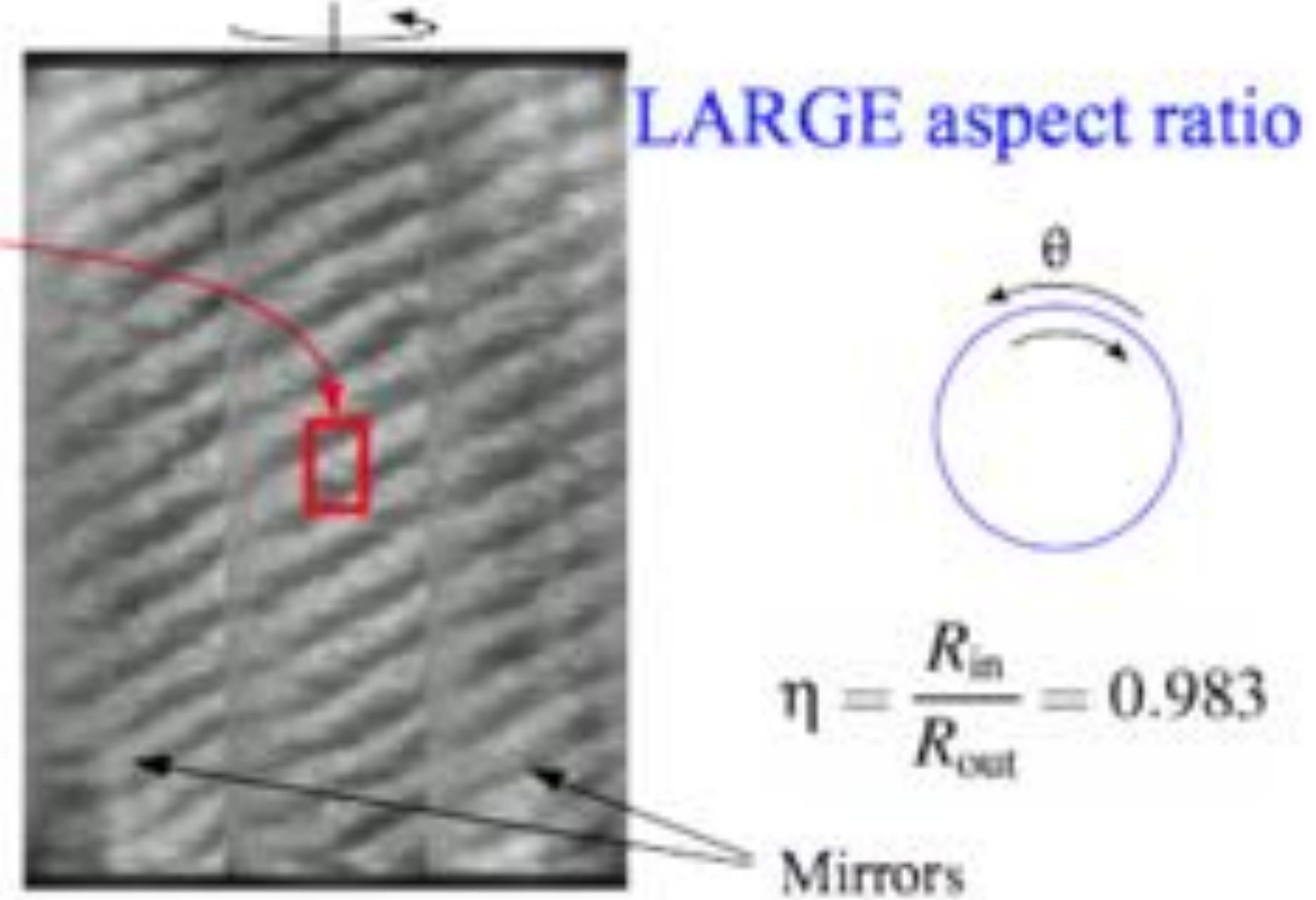

Spiral turbulence

# *Spiral Turbulence in counter-rotating Taylor-Couette Flow*




- ***Coles JFM (1965)***
- ***van Atta JFM (1966)***
- ***Andereck et al. JFM (1986)***

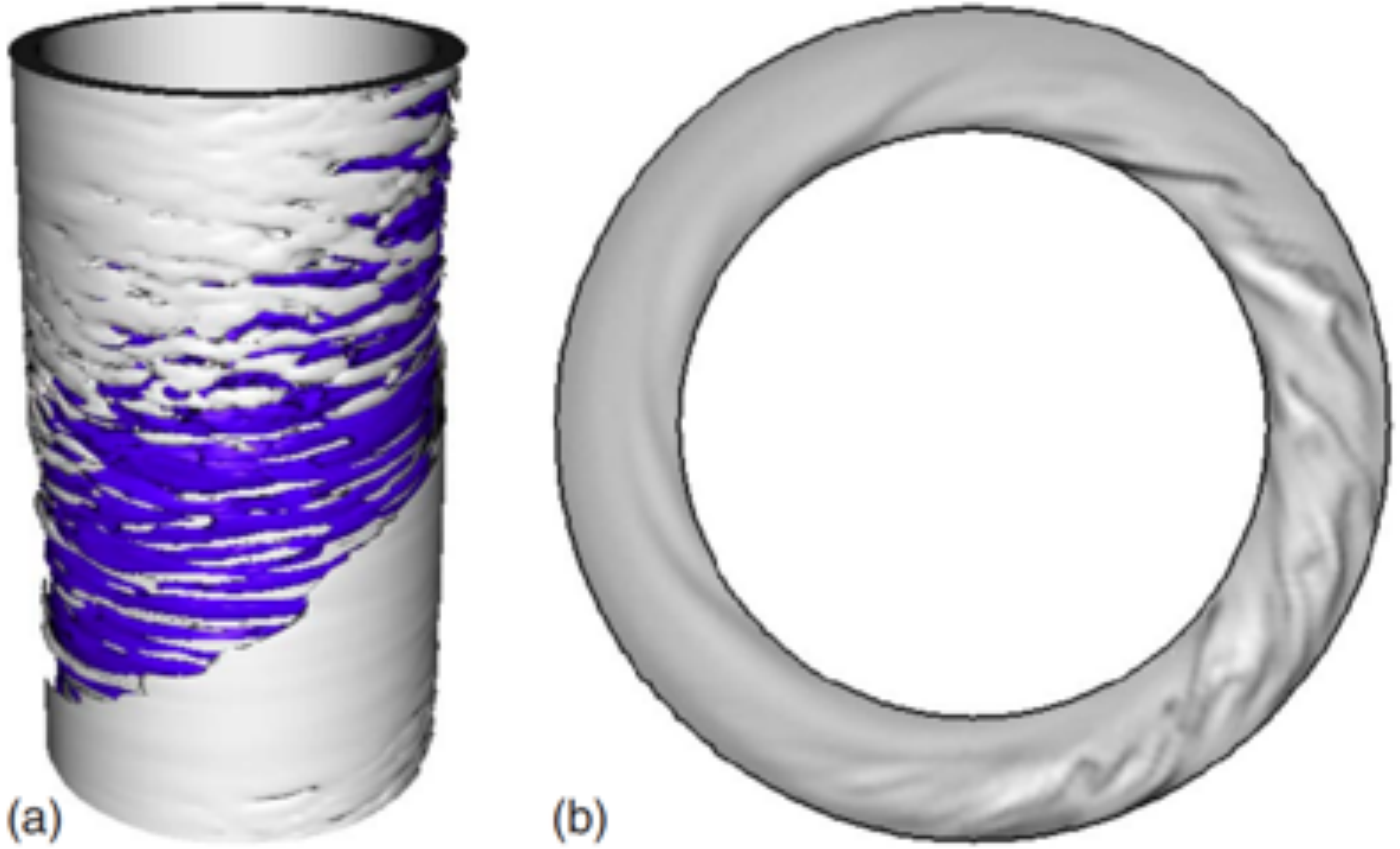
***Prigent & Dauchot PRL (2002)***



PHYSICAL REVIEW E **80**, 046315 (2009)

# Instability mechanisms and transition scenarios of spiral turbulence in Taylor-Couette flow

Alvaro Meseguer,[1,*] Fernando Mellibovsky,[1] Marc Avila,[2] and Francisco Marques[1]
[1]*Departament de Física Aplicada, Universitat Politècnica de Catalunya, 08034 Barcelona, Spain*
[2]*Max Planck Institute for Dynamics and Self-Organization, 37073 Göttingen, Germany*



# Transition to turbulence in counter-rotating Taylor-Couette flow:

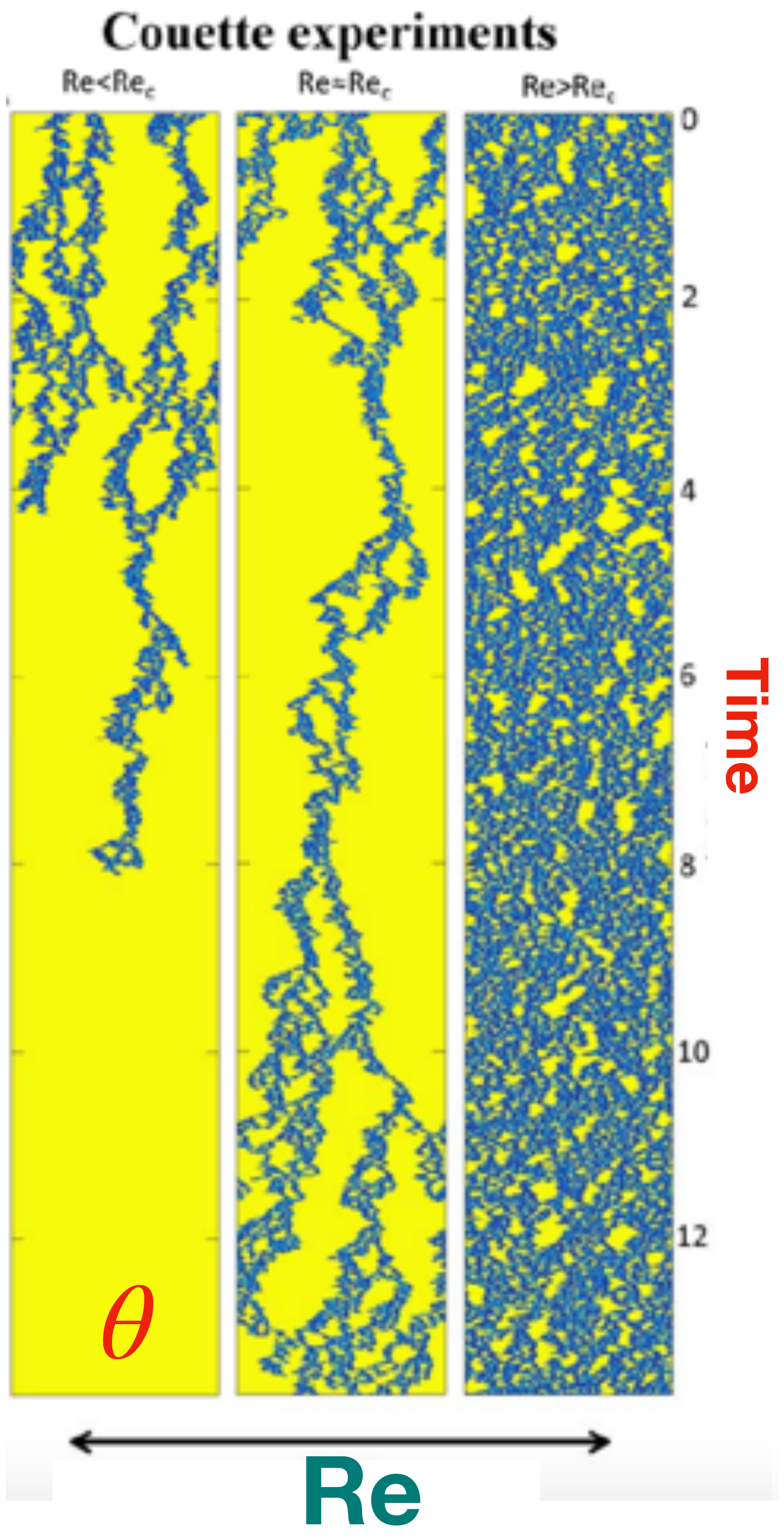
## Universal scenario of directed percolation (DP)

Very short Taylor-Couette apparatus:
only extended direction is azimuthal

$$\eta = \frac{r_{\text{in}}}{r_{\text{out}}} = 0.998$$

height-to-gap 16

circumference-to-gap 5500





LETTERS
PUBLISHED ONLINE: 15 FEBRUARY 2016 | DOI: 10.1038/NPHYS3675
nature physics


**Directed percolation phase transition to sustained turbulence in Couette flow**

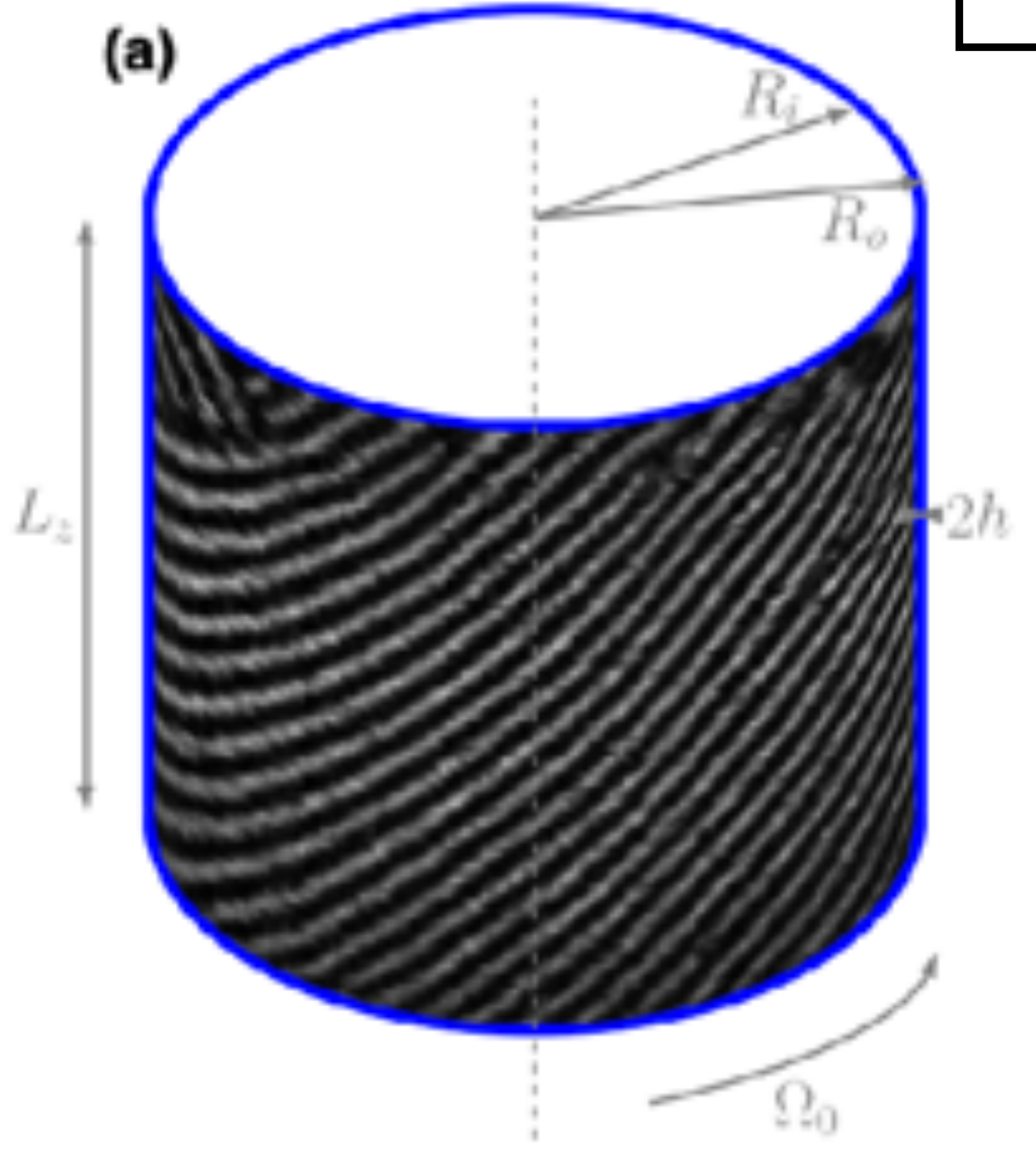

Grégoire Lemoult[1†], Liang Shi[1,2†], Kerstin Avila[1,2], Shreyas V. Jalikop[1], Marc Avila[3] and Björn Hof[1*]

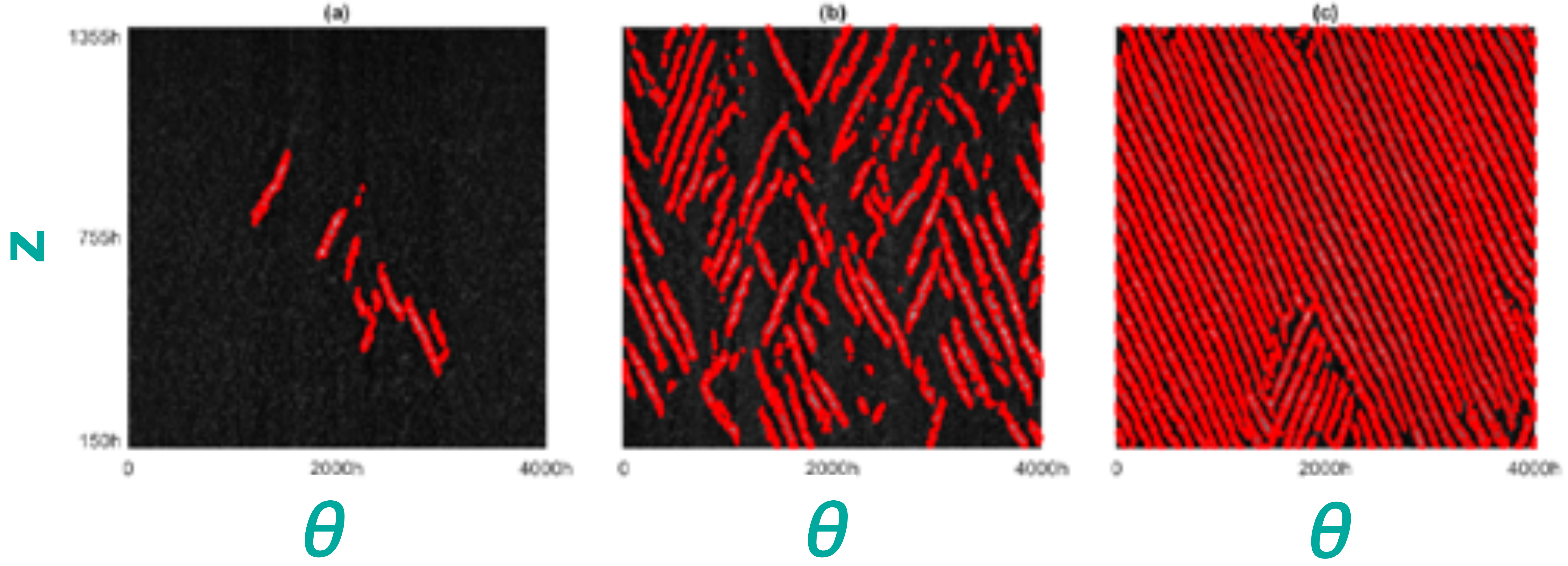
# Directed Percolation in TC with two extended directions

PHYSICAL REVIEW LETTERS 128, 014502 (2022)

**Phase Transition to Turbulence in Spatially Extended Shear Flows**

Lukasz Klotz,[1,2] Grégoire Lemoult,[3] Kerstin Avila,[4,5] and Björn Hof[1,*]

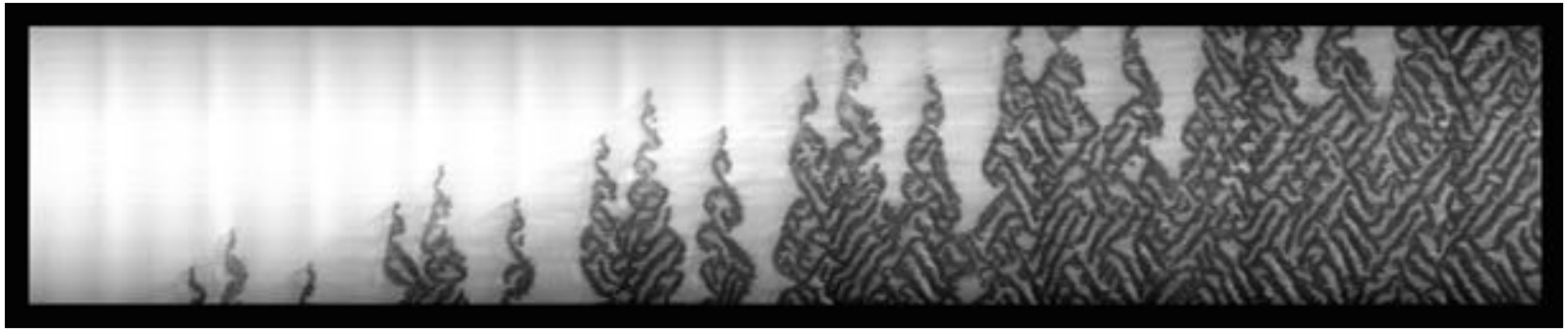



**Re→**

## Transition to turbulence in counter-rotating Taylor-Couette flow

## Universal scenario of directed percolation (DP)



$\uparrow$ **t**

$\theta \rightarrow$

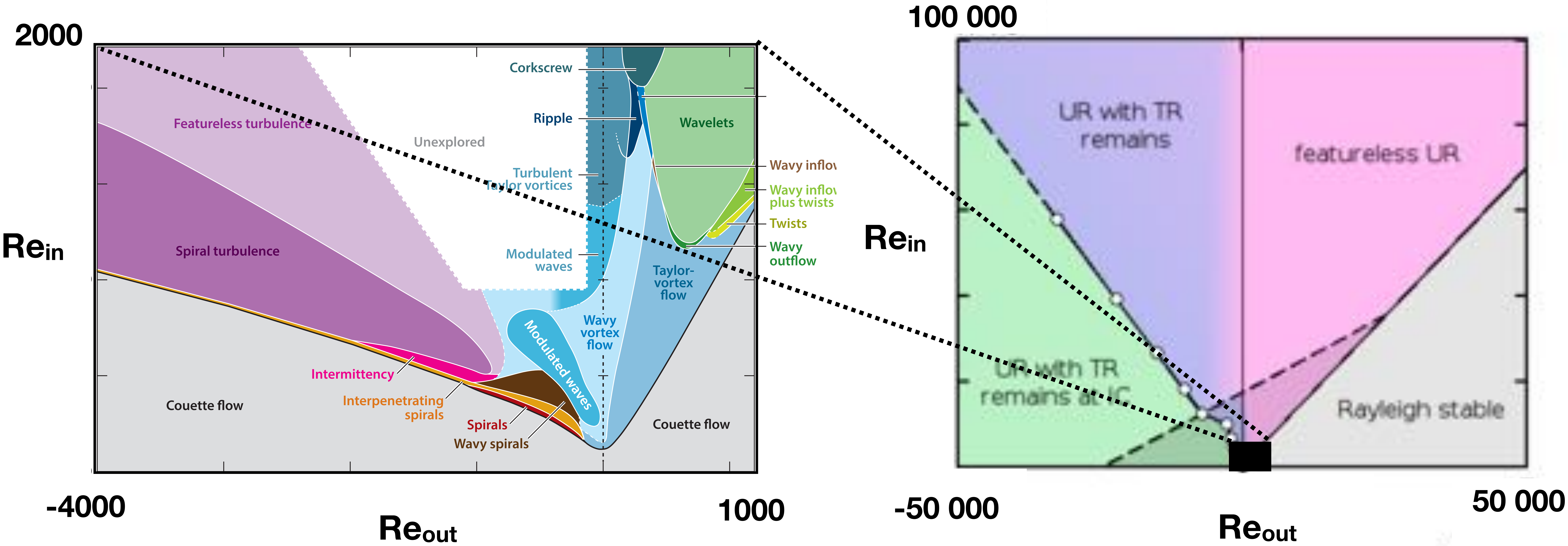
$\eta = r_{\text{in}}/r_{\text{out}} = 0.99$

*Lemoult, Maier, Hof, 2014*

PHYSICAL REVIEW FLUIDS 1, 024401 (2016)

# Exploring the phase space of multiple states in highly turbulent Taylor-Couette flow

Roeland C. A. van der Veen,[1] Sander G. Huisman,[1] On-Yu Dung (董安儒),[1] Ho L. Tang,[1]
Chao Sun,[2,1,*] and Detlef Lohse[1,3,†]



# 1985 International Couette-Taylor Workshop in Karlsruhe