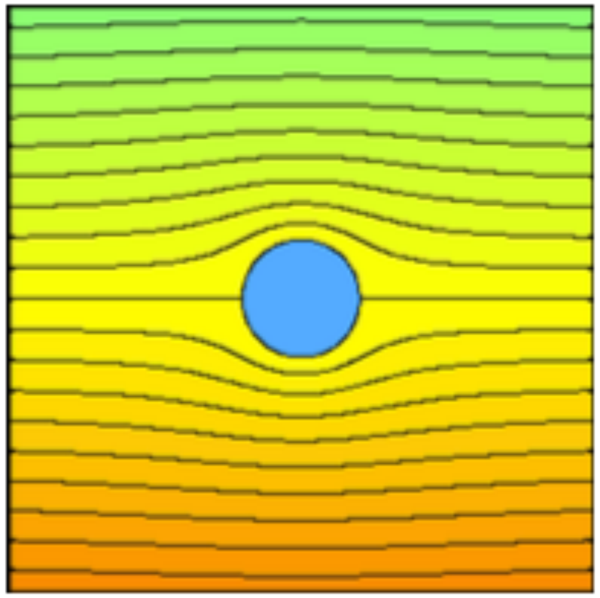
Mean flows and frequency prediction

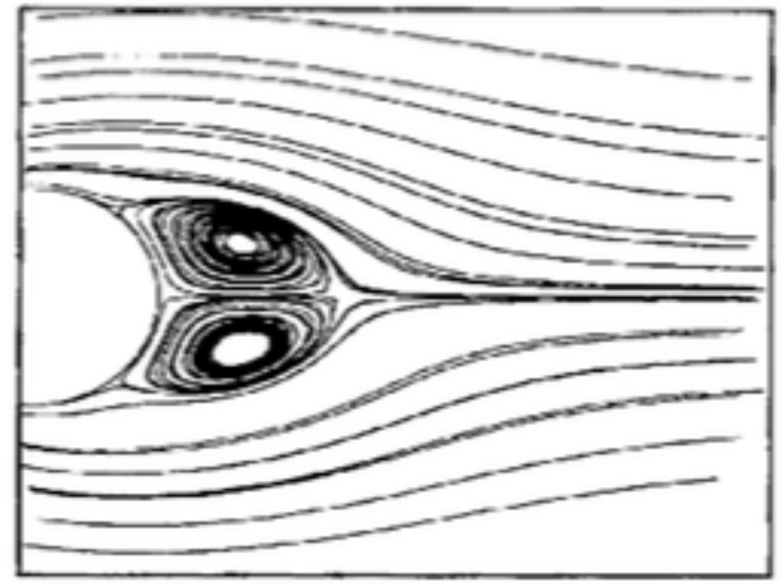
Sam Turton (Cambridge, Part III —> MIT)
Yacine Bengana (PMMH-CNRS-ESPCI)
Nicolas Périnet (Univ. of Santiago)
Laurette Tuckerman (PMMH-CNRS-ESPCI)
Dwight Barkley (University of Warwick)

Cylinder wake

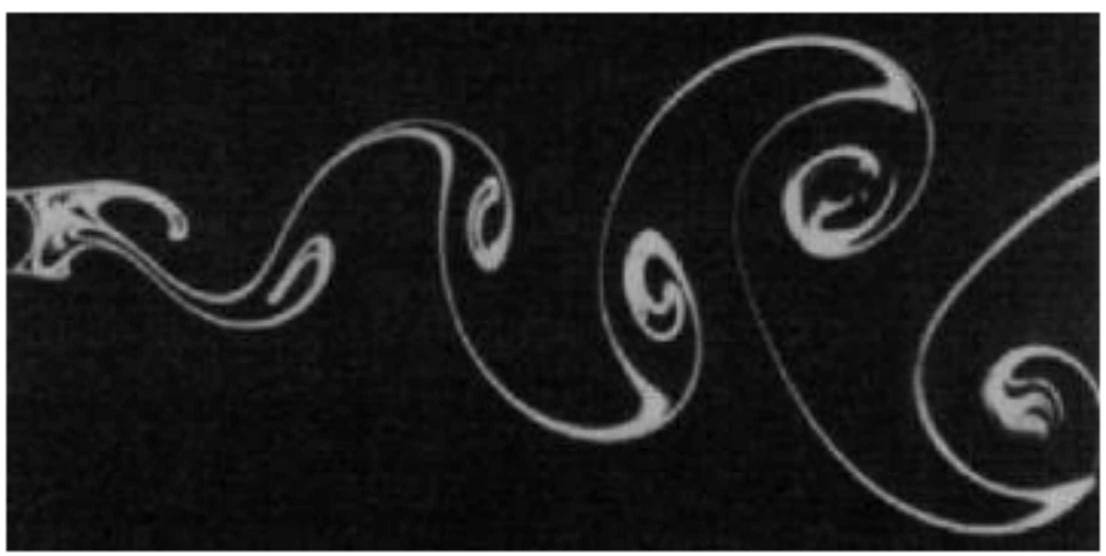
Ideal flow



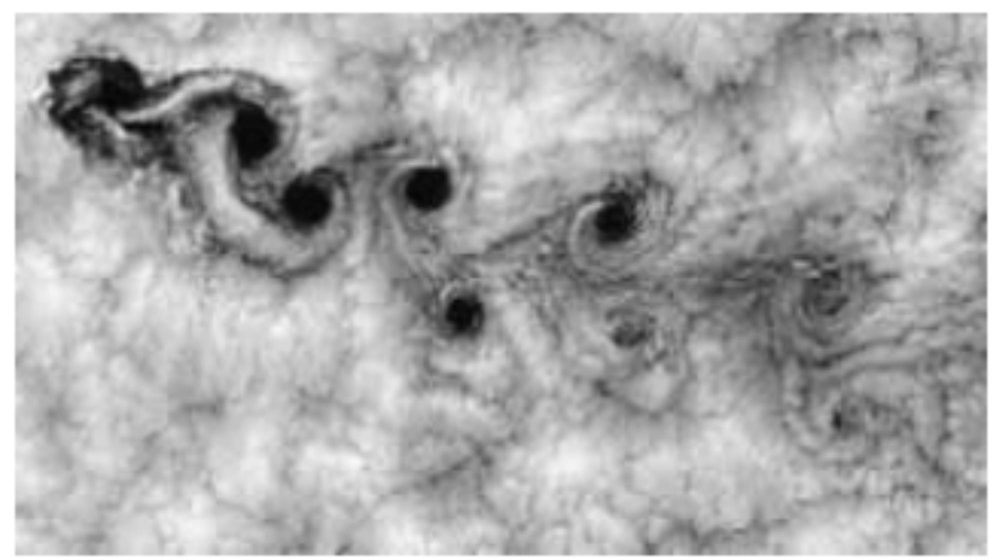
with downstream recirculation zone



Von Kármán vortex street ($\mathrm{Re} \geq 46$)

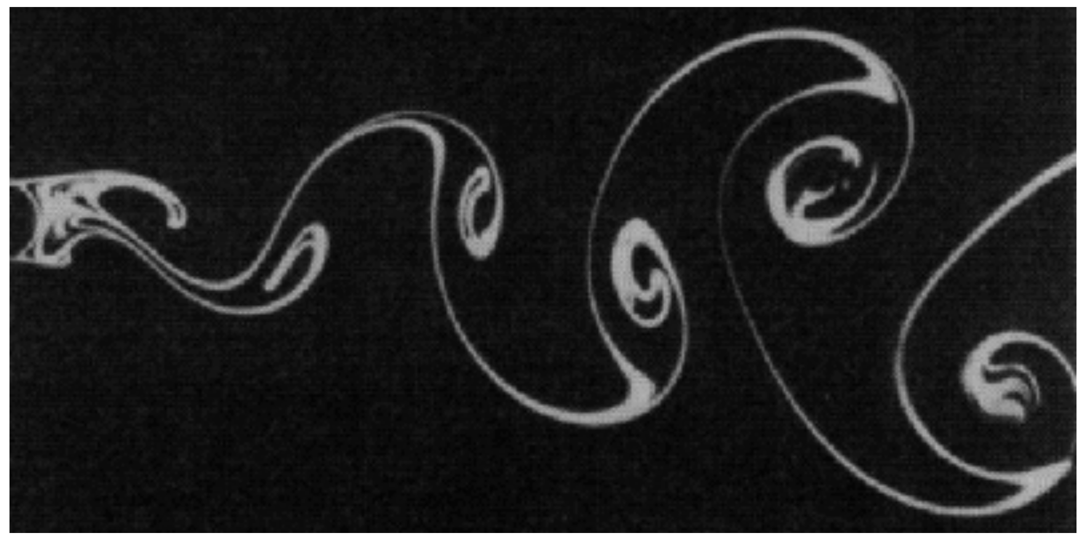


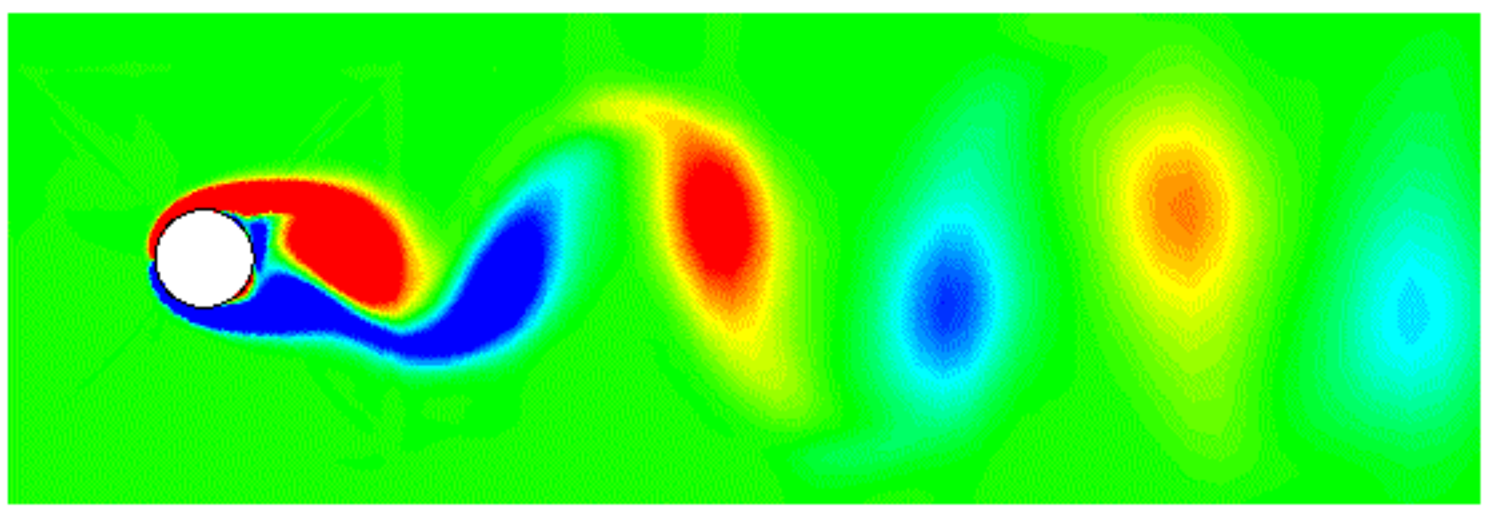
Laboratory experiment
(Taneda, 1982)



Off Chilean coast
past Juan Fernandez islands

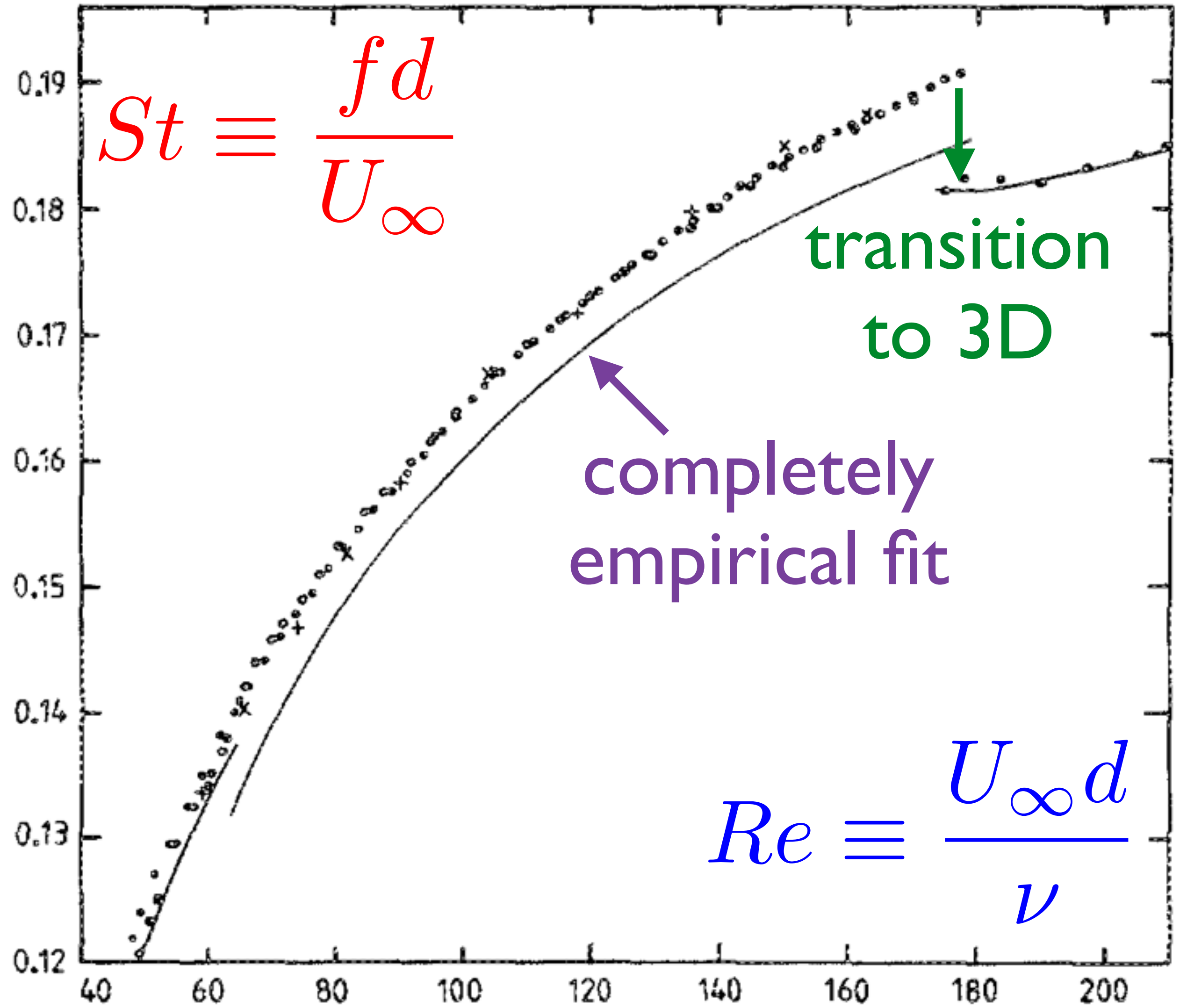
Vortex-shedding frequency of cylinder wake





Defining a universal and continuous Strouhal–Reynolds number relationship for the laminar vortex shedding of a circular cylinder

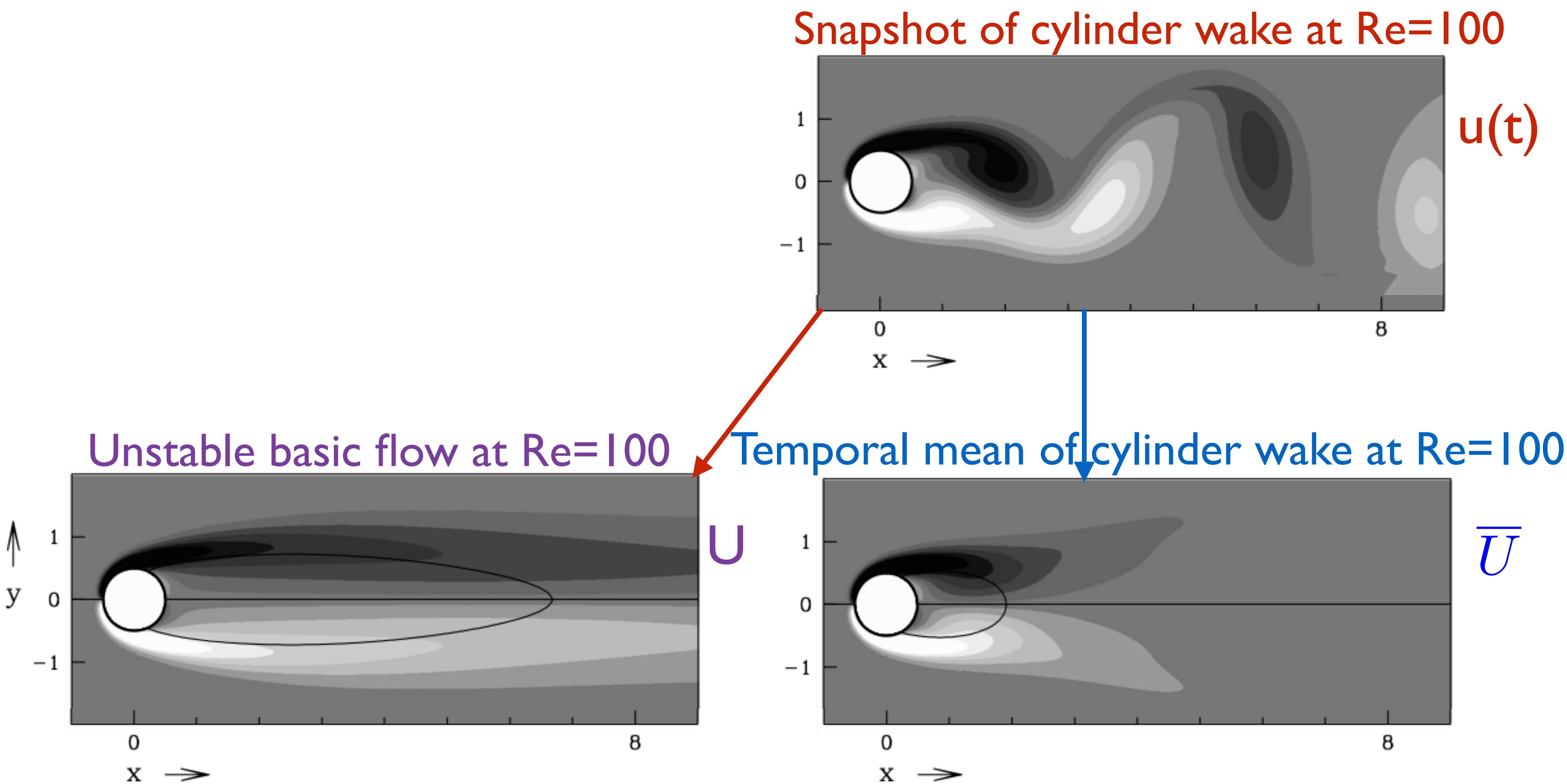
C. H. K. Williamson Physics of Fluids **31**, 2742 (1988)



Seek prediction from equations/physics

Linear analysis of the cylinder wake mean flow

D. Barkley(*) *Europhys. Lett.*, **75** (5), pp. 750–756 (2006)



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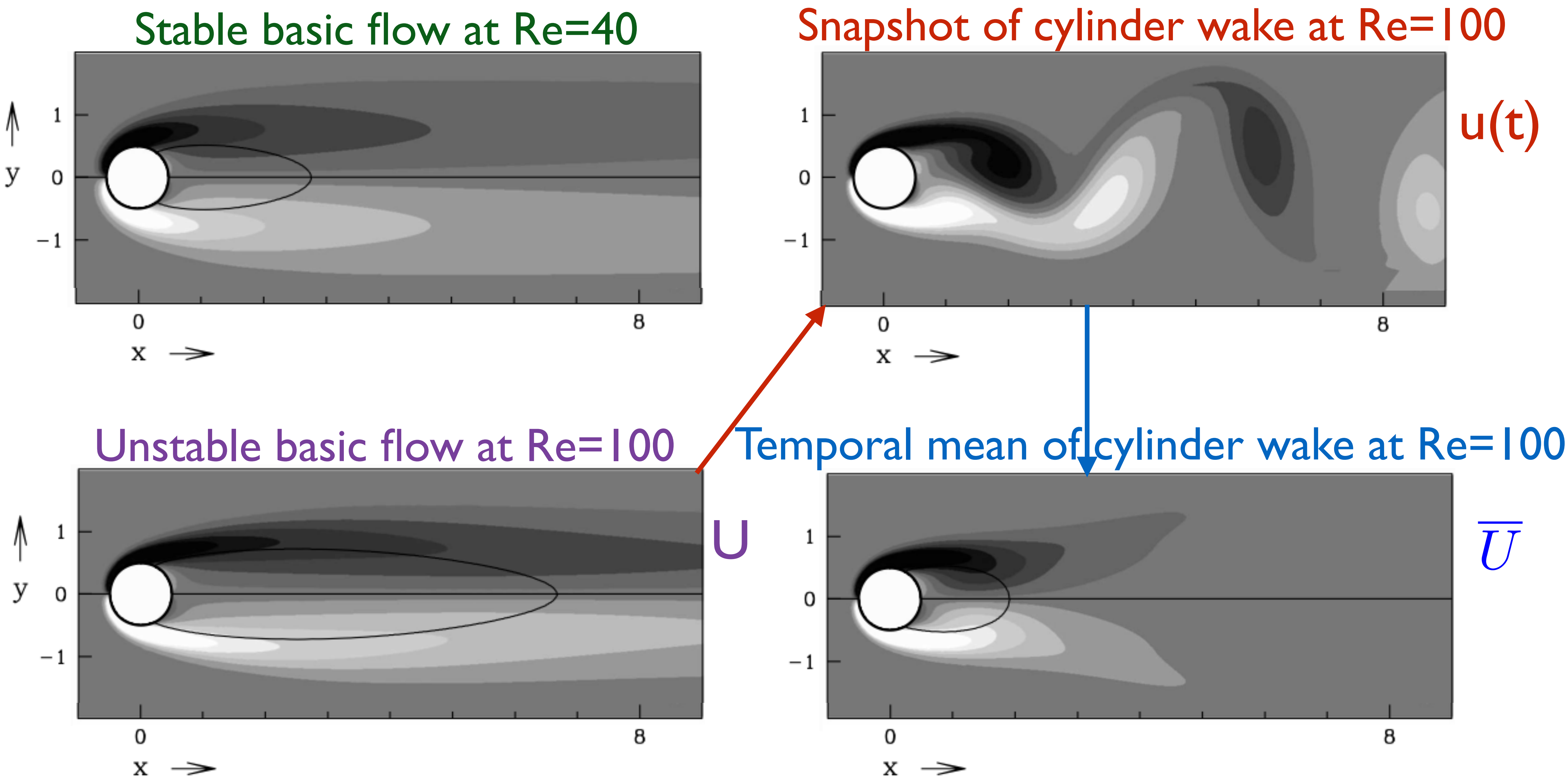


Basic flow

$$0 = -(U \cdot \nabla)U - \nabla P + \frac{1}{Re}\nabla^2 U$$

Temporally periodic wake flow

$$\partial_t u = -(u \cdot \nabla)u - \nabla p + \frac{1}{Re}\nabla^2 u$$

Temporal mean

$$\overline{U} \equiv \frac{1}{T}\int_0^T u(t)dt$$

Linearise about steady base flow

$$\partial_t u = -(U \cdot \nabla)u - (u \cdot \nabla)U - \nabla p + \frac{1}{Re}\nabla^2 u$$

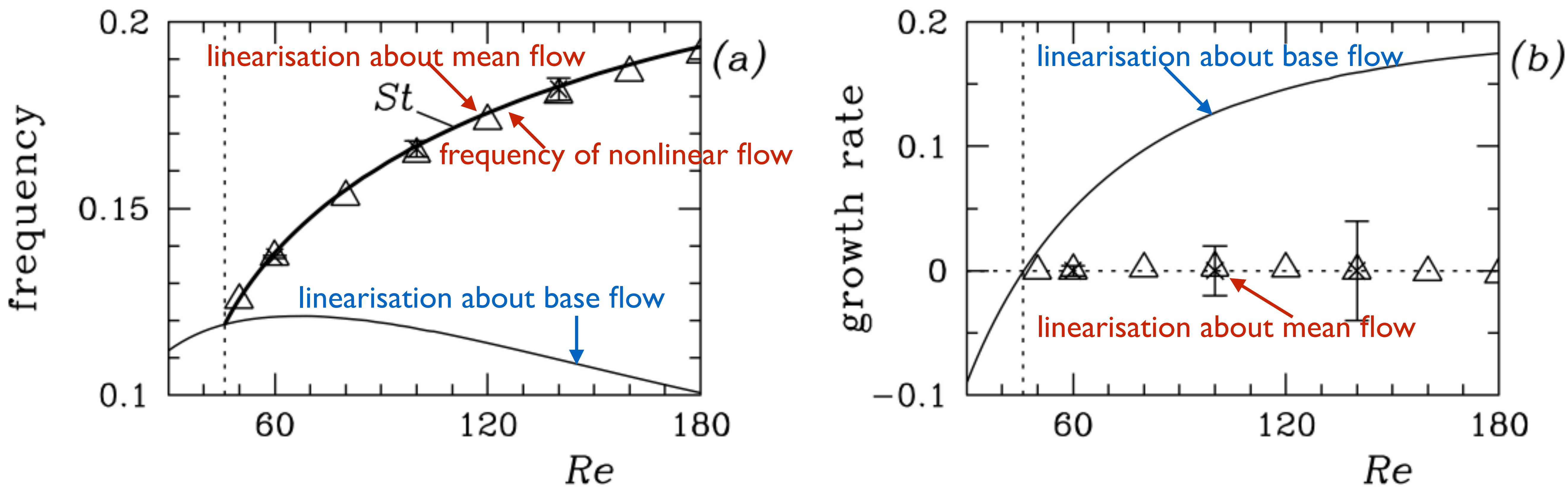
Linearise about temporal mean

$$\partial_t u = -(\overline{U} \cdot \nabla)u - (u \cdot \nabla)\overline{U} - \nabla p + \frac{1}{Re}\nabla^2 u$$

Strange and unjustified procedure, but quite successful !

Linear analysis of the cylinder wake mean flow

D. Barkley(*) *Europhys. Lett.*, **75** (5), pp. 750–756 (2006)



Mean flow eigenvalue has **RZIF** property:

Real part is near **Zero**.

Imaginary part is near exact nonlinear **Frequency**.

Linear analysis of the cylinder wake mean flow

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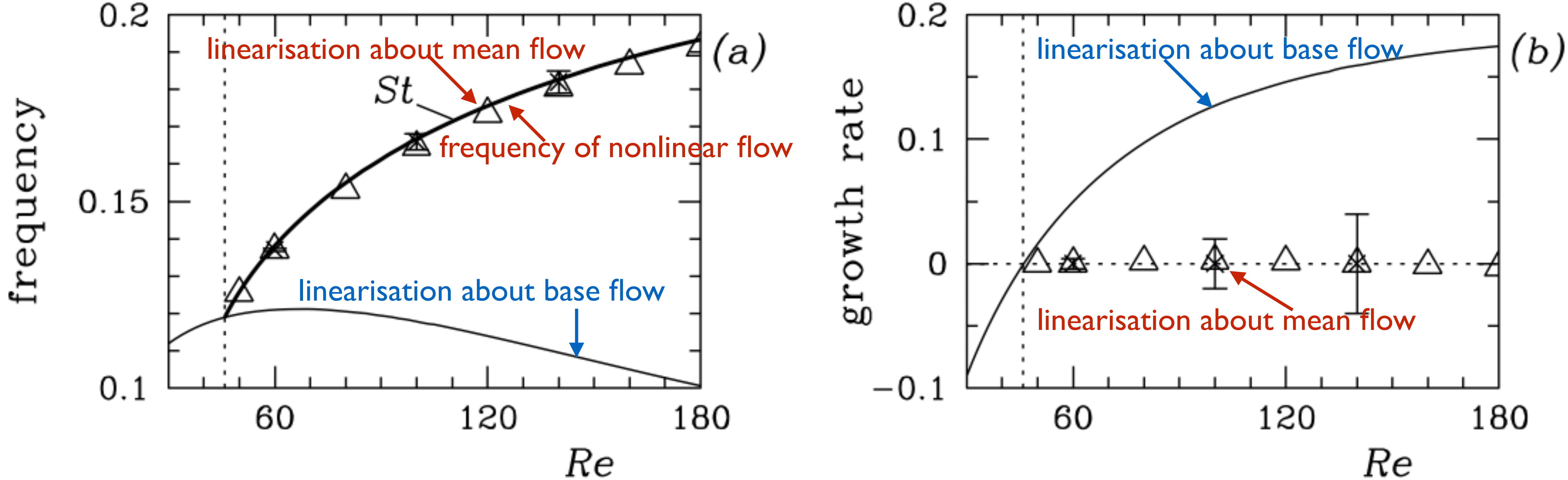
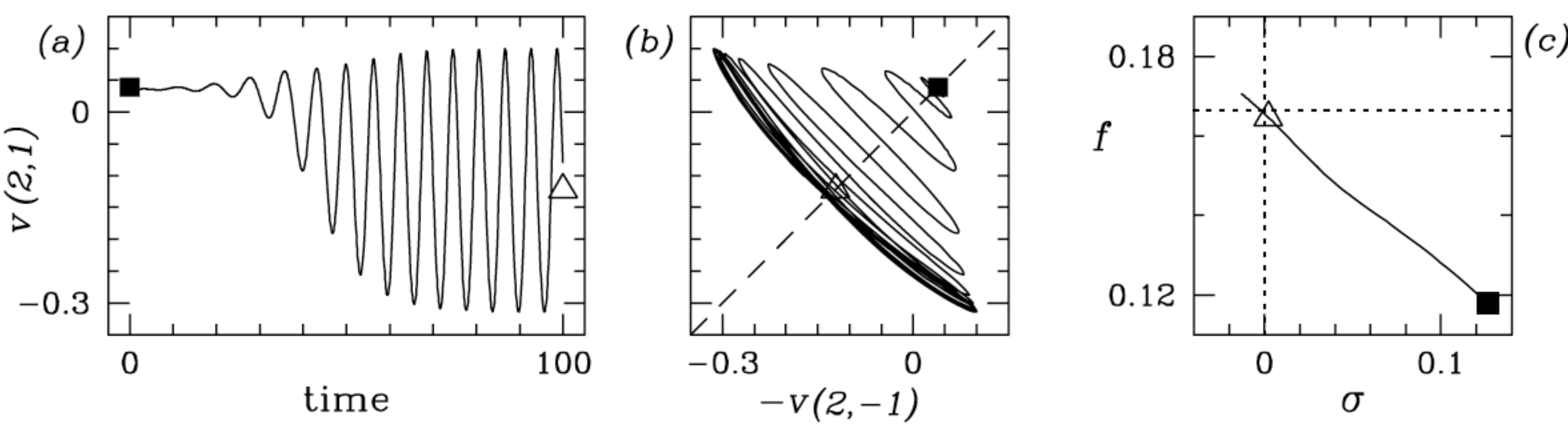




Malkus theory: Temporal mean of turbulent flow should be marginally stable

Outline of a theory of turbulent shear flow

W. V. R. Malkus

Journal of Fluid Mechanics / Volume 1 / Issue 05 / November 1956, pp 521 - 539

VOLUME 79, NUMBER 20 PHYSICAL REVIEW LETTERS 17 NOVEMBER 1997

Strongly Nonlinear Effect in Unstable Wakes

B. J. A. Zielinska,[2,1] S. Goujon-Durand,[1,3] J. Dušek,[4] and J. E. Wesfreid[1]
[1]*Ecole Supérieure de Physique et Chimie Industrielles de Paris (ESPCI), PMMH-URA CNRS No. 857,*

J. Fluid Mech. (2002), *vol.* 458, *pp.* 407–417.
DOI: 10.1017/S0022112002008054 Printed in the United Kingdom

On the frequency selection of finite-amplitude vortex shedding in the cylinder wake

By BENOÎT PIER
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, UK

J. Fluid Mech. (2003), *vol.* 497, *pp.* 335–363.
DOI: 10.1017/S0022112003006694 Printed in the United Kingdom

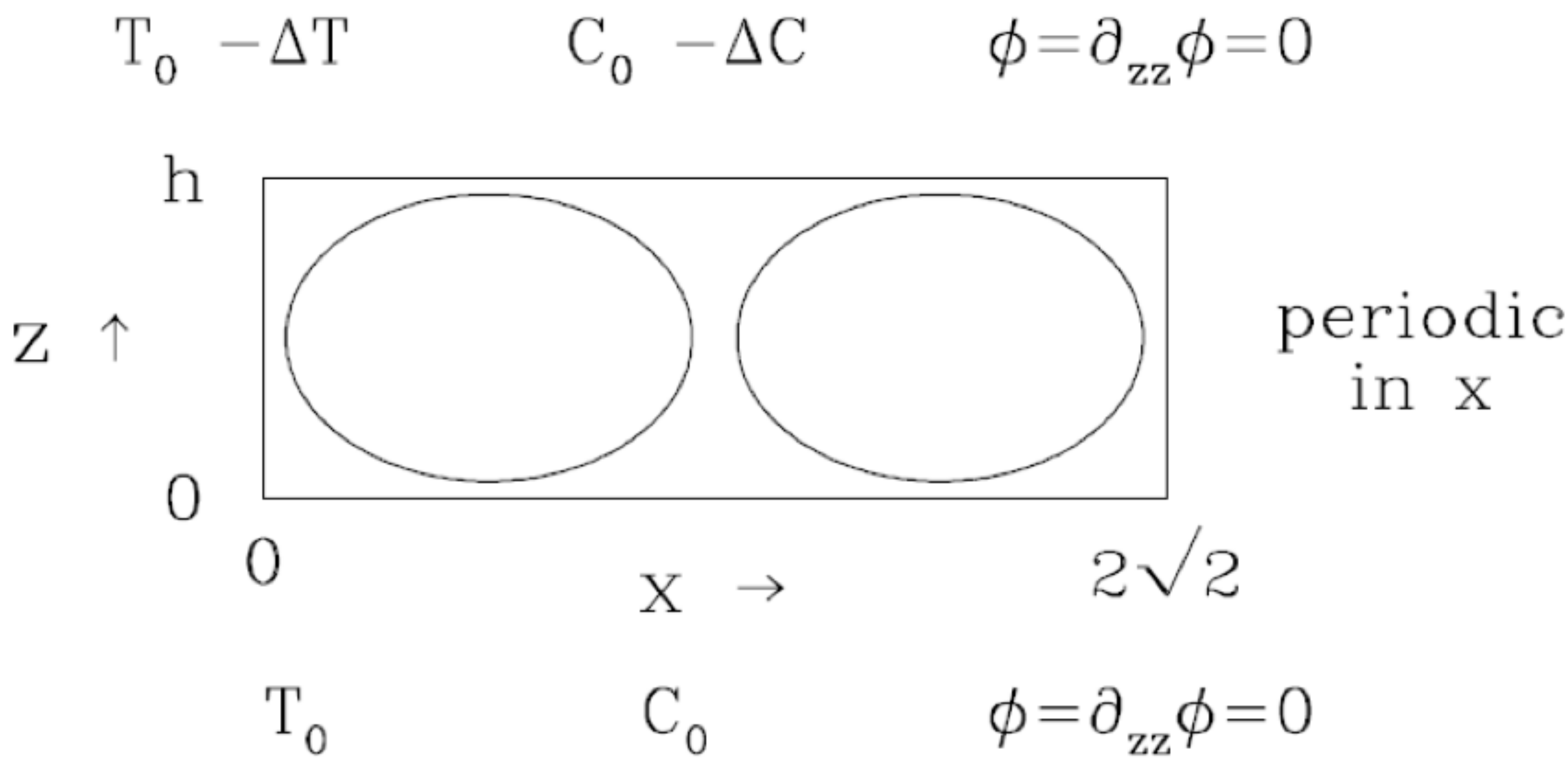
A hierarchy of low-dimensional models for the transient and post-transient cylinder wake

By BERND R. NOACK[1]†, KONSTANTIN AFANASIEV[2], MAREK MORZYŃSKI[3], GILEAD TADMOR[4] AND FRANK THIELE[1]

When is RZIF satisfied ?

Why is RZIF satisfied?

Simple Model: 2D Thermosolutal Problem



Vertical thermal and solutal gradients imposed at $z = 0, 1$

Boundary conditions: free-slip at $z = 0, 1$; periodic in x with length $2\sqrt{2}$

Streamfunction $\mathbf{U} = \boldsymbol{\nabla} \times \phi(x, z)\mathbf{e}_y$

Density: $\rho(T,C) = \rho_0 + \rho_T(T - T_0) + \rho_C(C - C_0)$

Diffusivities: κ_T (thermal), κ_C (solutal), ν (momentum)

Conductive solution:

$T = T_0 - z\Delta T/h, \qquad C = C_0 - z\Delta C/h, \qquad \mathrm{U} = \nabla \times \phi \mathrm{e}_y = 0$

Four nondimensional parameters:

Fix: Lewis number $L \equiv \frac{\kappa_C}{\kappa_T} \ll 1$ Prandtl number $P \equiv \frac{\nu}{\kappa_T} \gg 1$.

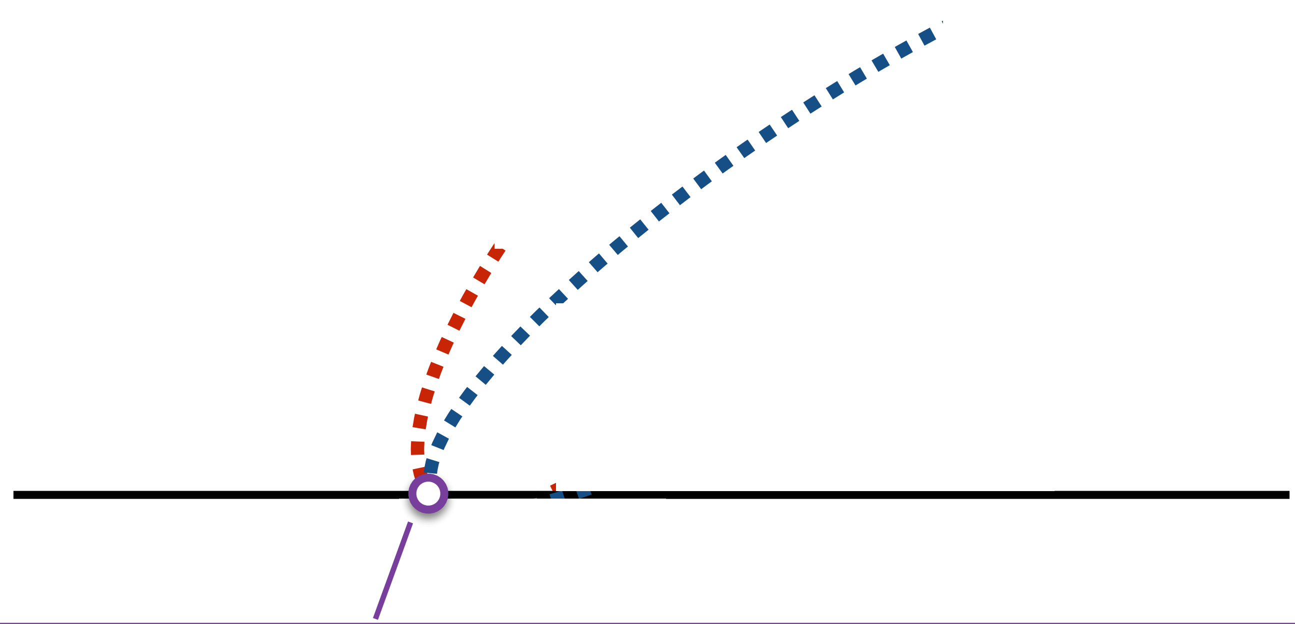
Vary: Rayleigh number $R \equiv \frac{g\rho_T \Delta T h^3}{\nu \kappa_T}$ Separation ratio $S \equiv \frac{\rho_C \Delta C}{\rho_T \Delta T}$

Subtract conductive solution and nondimensionalize.

Governing Equations:

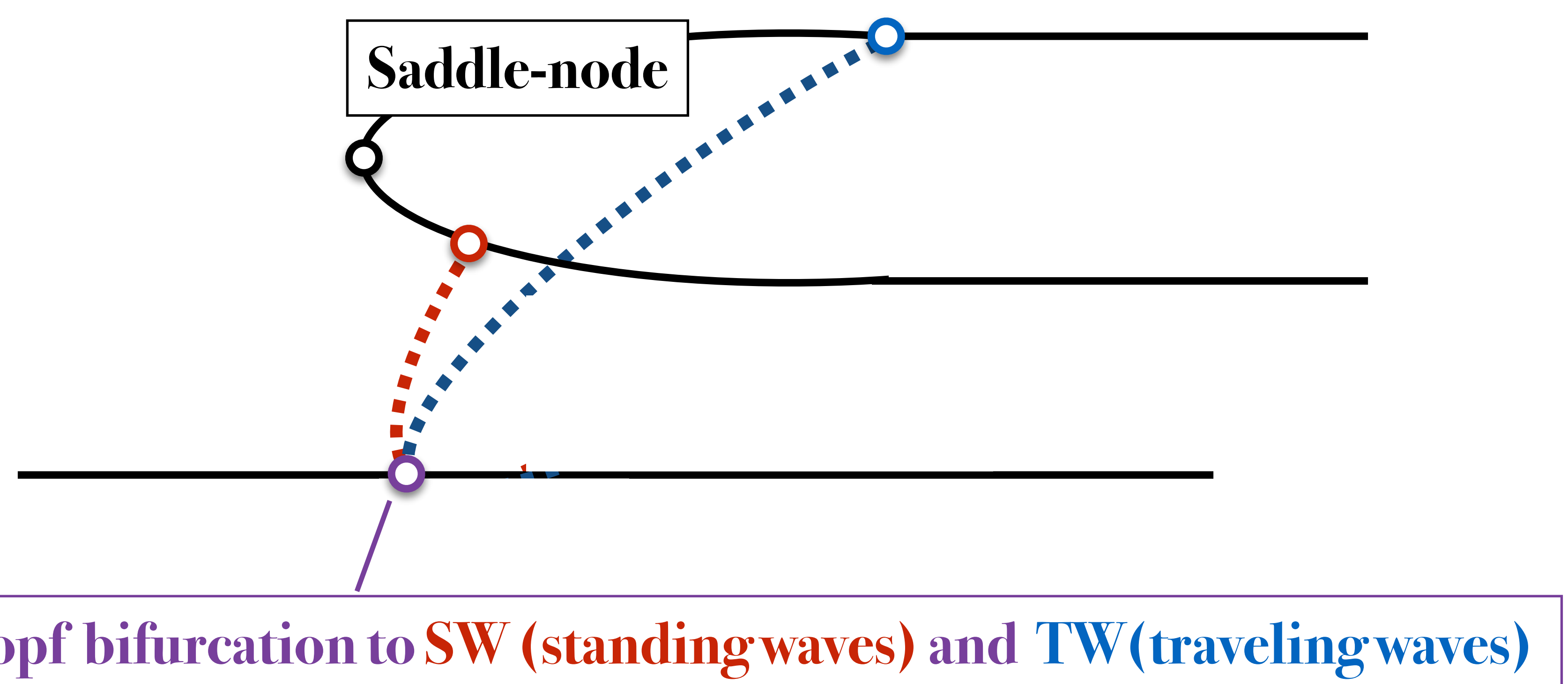
$$\begin{aligned}
\partial_t \tilde{T} &= \partial_x \tilde{\phi} + \mathrm{e_y} \cdot (\nabla \tilde{\phi} \times \nabla \tilde{T}) + \nabla^2 \tilde{T} \\
\partial_t \tilde{C} &= \partial_x \tilde{\phi} + \mathrm{e_y} \cdot (\nabla \tilde{\phi} \times \nabla \tilde{C}) + L\nabla^2 \tilde{C} \\
\partial_t \nabla^2 \tilde{\phi} &= PR\partial_x(\tilde{T} + S\tilde{C}) + \mathrm{e_y} \cdot (\nabla \tilde{\phi} \times \nabla \nabla^2 \tilde{\phi}) + P\nabla^4 \tilde{\phi}
\end{aligned}$$

E. Knobloch, J. Swift 1980s:
Hopf bifurcation breaking O(2) symmetry



Hopf bifurcation to SW (standing waves) and TW (traveling waves)

E. Knobloch, J. Swift 1980s:
Hopf bifurcation breaking O(2) symmetry



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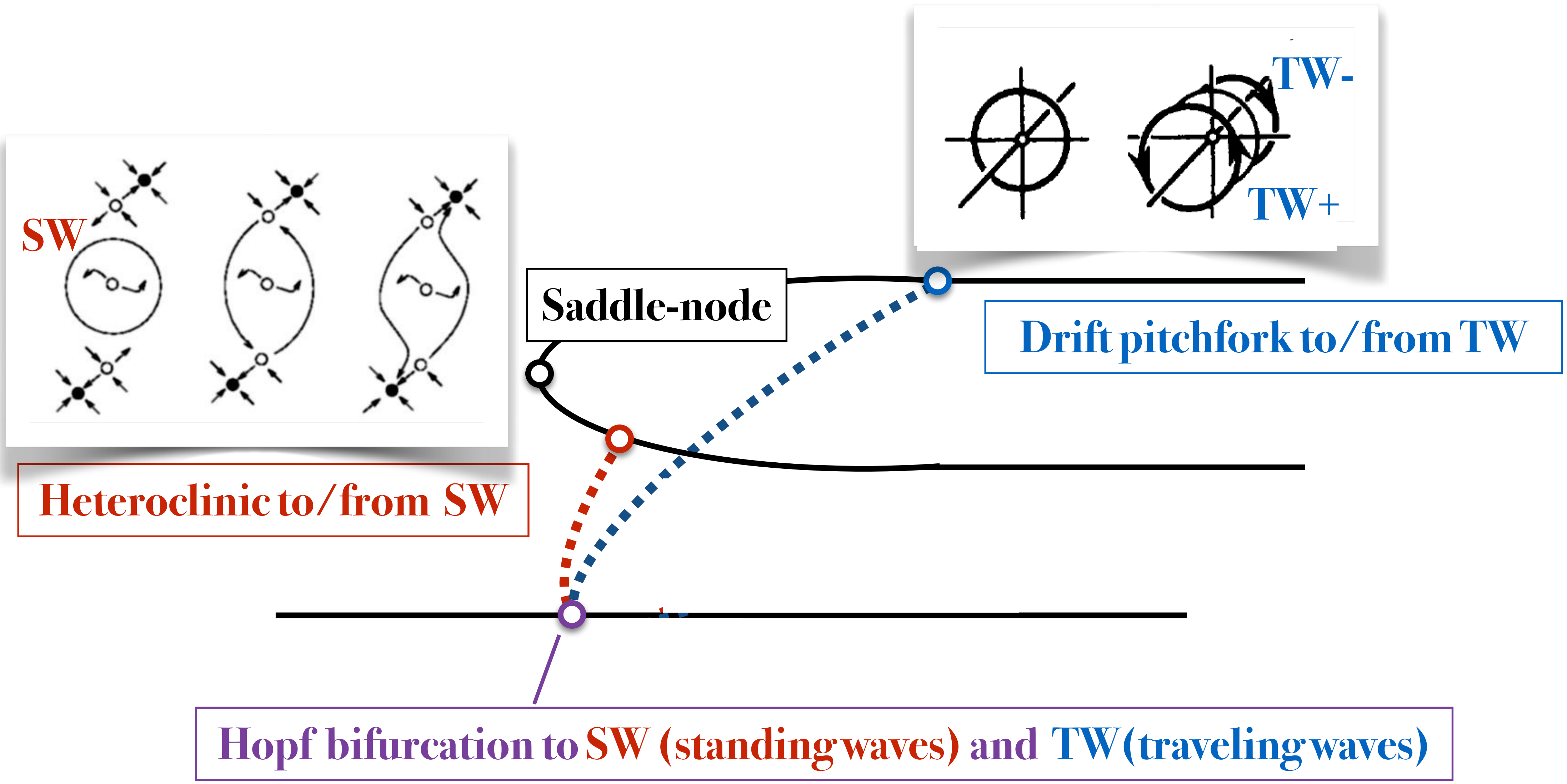
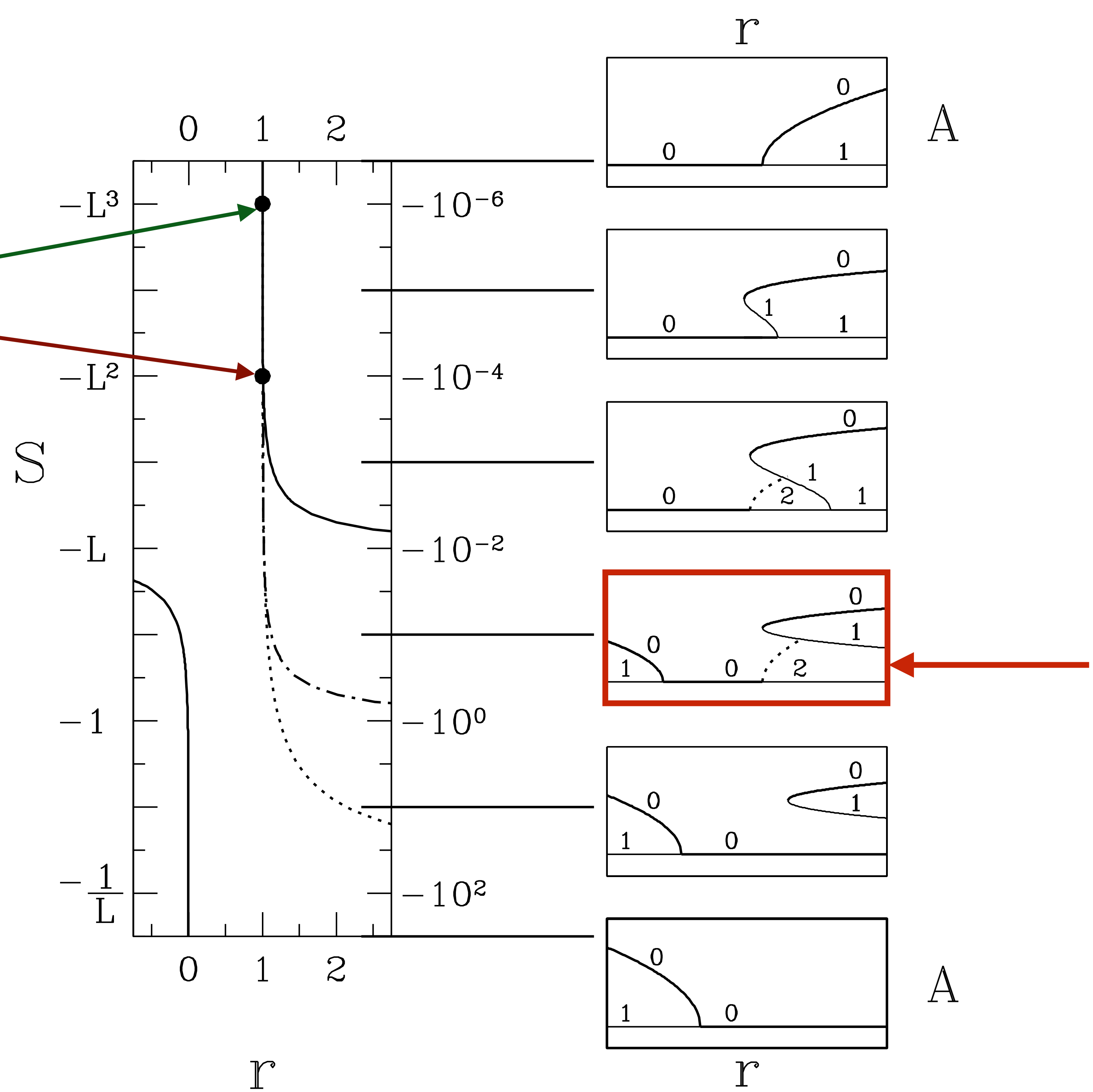


Thermosolutal Convection

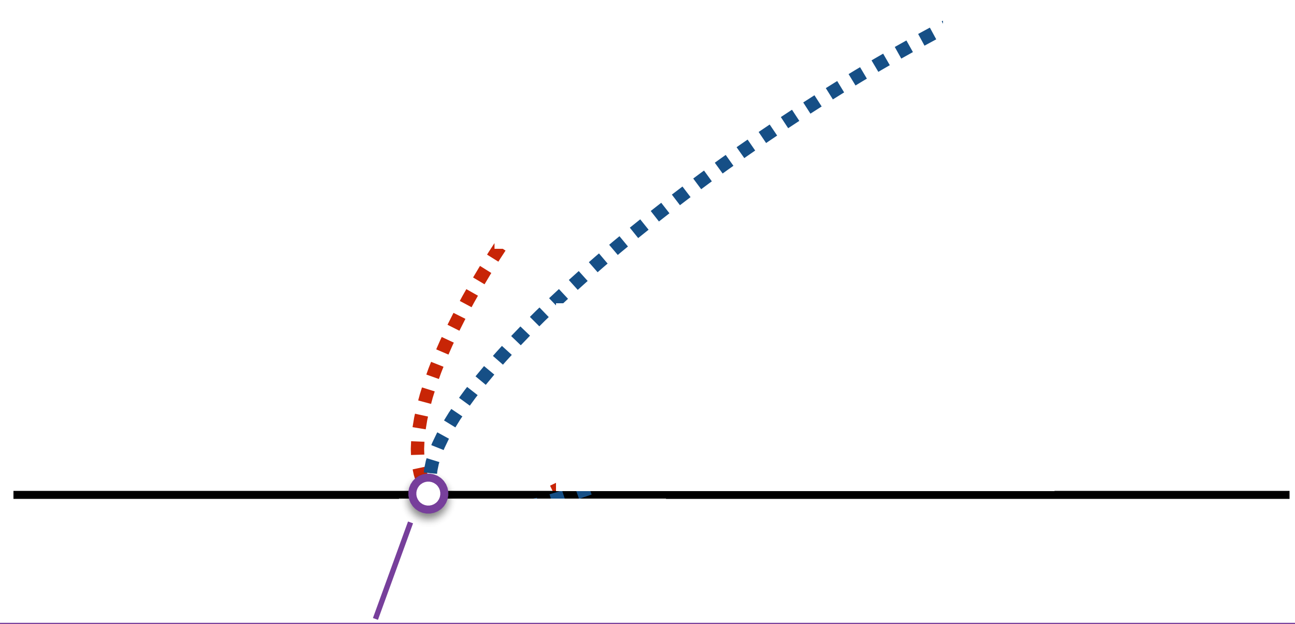
super/sub pitchfork
(nonlinear codim-2 point)

Hopf/pitchfork
Takens-Bogdanov
(linear codim-2 point)



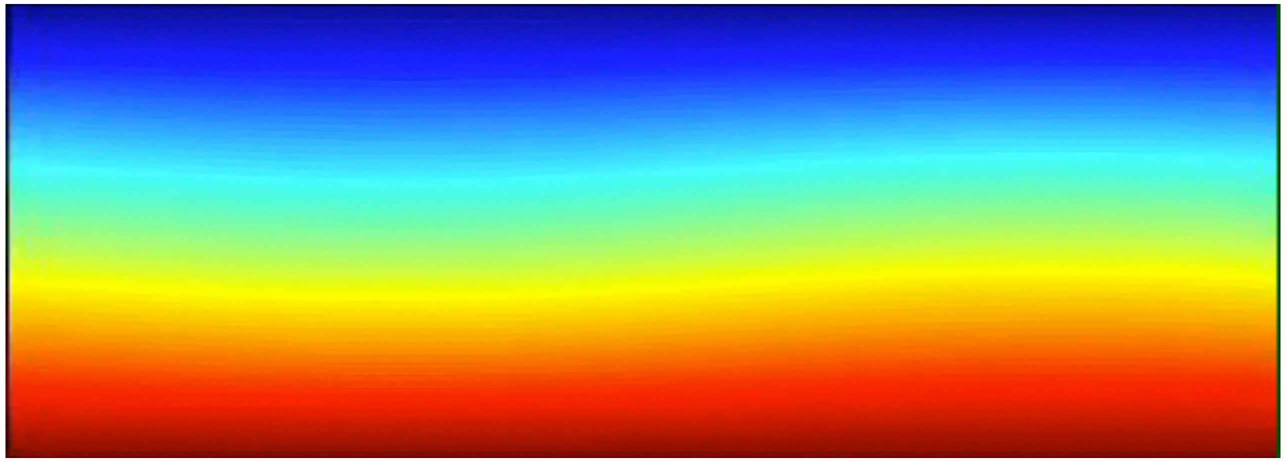
Tuckerman, Physica D (2001)

E. Knobloch, J. Swift 1980s:
Hopf bifurcation breaking O(2) symmetry

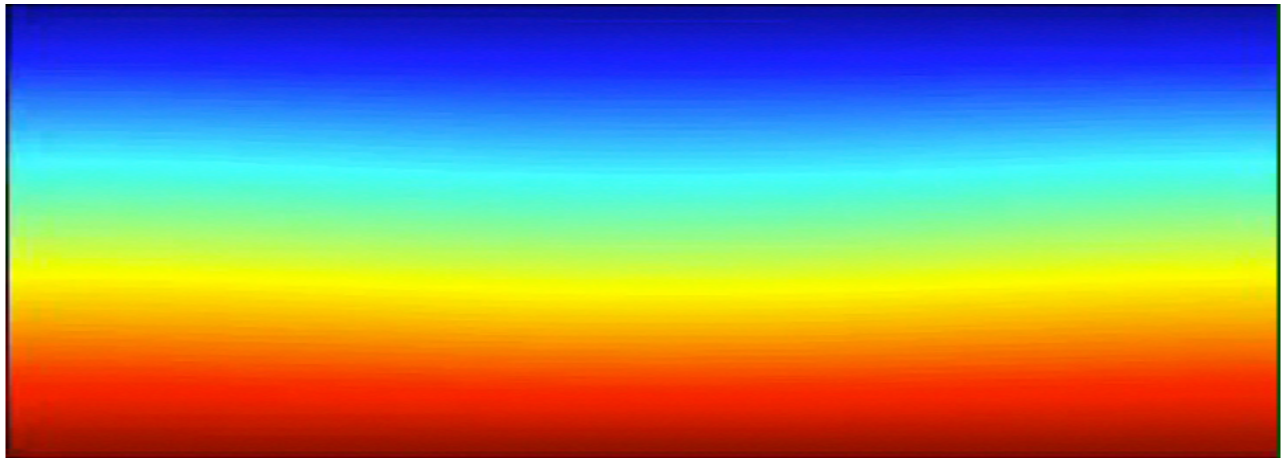


Hopf bifurcation to SW (standing waves) and TW (traveling waves)

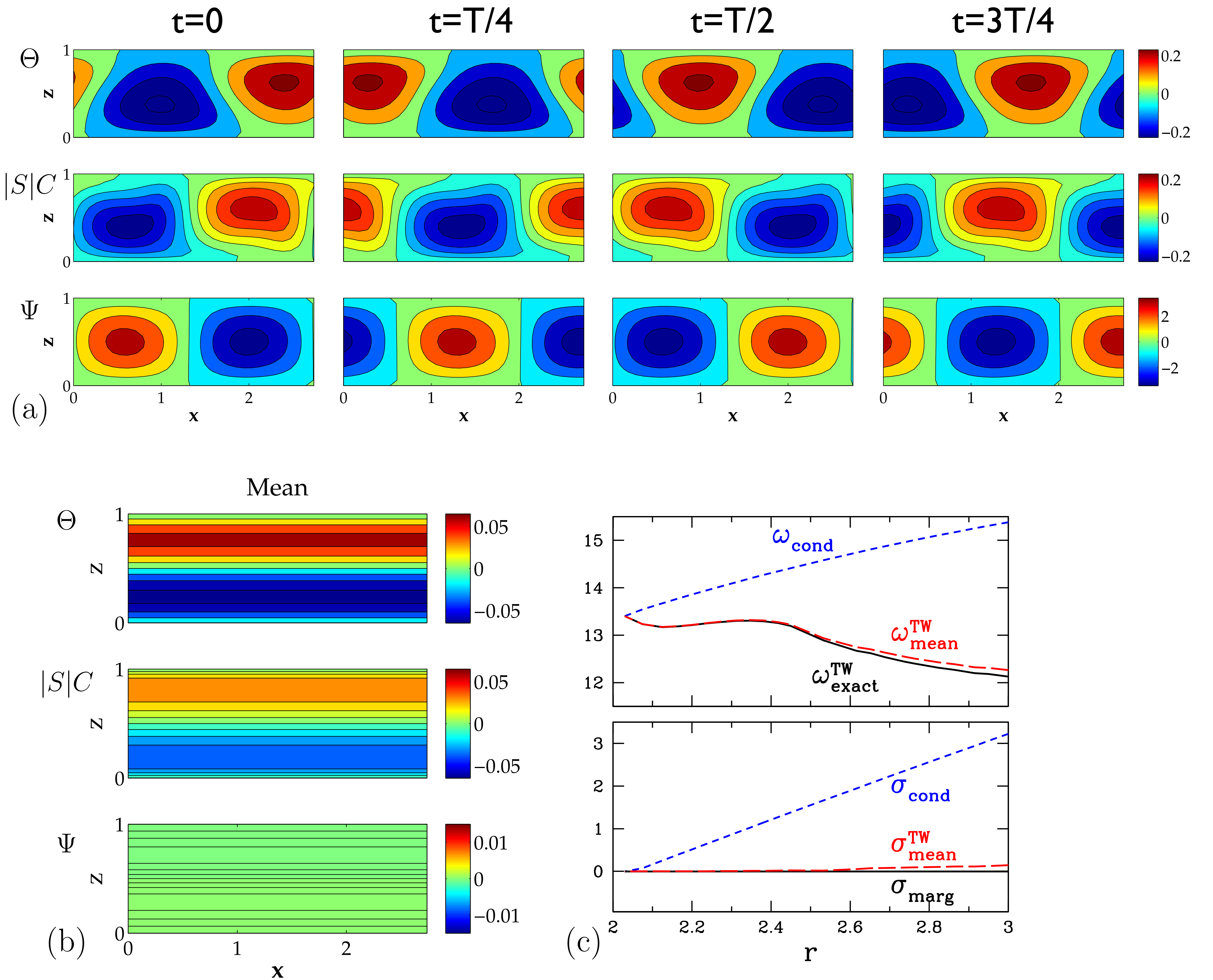
Traveling wave

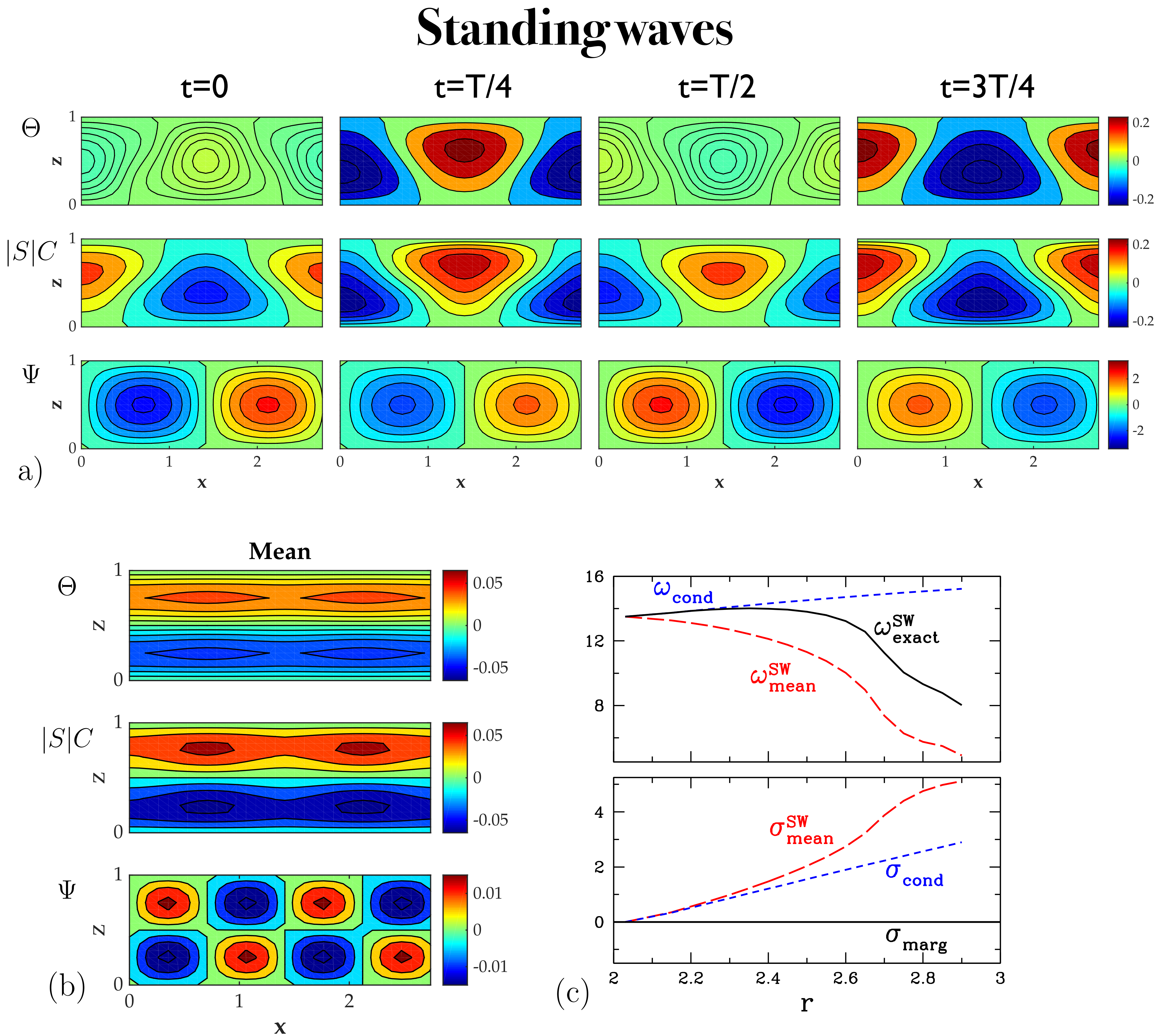


Standing wave



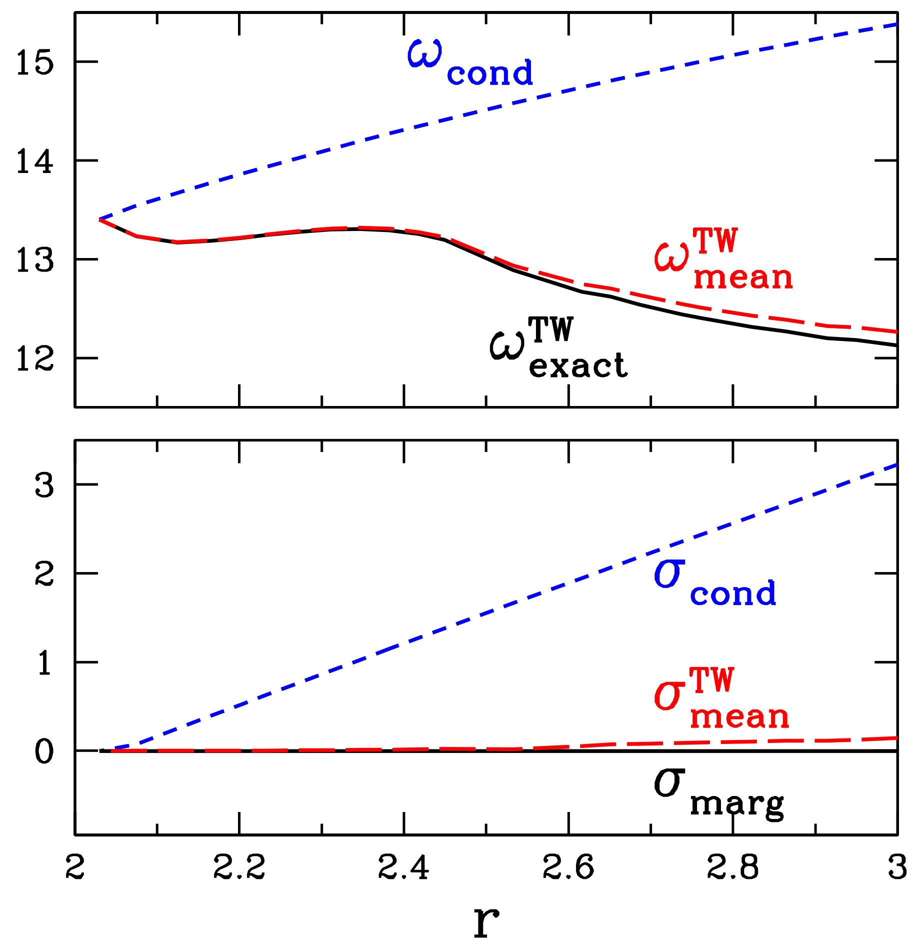
Traveling waves





Standing waves
t=0
t=T/4
t=T/2
t=3T/4
Θ
|S|C
Ψ
z
x
a)
Mean
(b)
(c)
ω cond
ω SW exact
ω SW mean
σ SW mean
σ cond
σ marg
r

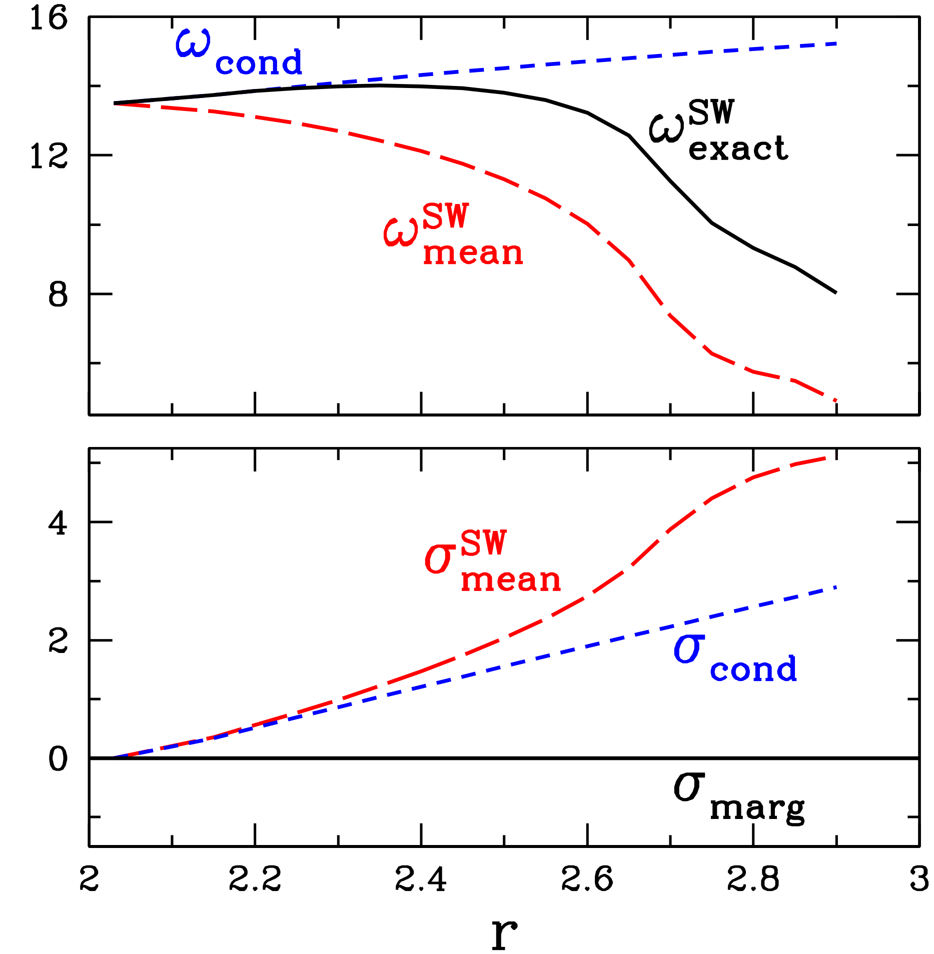
Traveling waves



mean = exact

RZIF

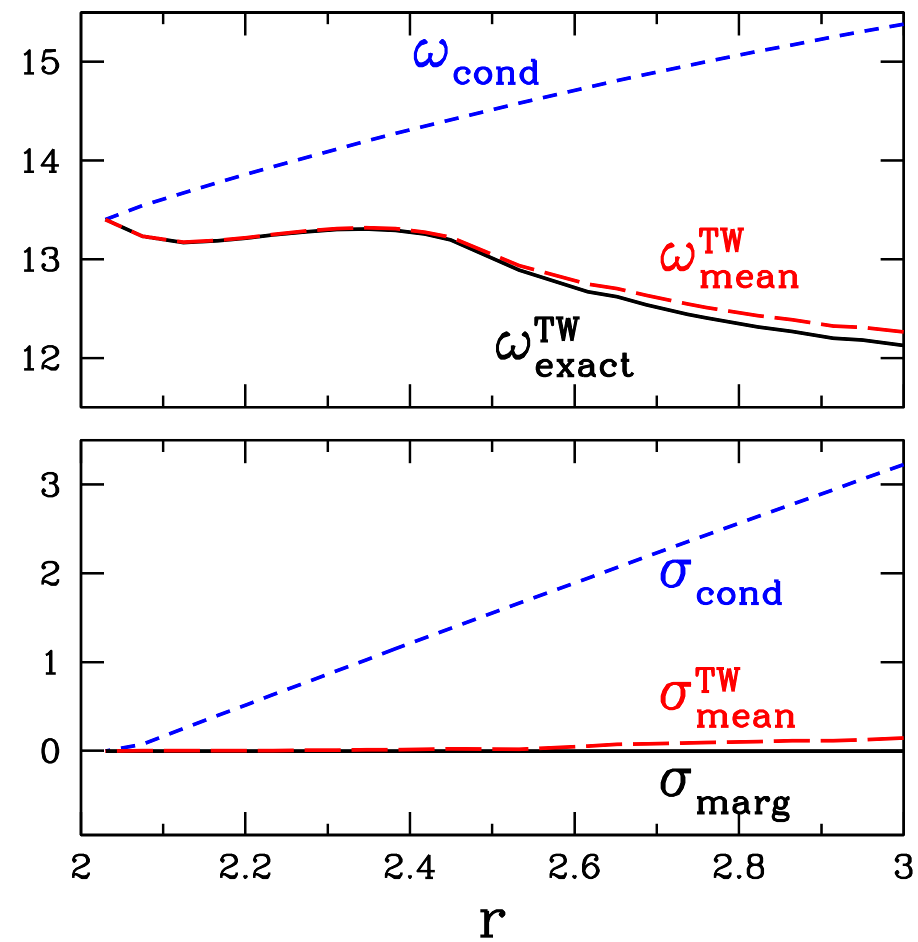
Standing waves



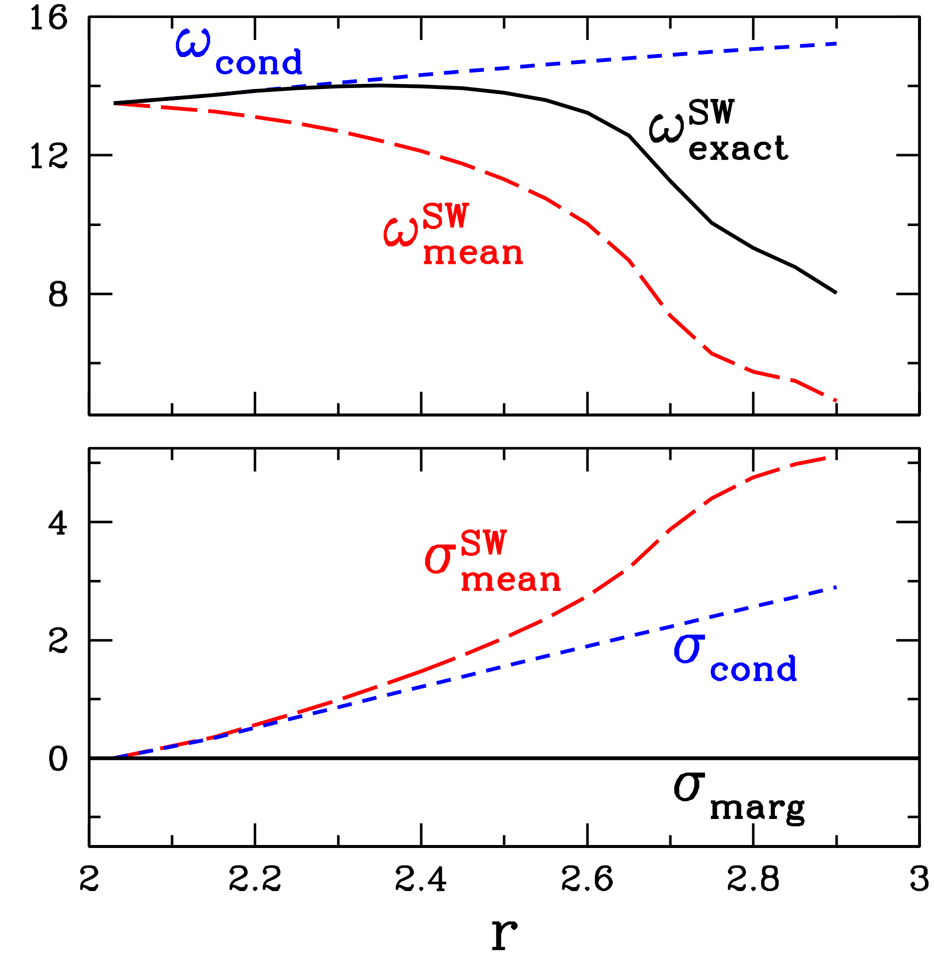
mean ≠ exact

~~**RZIF**~~

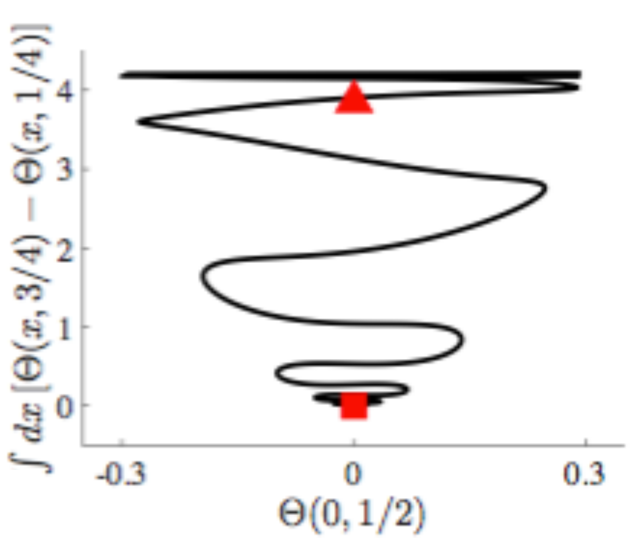
Traveling waves



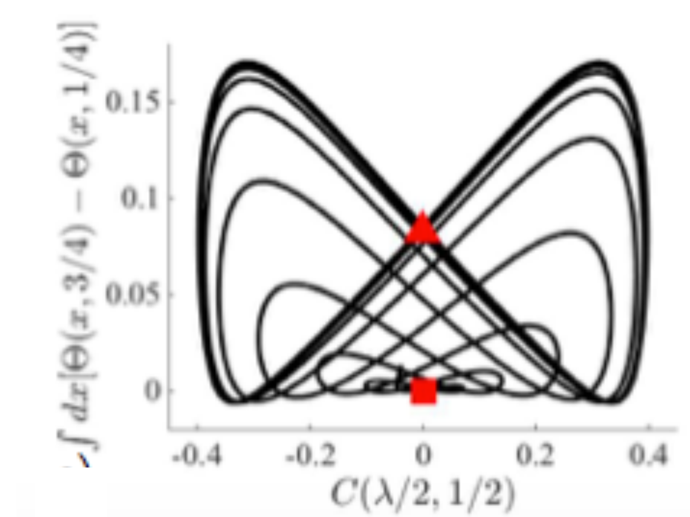
Standing waves



mean = exact



mean ≠ exact



Evolution equation:

$$\partial_t \mathbf{U} = \mathcal{L}\mathbf{U} + \mathcal{N}(\mathbf{U}, \mathbf{U})$$

Temporal Fourier decomposition:

$$\mathbf{U} = \overline{\mathbf{U}} + \sum_{n \neq 0} \mathbf{u}_n e^{in\omega t}$$

Substitute into evolution equation

Component 0:

$$0 = \mathcal{L}\overline{\mathbf{U}} + \mathcal{N}(\overline{\mathbf{U}}, \overline{\mathbf{U}}) + \sum_{m \neq 0} \mathcal{N}(\mathbf{u}_m, \mathbf{u}_{-m})$$

Component 1:

$$i\omega \mathbf{u}_1 = \underbrace{\mathcal{L}\mathbf{u}_1 + \mathcal{N}(\overline{\mathbf{U}}, \mathbf{u}_1) + \mathcal{N}(\mathbf{u}_1, \overline{\mathbf{U}})}_{\mathcal{L}_{\overline{\mathbf{U}}}\mathbf{u}_1} + \underbrace{\mathcal{N}(\mathbf{u}_2, \mathbf{u}_{-1}) + \mathcal{N}(\mathbf{u}_{-1}, \mathbf{u}_2) + \ldots}_{\text{small?}}$$

$$\mathcal{N}_1$$

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$$\mathcal{N}_1$$

Justification:

Assume: $||u_n|| \sim \epsilon^{|n|}$

Then: $$\underbrace{i\omega \mathbf{u}_1}_{\epsilon} - \underbrace{\mathcal{L}_{\overline{\mathbf{U}}}\mathbf{u}_1}_{\epsilon} = \underbrace{\mathcal{N}_1}_{\epsilon^3}$$

Applies only to component 1, since:

$$\underbrace{i2\omega \mathbf{u}_2 - \mathcal{L}_{\overline{\mathbf{U}}}\mathbf{u}_2}_{\epsilon^2} = \underbrace{\mathcal{N}_2}_{\epsilon^2} \qquad \underbrace{i3\omega \mathbf{u}_3 - \mathcal{L}_{\overline{\mathbf{U}}}\mathbf{u}_3}_{\epsilon^3} = \underbrace{\mathcal{N}_3}_{\epsilon^3}$$

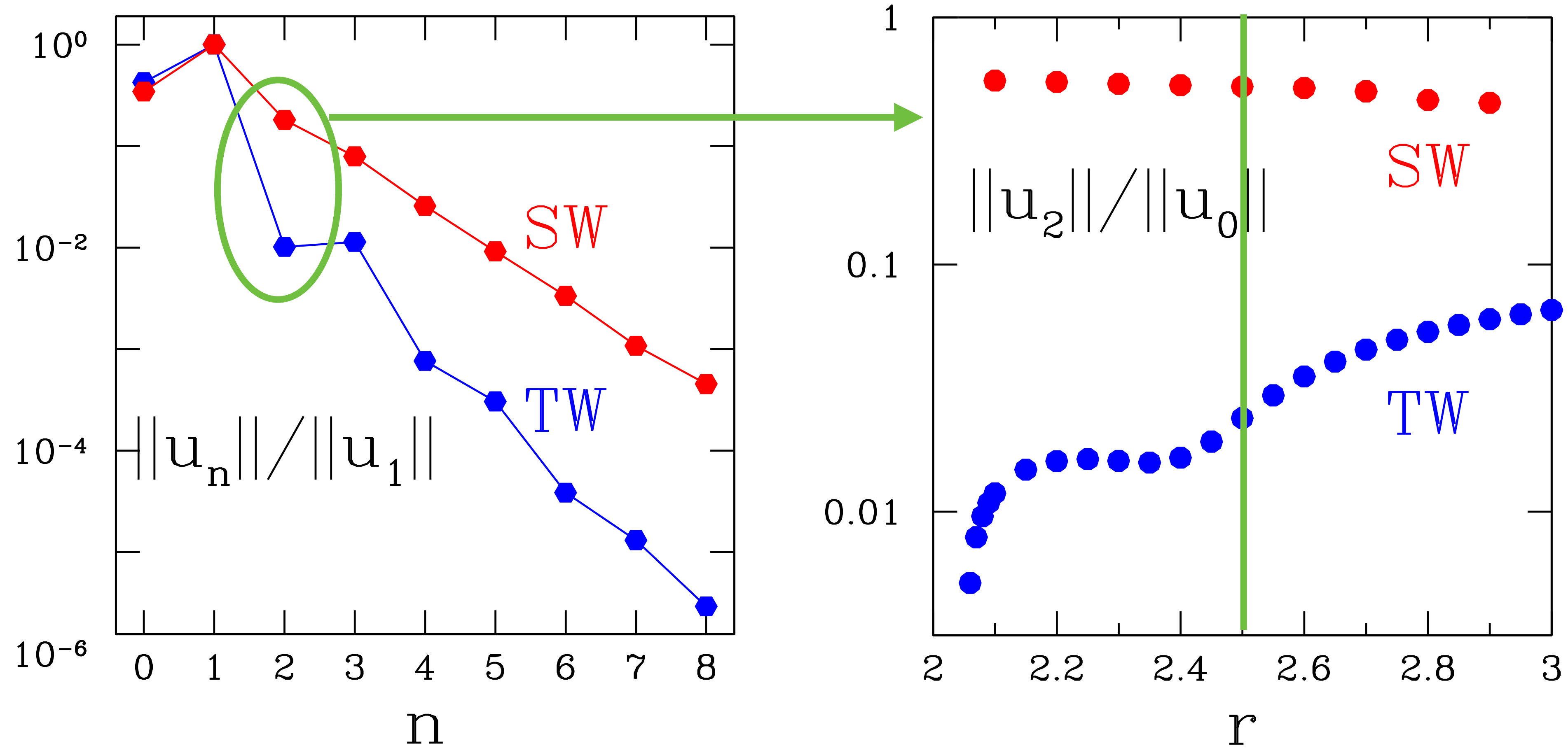
$$||u_n|| \sim \epsilon^{|n|} \implies \mathcal{N}_1 \ll (i\omega - \mathcal{L}_{\bar{U}})u_1$$

Mean flow eigenvalue has **RZIF** property:
Real part is near **Zero**.
Imaginary part is near exact nonlinear **Frequency**.

Do **TW** generally have **highly peaked** temporal spectra?
Do **SW** generally have **broad** temporal spectra?

Yes!

temporal spectrum for r=2.5



And **WHY** do **TW** have **highly peaked** temporal spectra?

And **WHY** do **SW** have **broad** temporal spectra?

We've had **RZIF** = Real Zero Imaginary Frequency

Now introduce **SCM** = Self-Consistent Model

Other terms:

Harmonic balance:
decomposing equations and fields into Fourier components

Quasilinear and generalized quasilinear approximation
quasilinear ≈ SCM

PRL 113, 084501 (2014) PHYSICAL REVIEW LETTERS week ending 22 AUGUST 2014

Self-Consistent Mean Flow Description of the Nonlinear Saturation of the Vortex Shedding in the Cylinder Wake

Vladislav Mantič-Lugo,* Cristóbal Arratia,† and François Gallaire
Laboratory of Fluid Mechanics and Instabilities, École Polytechnique Fédérale de Lausanne, EPFL-STI-IGM-LFMI, Lausanne CH-1015, Switzerland
(Received 19 December 2013; published 20 August 2014)

Mean flows without time integration:

$$0 = \mathcal{L}\overline{\mathbf{U}} + \mathcal{N}(\overline{\mathbf{U}}, \overline{\mathbf{U}}) + \sum_{m \neq 0} \mathcal{N}(\mathbf{u}_m, \mathbf{u}_{-m}) \rightarrow \mathcal{N}(\mathbf{u}_1, \mathbf{u}_{-1})$$

$$(\sigma + i\omega)\mathbf{u}_1 = \mathcal{L}_{\bar{U}}\mathbf{u}_1 + \mathcal{N}_1 \qquad \|u_1\| = A$$

Find A such that $\sigma = 0$

Yields almost exact $\bar{\mathbf{U}}, \mathbf{u}_1, \omega$!

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Solve for $\bar{\mathbf{U}}$ with Newton's method

$$(\sigma + i\omega)\mathbf{u}_1 = \mathcal{L}_{\bar{U}}\mathbf{u}_1 + \mathcal{N}_1 \qquad \|u_1\| = A$$

Solve for ($\mathbf{u}_1, \sigma, \omega$) by diagonalisation

Find A such that $\sigma = 0$

Yields almost exact $\bar{\mathbf{U}}, \mathbf{u}_1, \omega$!

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$$0 = \mathcal{L}\overline{\mathbf{U}} + \mathcal{N}(\overline{\mathbf{U}}, \overline{\mathbf{U}}) + \sum_{m \neq 0} \mathcal{N}(\mathbf{u}_m, \mathbf{u}_{-m}) \quad \rightarrow \quad \mathcal{N}(\mathbf{u}_1, \mathbf{u}_{-1})$$

~~Solve for $\bar{\mathrm{U}}$ with Newton's method~~

$$(\sigma + i\omega)\mathbf{u}_1 = \mathcal{L}_{\bar{U}}\mathbf{u}_1 + \mathcal{N}_1 \qquad \|u_1\| = A$$

~~Solve for ($\mathbf{u}_1, \sigma, \omega$) by diagonalisation~~

Find A such that $\sigma = 0$

Yields almost exact $\bar{\mathrm{U}}, \mathbf{u}_1, \omega$!

Much better to solve single coupled problem with Newton's method

PRL 113, 084501 (2014) PHYSICAL REVIEW LETTERS week ending 22 AUGUST 2014

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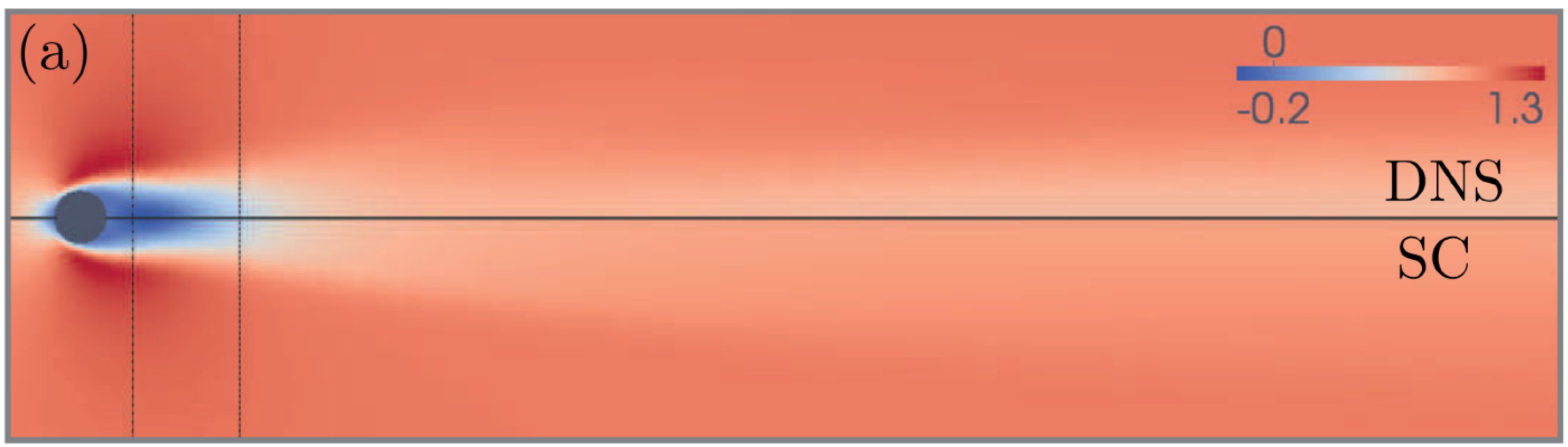

PRL 113, 084501 (2014) PHYSICAL REVIEW LETTERS week ending 22 AUGUST 2014

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(Received 19 December 2013; published 20 August 2014)

J. Fluid Mech. (2016), *vol.* 793, *pp.* 777–797. © Cambridge University Press 2016 777
doi:10.1017/jfm.2016.109

Self-consistent model for the saturation mechanism of the response to harmonic forcing in the backward-facing step flow

V. Mantič-Lugo[1,†] and F. Gallaire[1]

J. Fluid Mech. (2016), *vol.* 800, *pp.* 327–357. © Cambridge University Press 2016 327
doi:10.1017/jfm.2016.390

A self-consistent formulation for the sensitivity analysis of finite-amplitude vortex shedding in the cylinder wake

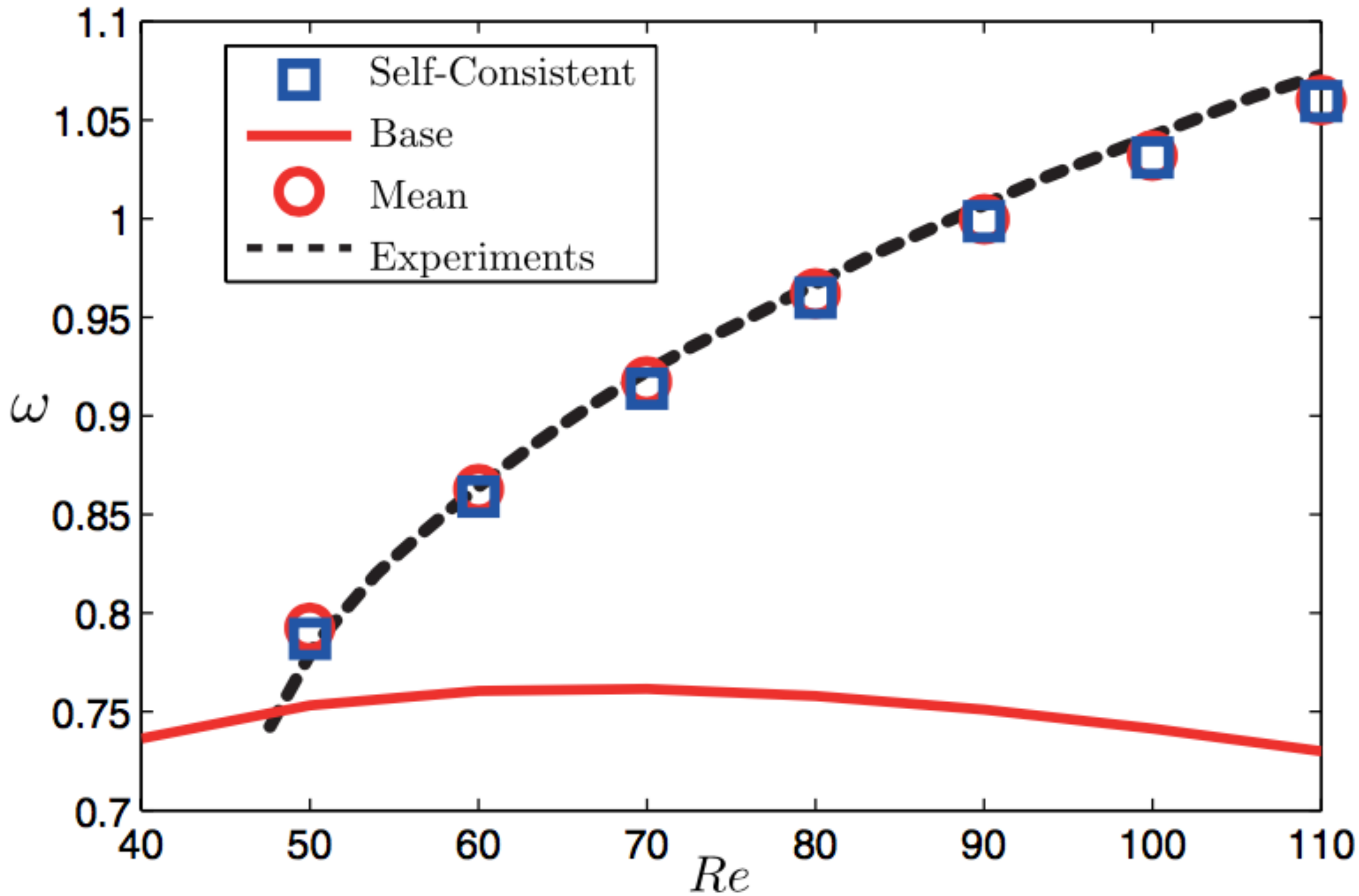
P. Meliga[1,†], E. Boujo[2,‡] and F. Gallaire[2]

PHYSICS OF FLUIDS **27**, 074103 (2015)

A self-consistent model for the saturation dynamics of the vortex shedding around the mean flow in the unstable cylinder wake

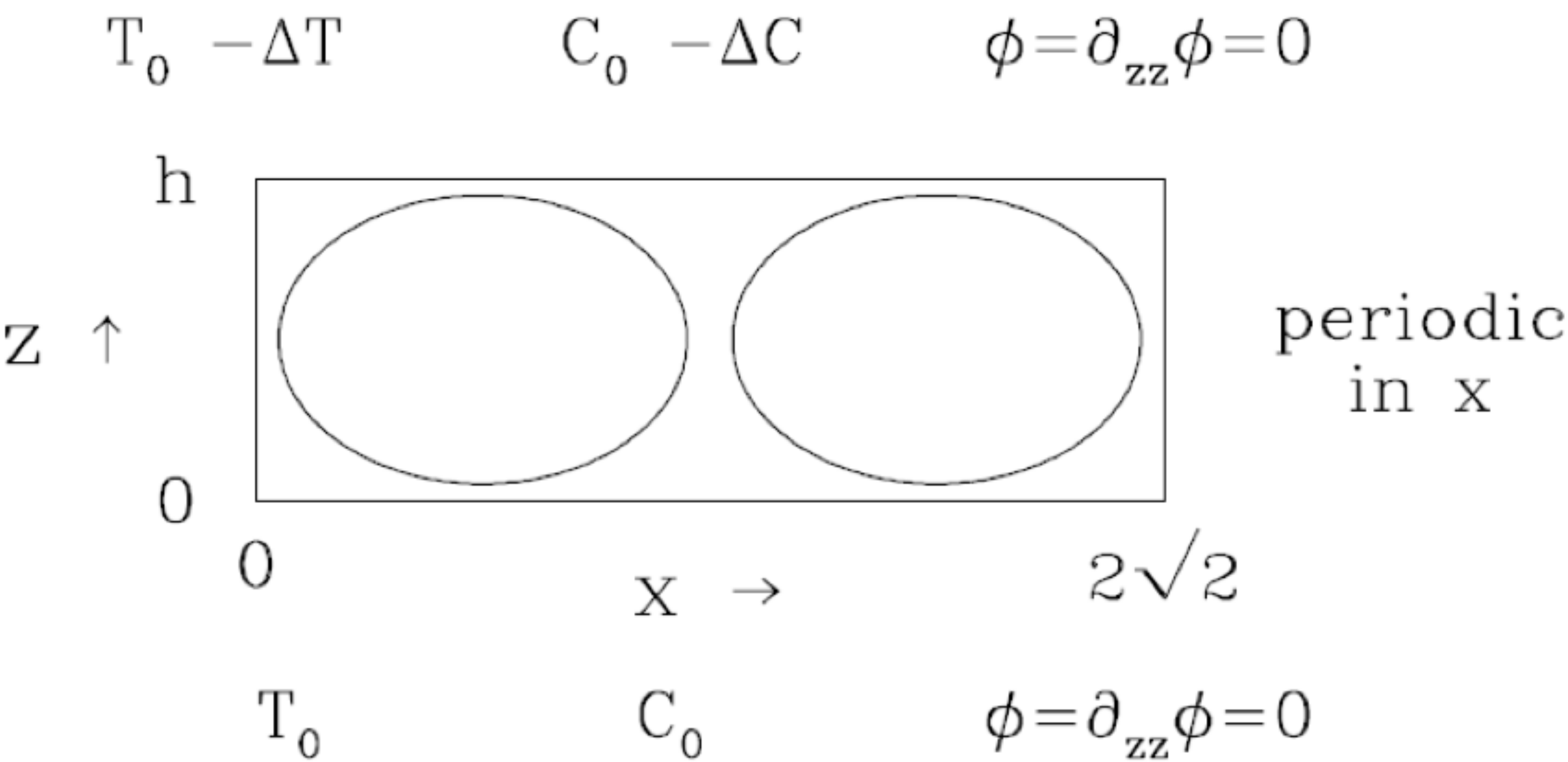
Vladislav Mantič-Lugo,[1,a)] Cristóbal Arratia,[1,2,b)] and François Gallaire[1,c)]

SCM vs RZIF vs exact for cylinder wake



Exact nonlinear frequency RZIF and SCM all agree

Simple Model: 2D Thermosolutal Problem

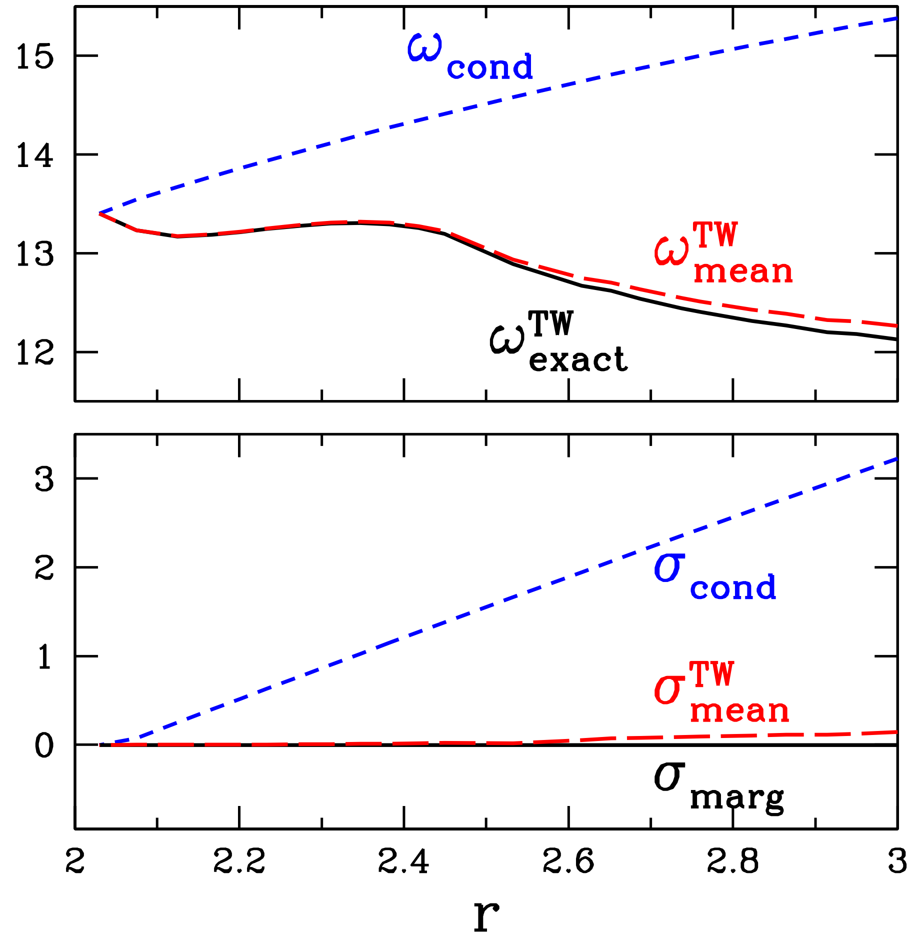


Vertical thermal and solutal gradients imposed at $z = 0, 1$

Boundary conditions: free-slip at $z = 0, 1$; periodic in x with length $2\sqrt{2}$

Streamfunction $\mathrm{U} = \boldsymbol{\nabla} \times \phi(x, z)\mathbf{e}_y$

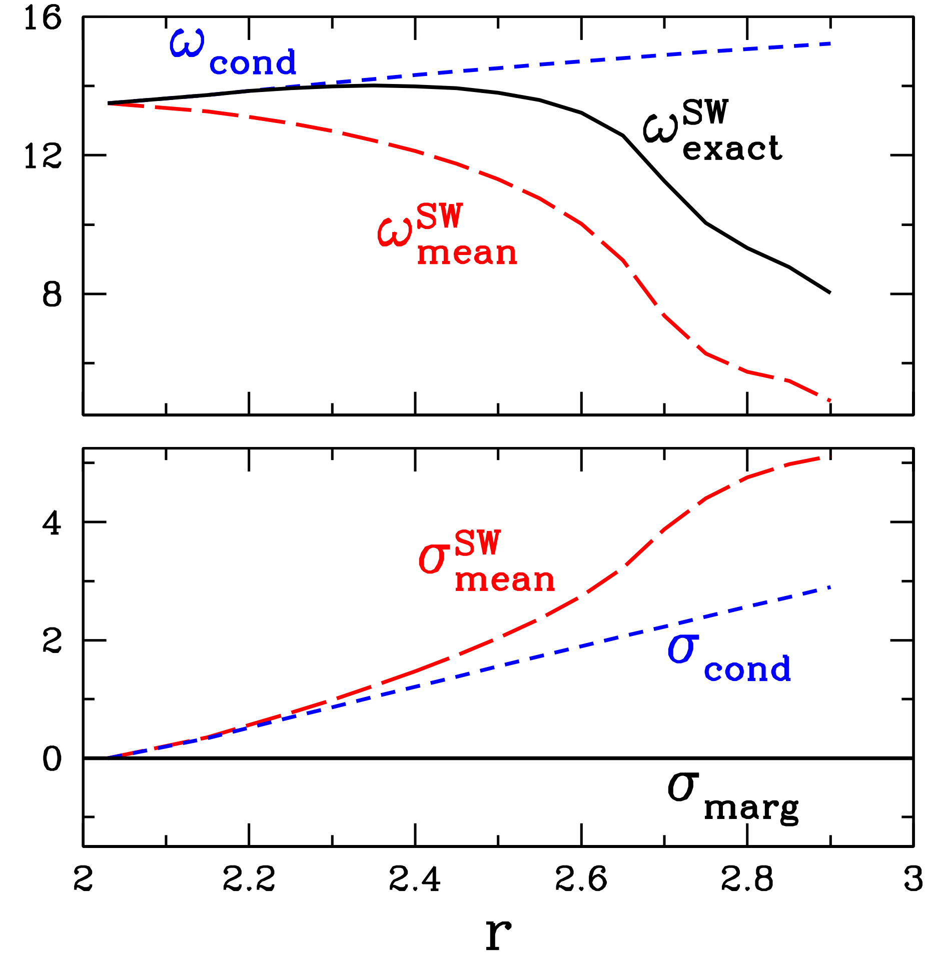
Traveling waves



mean = exact

RZIF

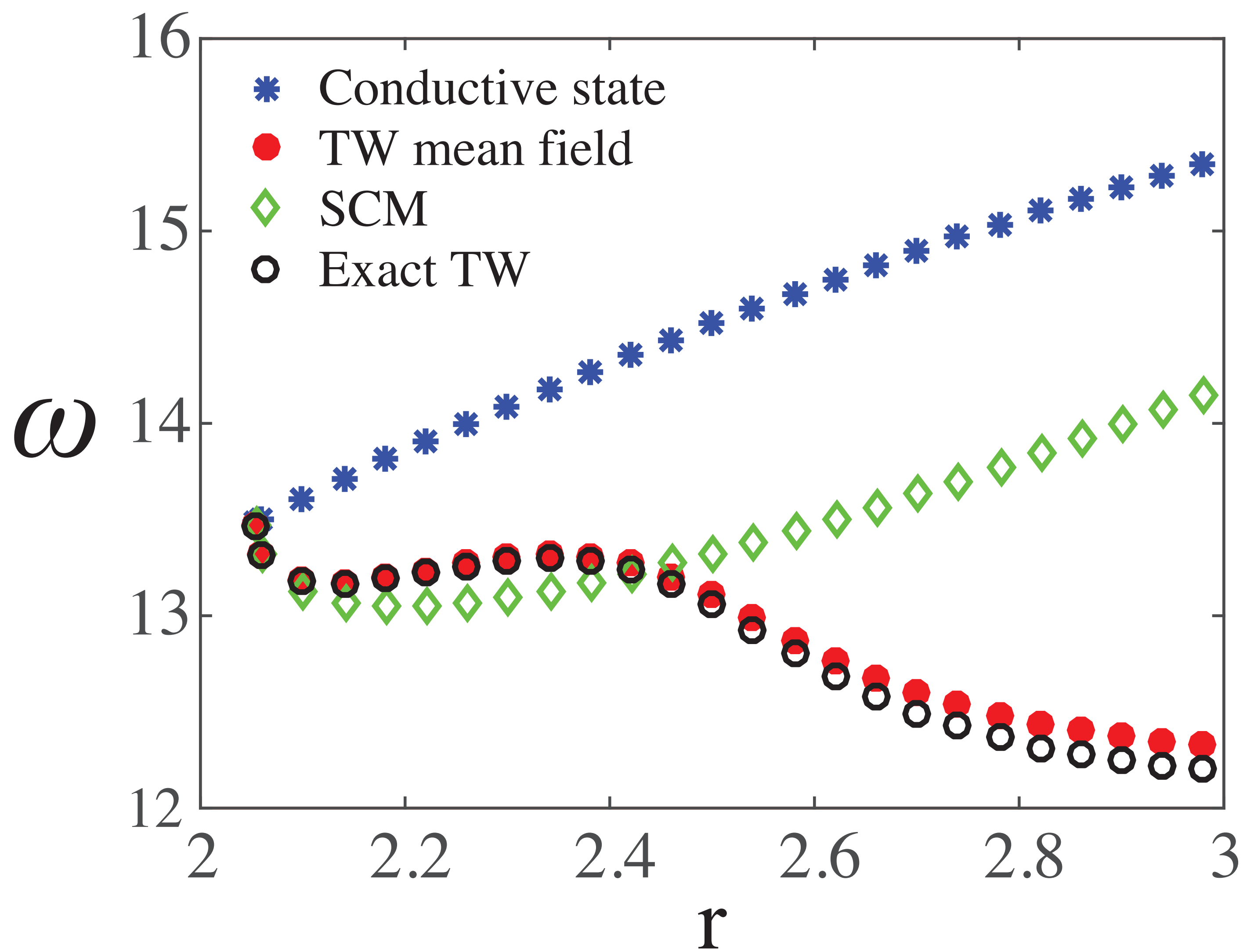
Standing waves



mean ≠ exact

~~**RZIF**~~

SCM vs RZIF vs exact for thermosolutal TW



SCM: linearization about mean flow generated by $\mathcal{N}(u_1, u_{-1})$ only

RZIF: linearization about exact mean flow

Exact nonlinear frequency

Need u_2 for its effect on meanflow, but not for its effect on u_1

RZIF includes terms like $\mathcal{N}(u_2, u_{-2})$ but not like $\mathcal{N}(u_2, u_{-1})$

SCM contains neither

Higher-order SCM (include u_2, u_3, …)

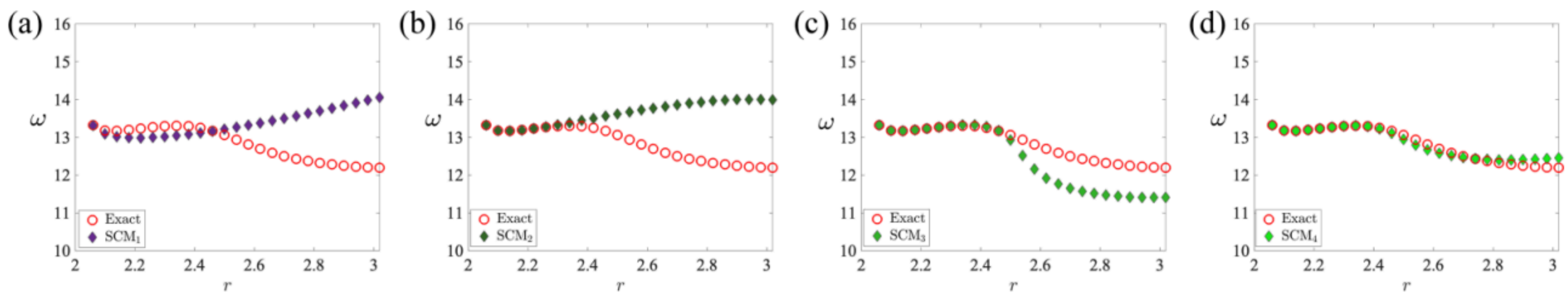


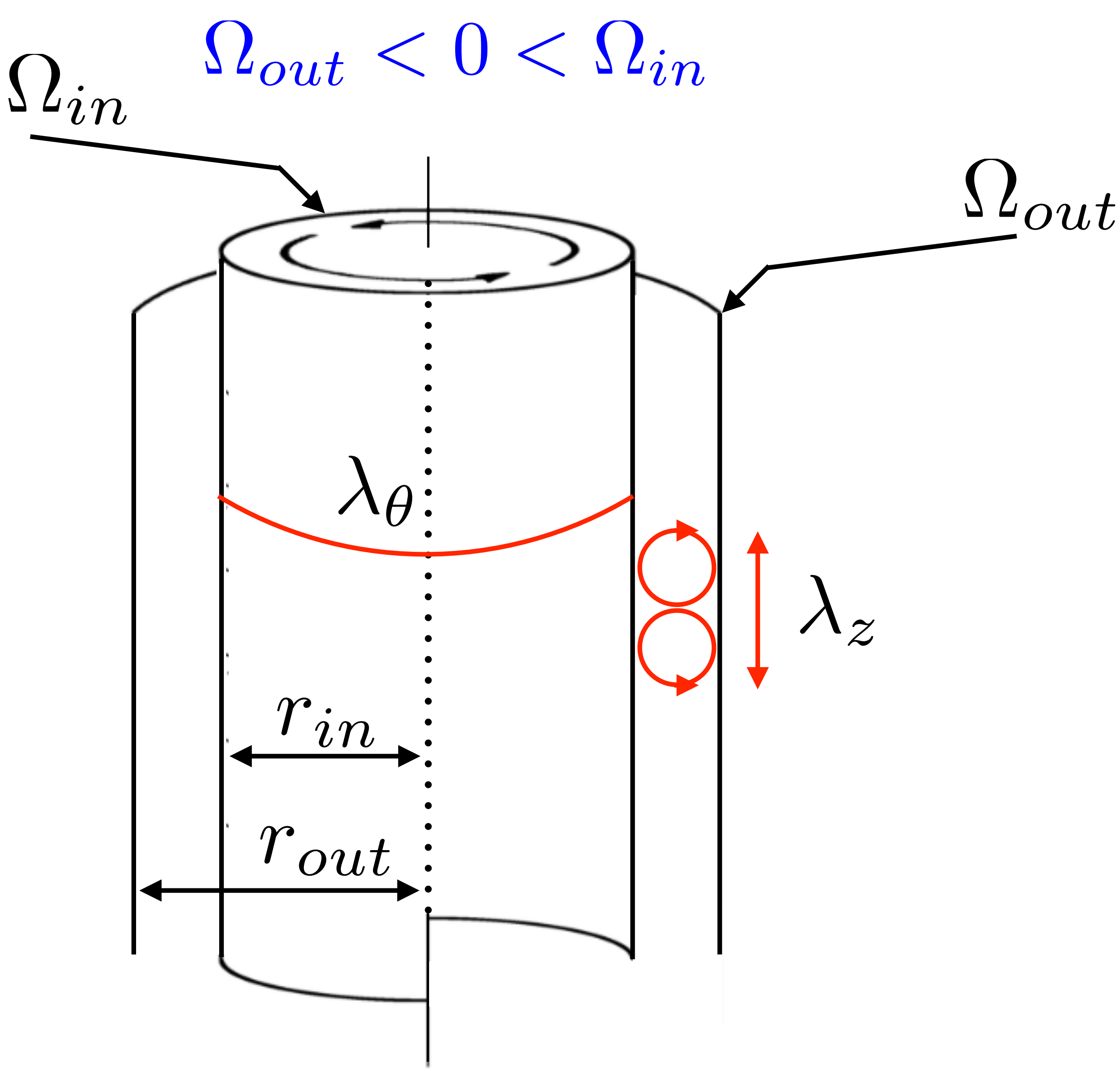
FIG. 7. Frequency calculated by SCM methods of increasing order as a function of Rayleigh number. Exact frequencies are shown by open circles (○), while those predicted by the SCM are shown by diamonds: (a) first order (♦), (b) second order (♦), (c) third order (♦), and (d) fourth order (♦).

Other Flows

Counter-rotating Taylor-Couette flow
Shear-driven and square cavity

(seeking more counter-examples)

Counter-rotating Taylor-Couette flow



Periodic in z and θ

$$\lambda_\theta = \pi \quad \lambda_z = 1.2$$

$$\eta = \frac{r_{in}}{r_{out}} = 0.5$$

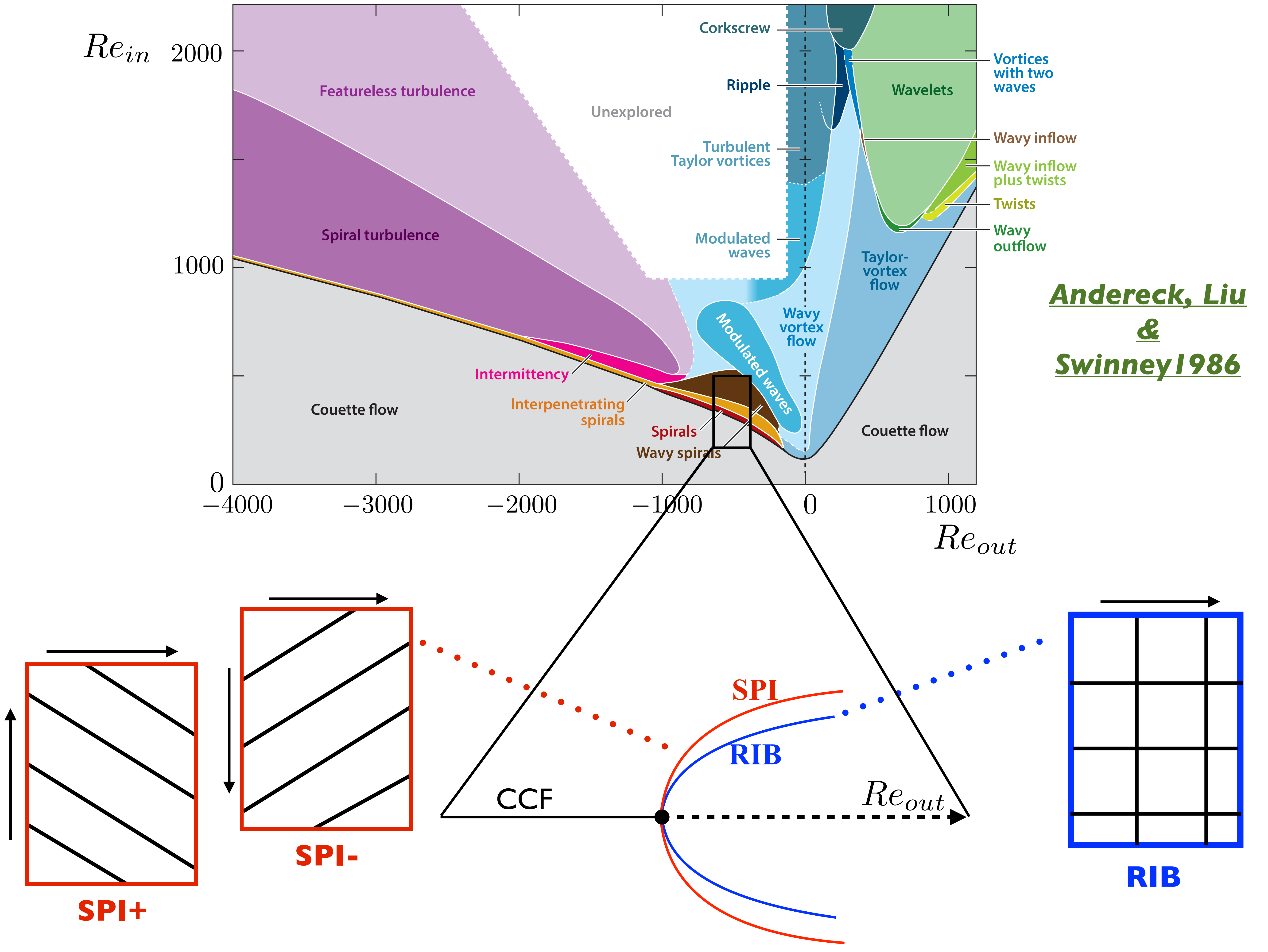
$$d \equiv (r_{out} - r_{in})$$

$$Re_{in} \equiv \frac{\Omega_{in} r_{in}\, d}{\nu} = 240$$

$$Re_{out} \equiv \frac{\Omega_{out} r_{out}\, d}{\nu} \in [-585, -480]$$

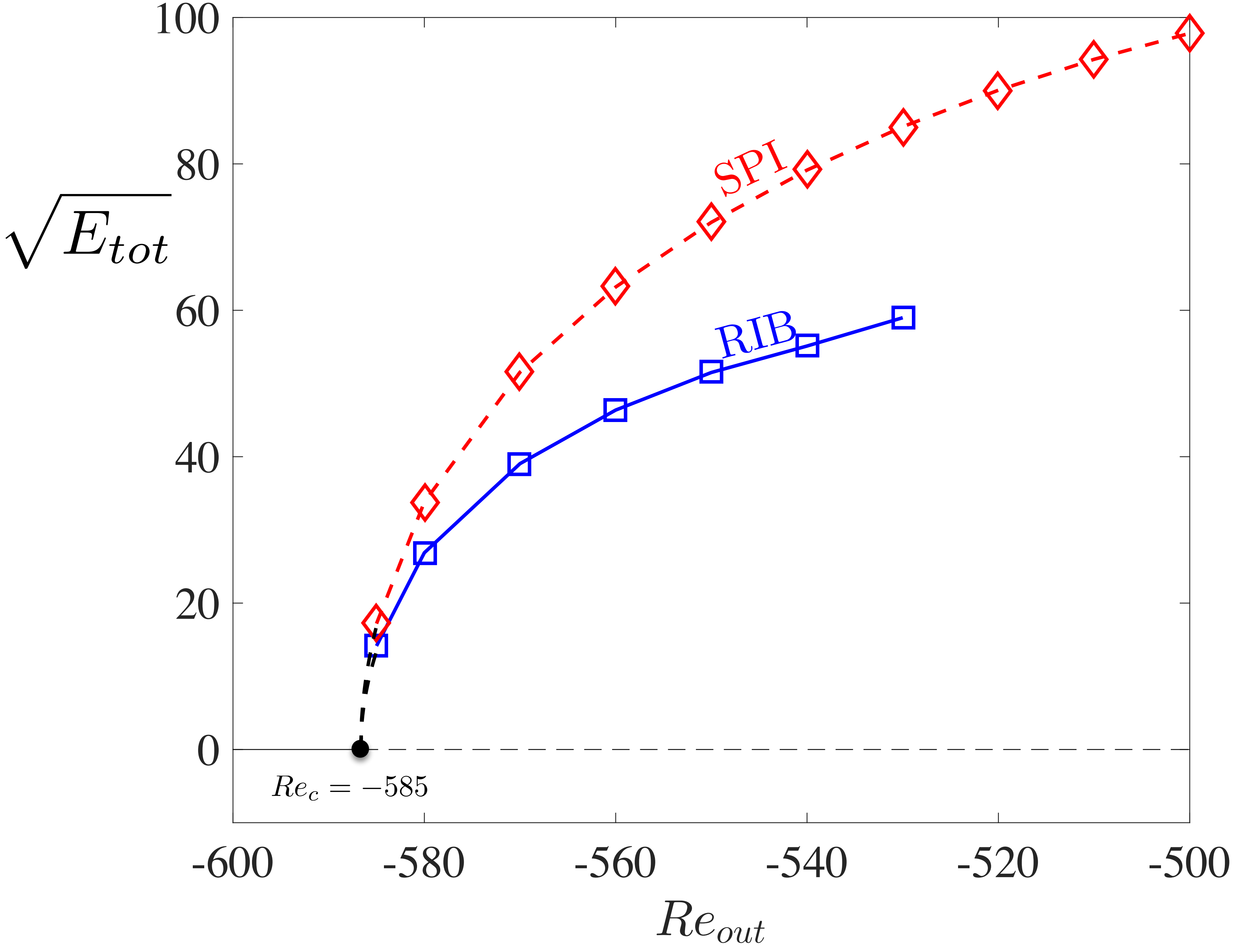
Pinter, Lücke, Hoffmann, 2008
Altmeyer & Hoffmann, 2010

Willis, SoftwareX 6, 2017



Re_in
2000
1000
0
−4000
−3000
−2000
−1000
0
1000
Re_out
Featureless turbulence
Unexplored
Spiral turbulence
Corkscrew
Ripple
Turbulent Taylor vortices
Modulated waves
Vortices with two waves
Wavelets
Wavy inflow
Wavy inflow plus twists
Twists
Wavy outflow
Taylor-vortex flow
Wavy vortex flow
Modulated waves
Intermittency
Interpenetrating spirals
Spirals
Wavy spirals
Couette flow
Couette flow
Andereck, Liu & Swinney1986
SPI
RIB
CCF
Re_out
SPI+
SPI-
RIB

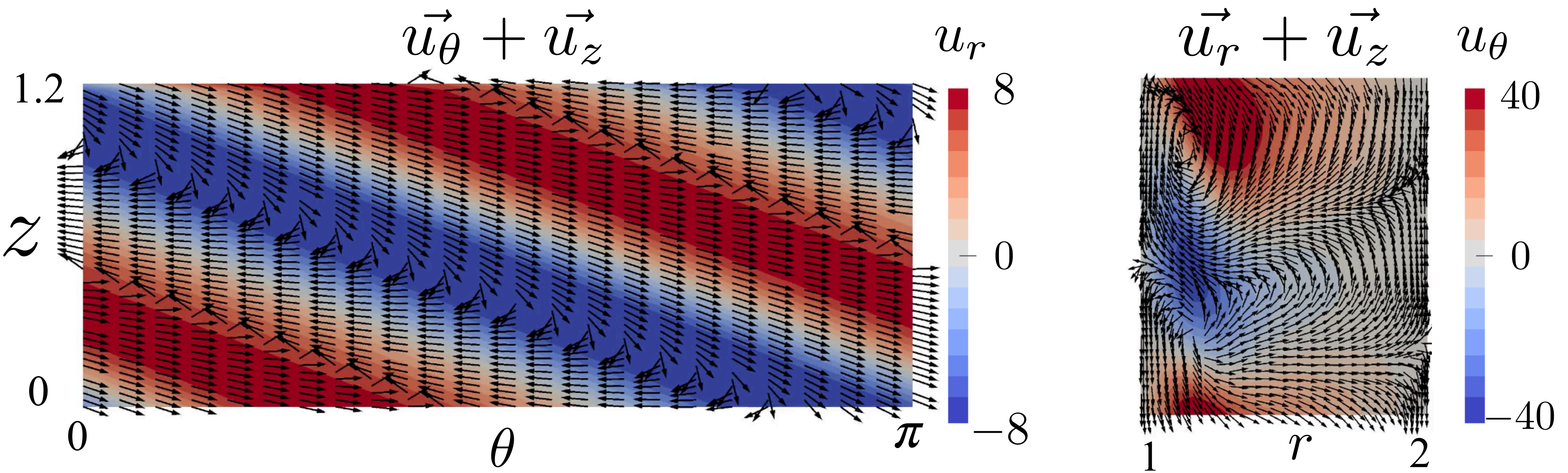
Bifurcation diagram



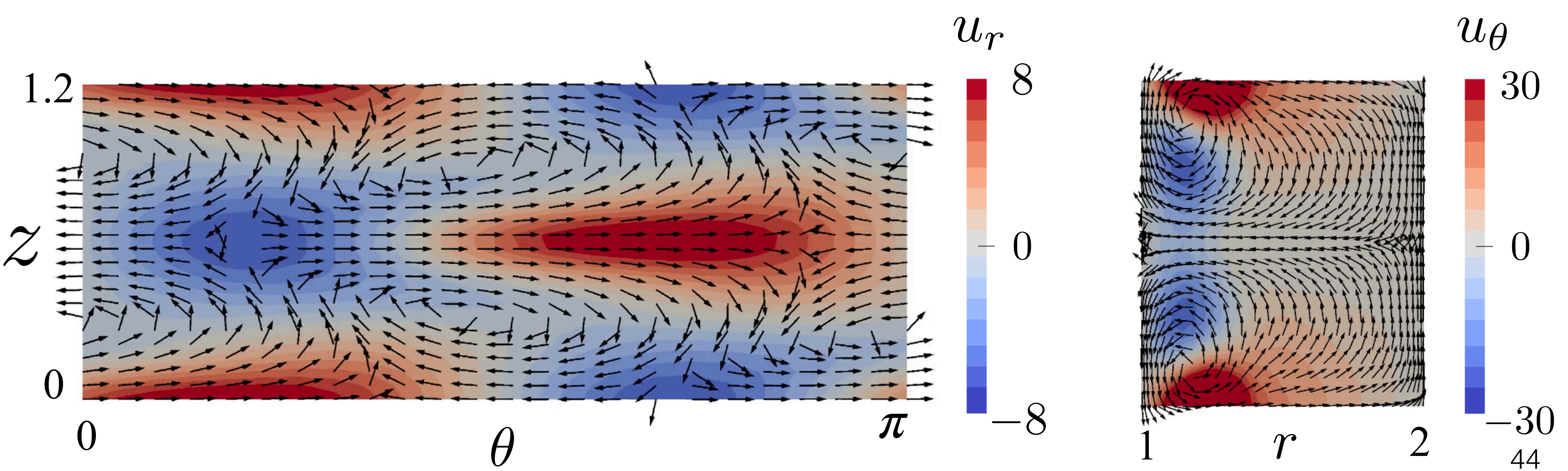
The spirals (SPI) are unstable: obtained by imposing their symmetry

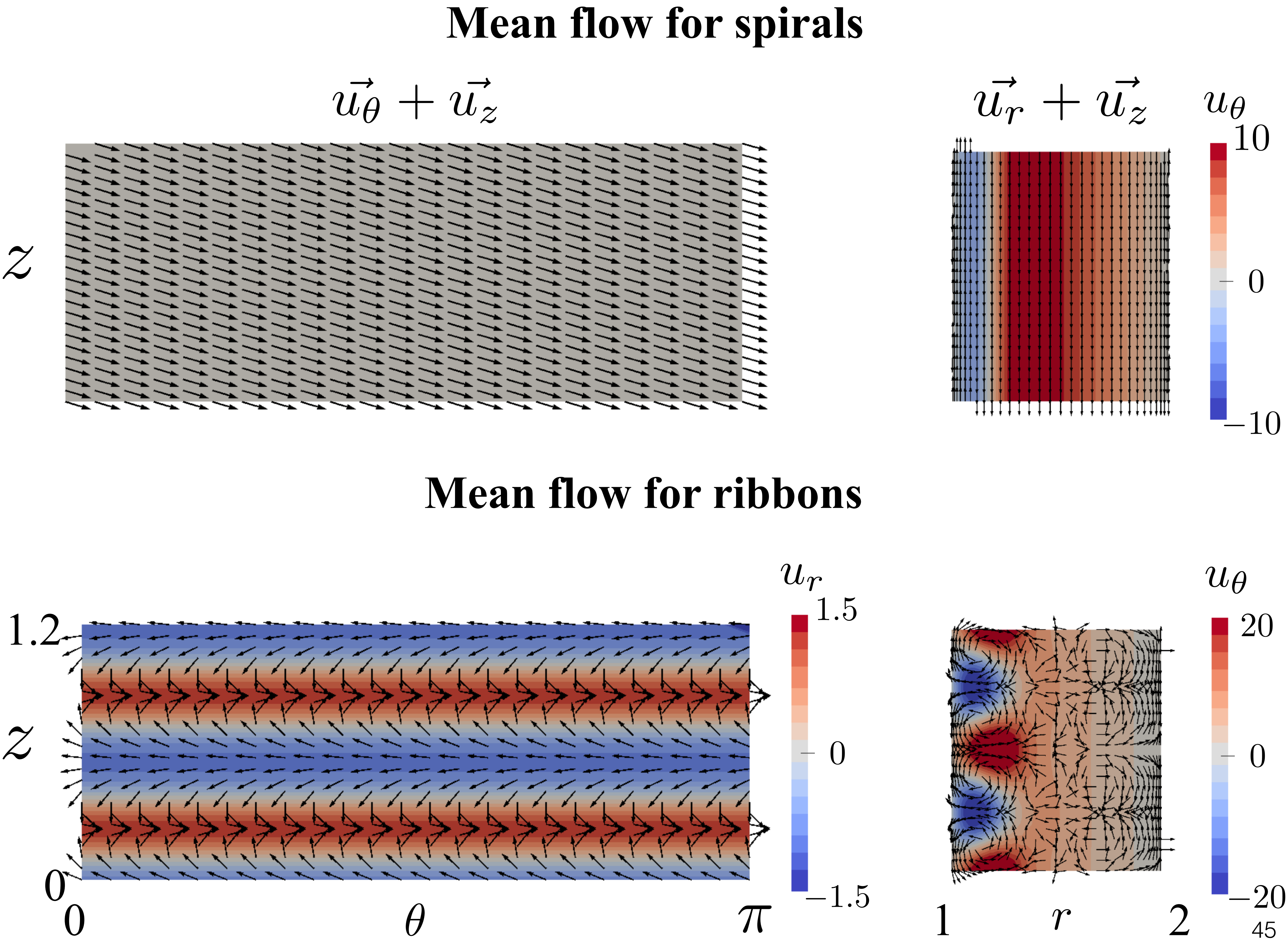
The ribbons (RIB) are stable within this range of Re

Nonlinear spirals

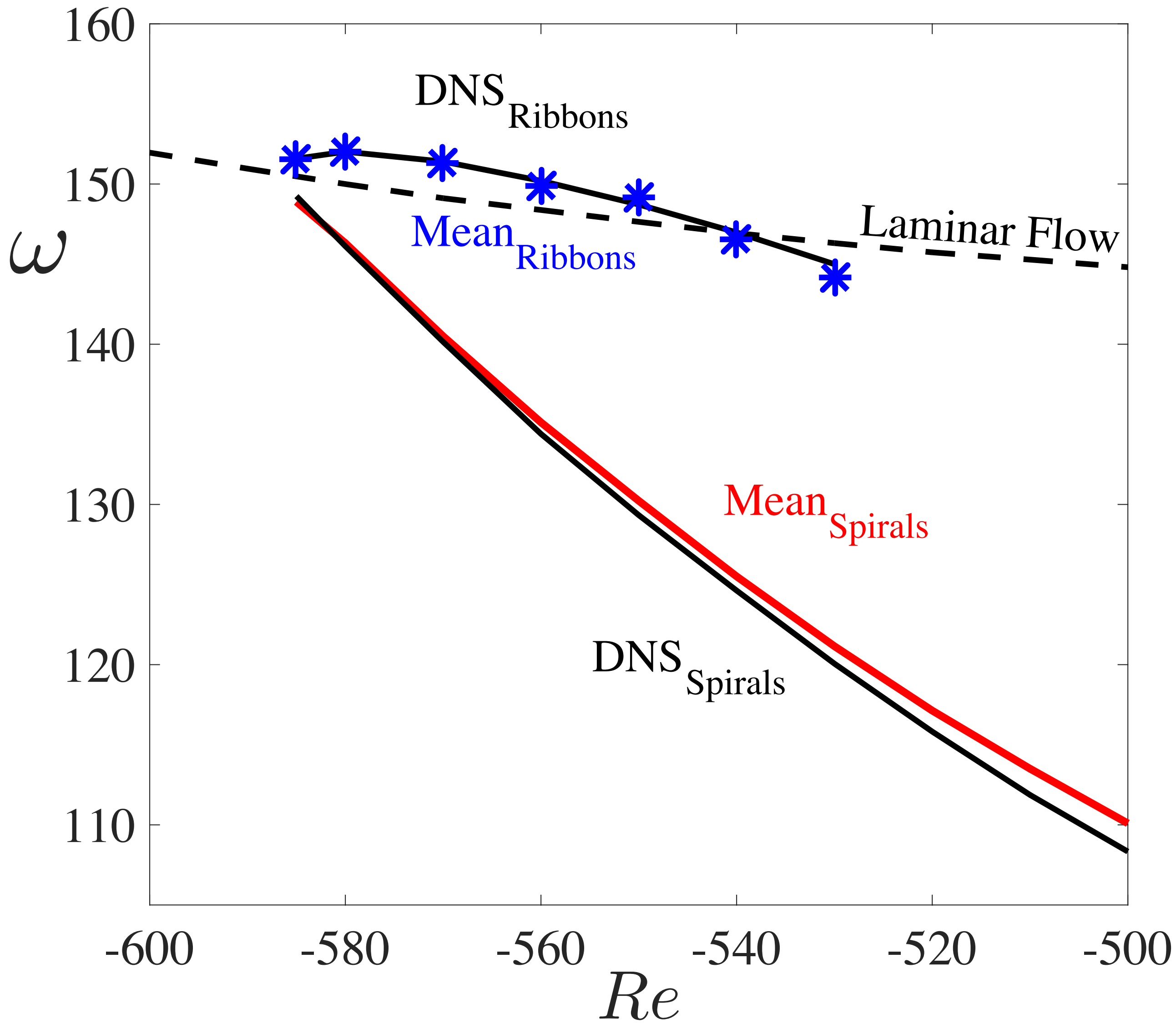


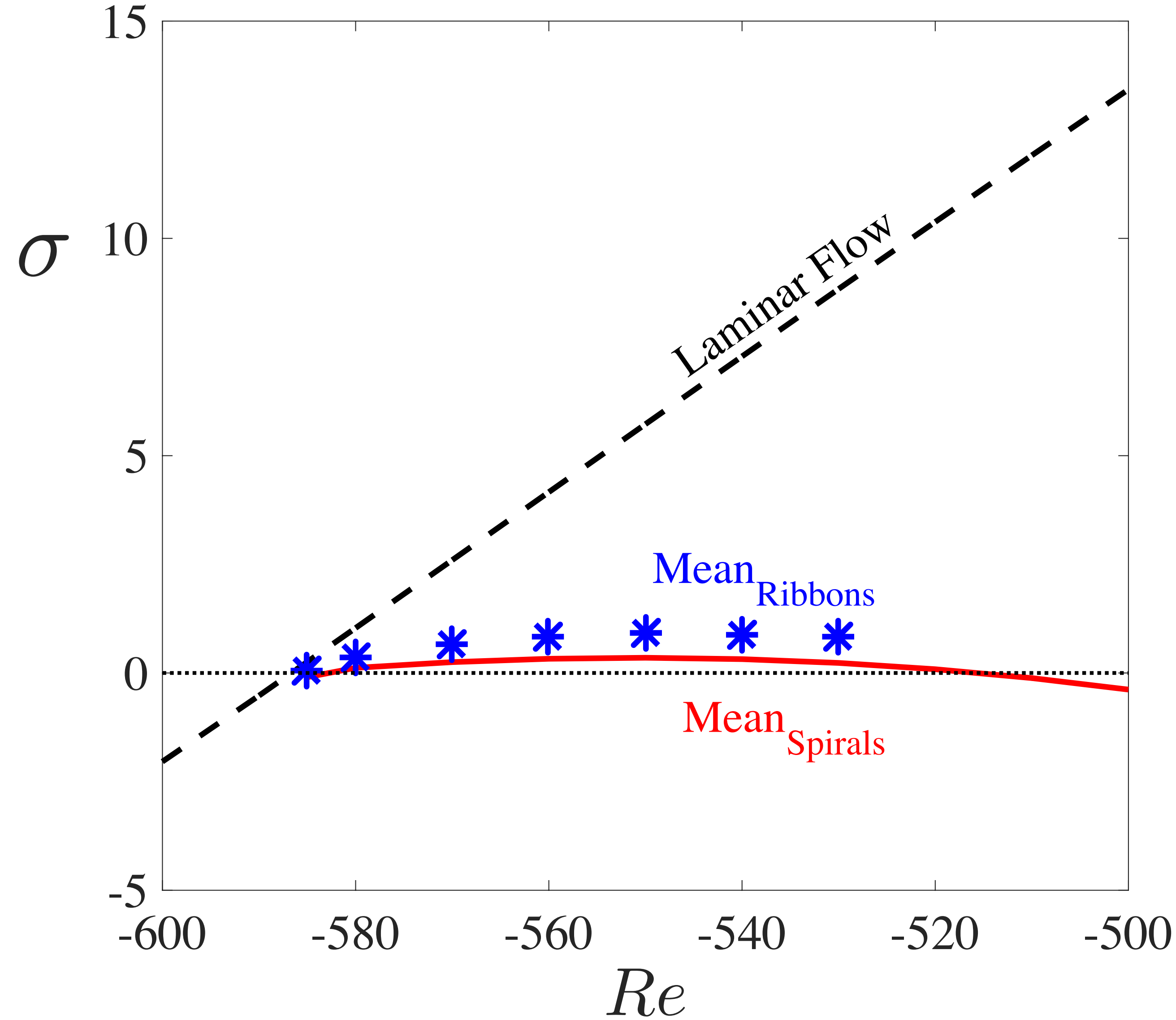
Nonlinear ribbons




Mean flow for spirals
$\vec{u_\theta} + \vec{u_z}$
$\vec{u_r} + \vec{u_z}$
u_θ
10
0
−10
z
Mean flow for ribbons
u_r
1.5
0
−1.5
u_θ
20
0
−20
1.2
z
0
0
θ
π
1
r
2

Stability analysis about laminar and mean flows

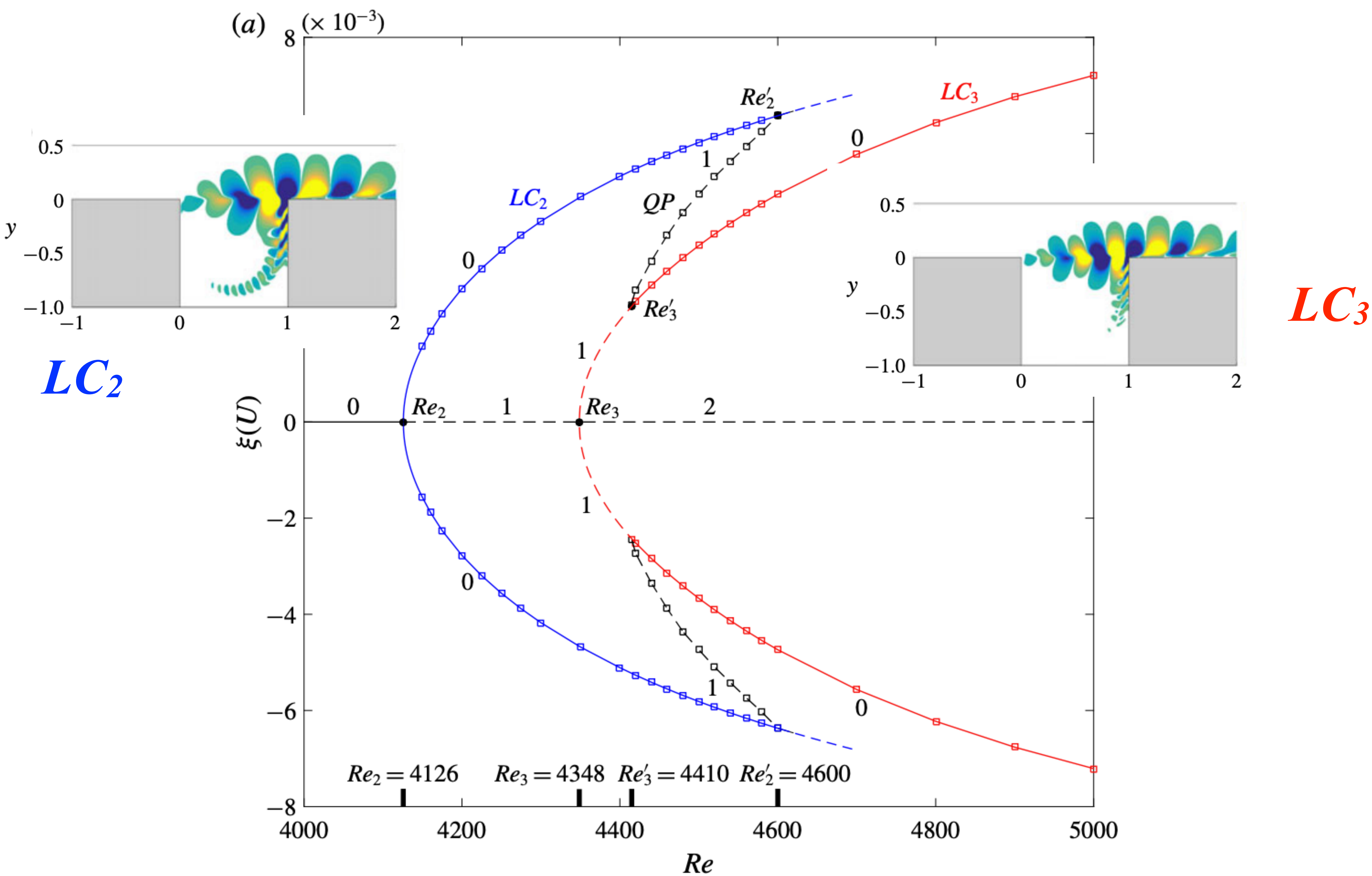




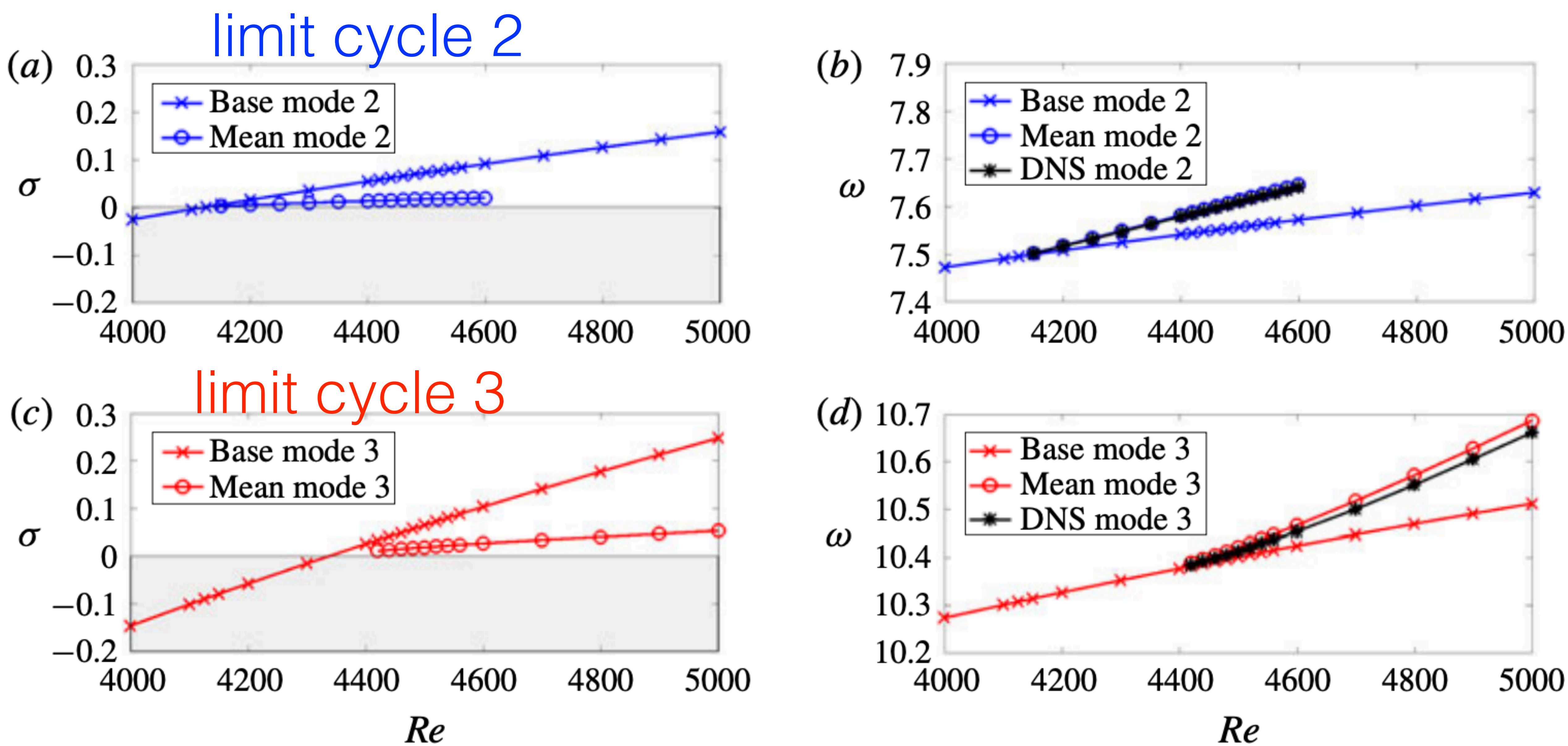
mean = exact : **RZIF for spirals and ribbons !**

no new counter example

shear-driven and square cavity



Linear stability about base flow and about mean flow



Resolvent Analysis

McKeon & Sharma, JFM 2010

$$i\omega \mathbf{u}_1 = \mathcal{L}_{\bar{U}} \mathbf{u}_1 + \mathcal{N}_1$$

$$\mathbf{u}_1 = \left(i\omega - \mathcal{L}_{\overline{U}}\right)^{-1} \mathcal{N}_1 \equiv \mathcal{R}(\omega)\mathcal{N}_1$$

More generally: $\mathbf{u}(\omega) = \mathcal{R}(\omega)\mathcal{N}(\omega)$

Singular value decomposition: $\mathcal{R}(\omega)\phi_j(\mathbf{x},\omega) = \mu_j(\omega)\psi_j(\mathbf{x},\omega)$

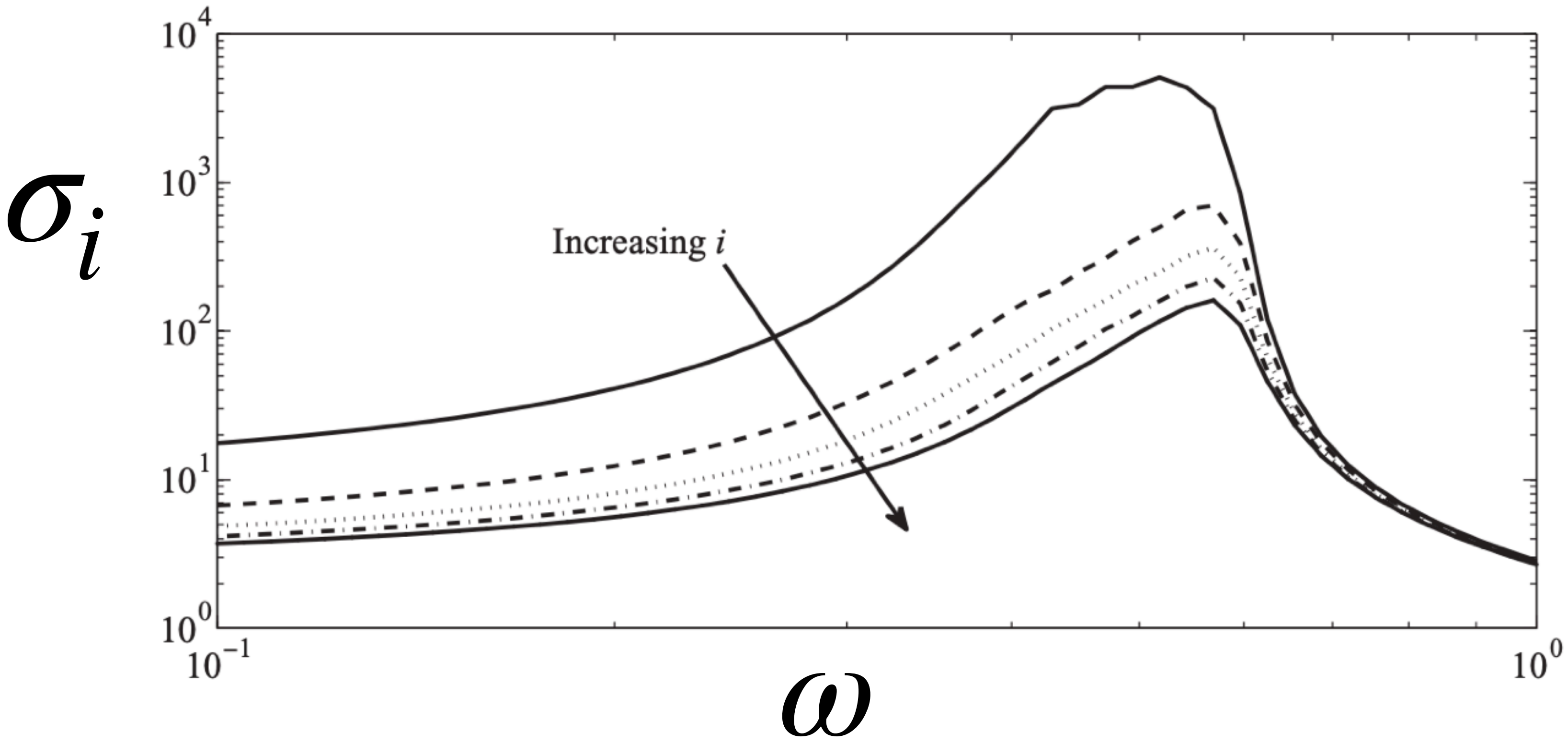
If resolvent has a highly dominant singular value μ_{dom},

then $\mathcal{R}$ extracts and amplifies the component of mode ϕ_{dom} in $\mathcal{N}$

Independent of the details of $\mathcal{N}$

$$\mathbf{u} = \mathcal{R}(\omega) \sum_j \langle \mathcal{N}, \phi_j \rangle \phi_j = \sum_j \langle \mathcal{N}, \phi_j \rangle \mu_j \psi_j \approx \underline{\langle \mathcal{N}, \phi_{\text{dom}} \rangle \mu_{\text{dom}}}\, \underline{\psi_{\text{dom}}}$$

$$\mathbf{u}(\mathbf{x},\omega) \approx \underbrace{\langle \mathcal{N}, \phi_{\text{dom}} \rangle(\omega)\ \mu_{\text{dom}}(\omega)}_{\text{scalar amplitude } \Lambda(\omega)}\ \underbrace{\psi_{\text{dom}}(\mathbf{x},\omega)}_{\text{spatial dependence}}$$



McKeon & Sharma, JFM 2010
A critical-layer framework for turbulent pipe flow

$$\mathbf{u}(\mathbf{x},\omega) \approx \underbrace{\langle \mathcal{N}, \phi_{\mathrm{dom}} \rangle(\omega)\, \mu_{\mathrm{dom}}(\omega)}_{\textbf{scalar amplitude } \Lambda(\omega)} \; \underbrace{\psi_{\mathrm{dom}}(\mathbf{x},\omega)}_{\textbf{spatial dependence}}$$

How to obtain mean flow $\overline{U}$?
One possibility is from experimental measurements
How to obtain scalar amplitude?
One possibility is by fitting correspondence between **u** and ψ

Also works for spatial wavenumbers and spatial averages.

J. Fluid Mech. (2016), *vol.* 798, *pp.* 485–504. 485
doi:10.1017/jfm.2016.331

Conditions for validity of mean flow stability analysis

Samir Beneddine[1,†], Denis Sipp[1], Anthony Arnault[2], Julien Dandois[2] and Lutz Lesshafft[3]

Fully turbulent flow with broad spectrum, rather than periodic flow with only $\omega, 2\omega, \ldots$

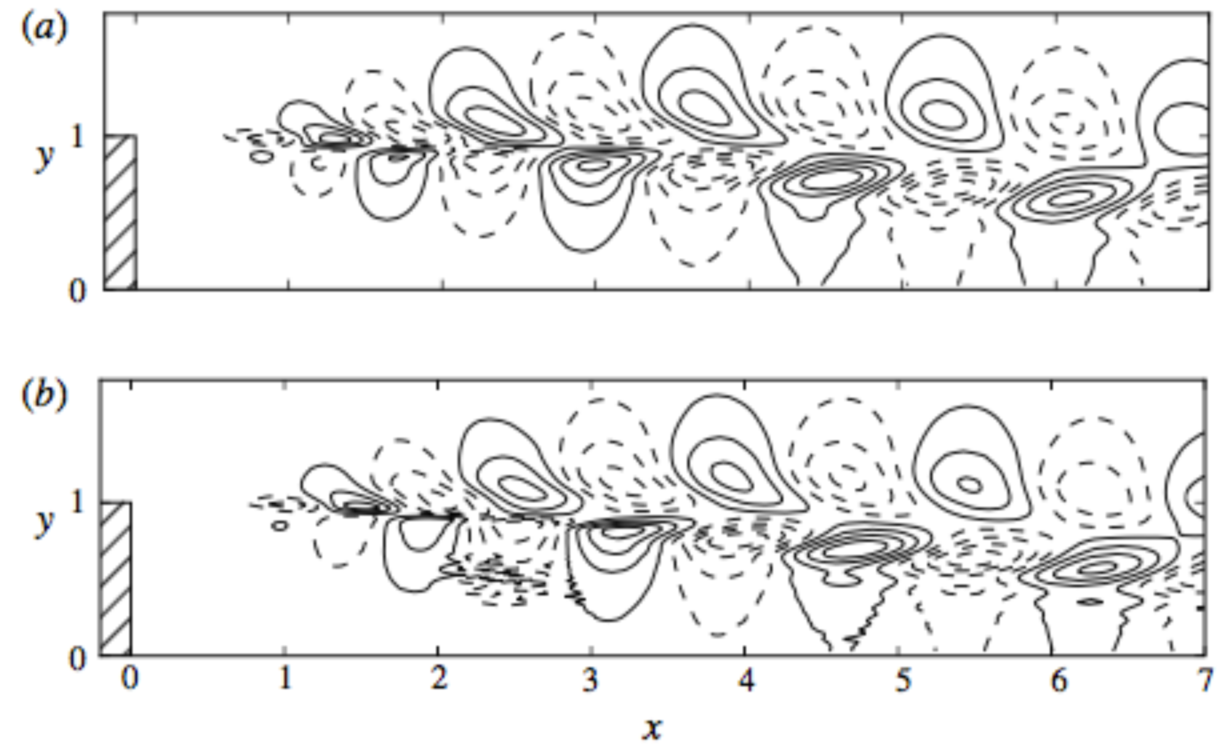
$$\mathcal{N}(\mathbf{x},\omega) \equiv -(\tilde{\mathbf{u}} \cdot \nabla)\tilde{\mathbf{u}} + \langle (\tilde{\mathbf{u}} \cdot \nabla)\tilde{\mathbf{u}} \rangle$$

$$\mathbf{u}(\mathbf{x},\omega) \approx \underbrace{\langle \mathcal{N}, \phi_{\text{dom}} \rangle(\omega)\, \mu_{\text{dom}}(\omega)}_{\textbf{scalar amplitude } \Lambda(\omega)} \; \underbrace{\psi_{\text{dom}}(\mathbf{x},\omega)}_{\textbf{spatial dependence}}$$

Spatial dependence

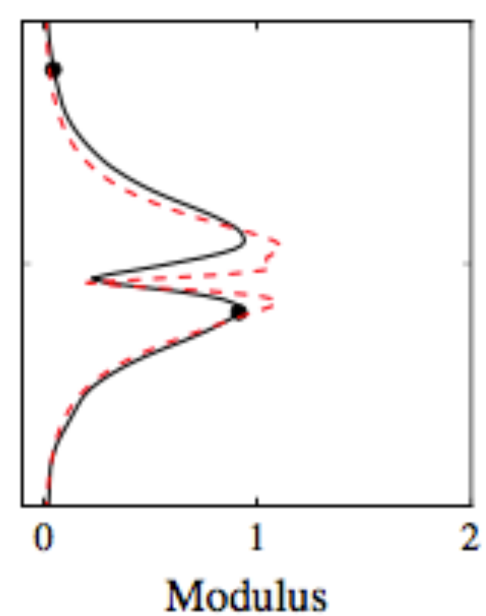
dominant optimal response of resolvent $\psi_{\mathrm{dom}}(\mathbf{x}, \omega)$

simulation $\mathbf{u}(\mathbf{x}, \omega)$

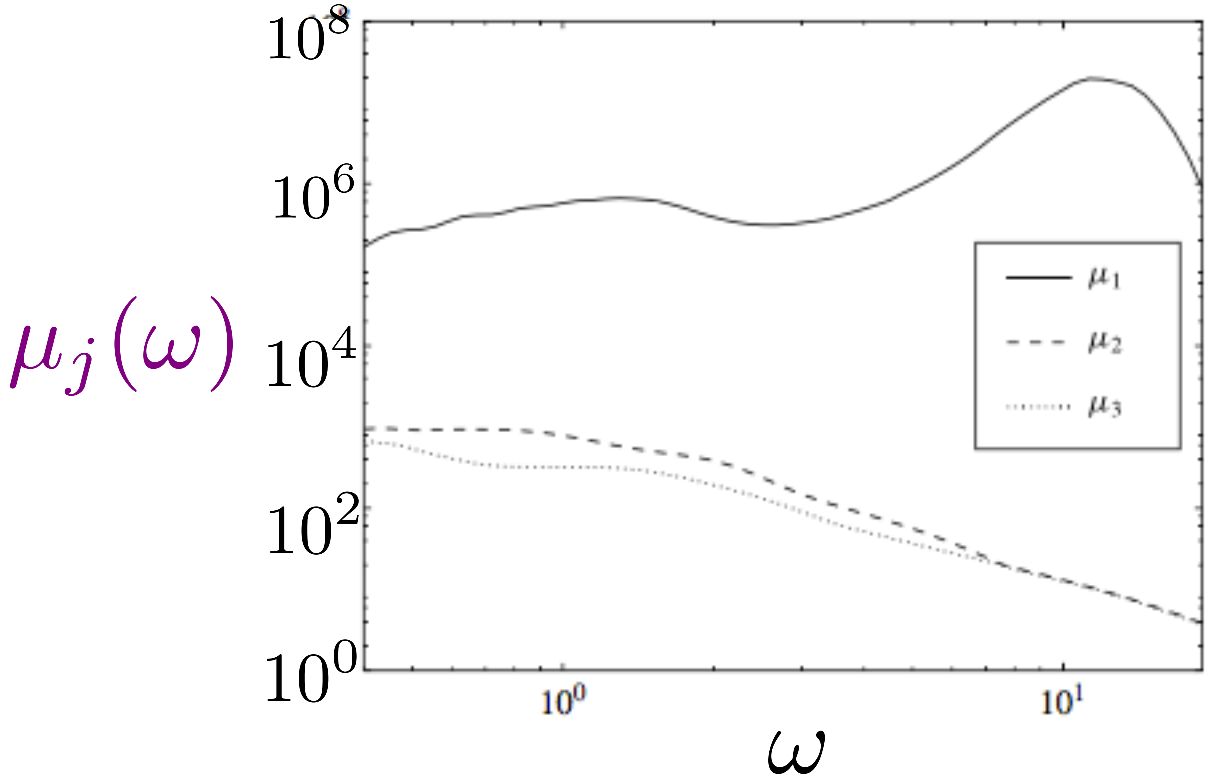


Scalar amplitude

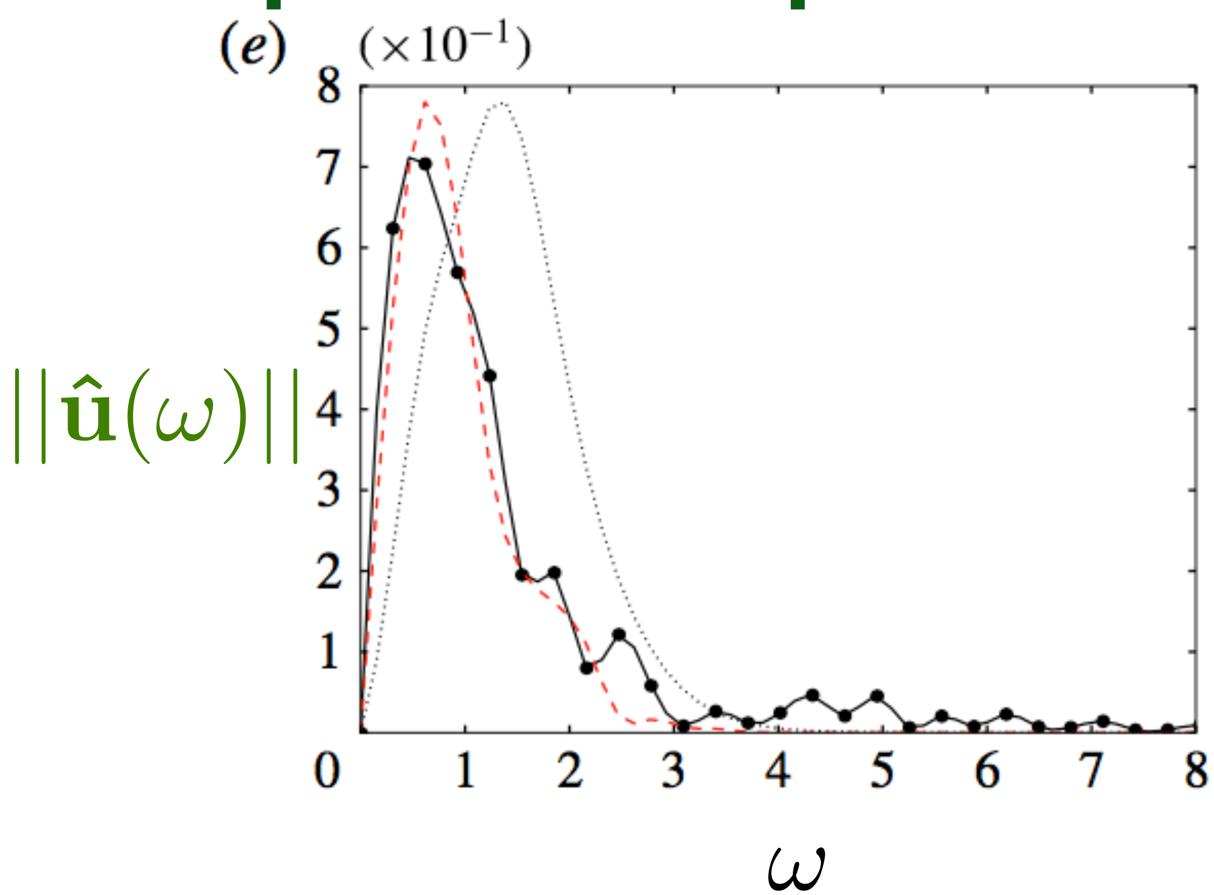
choose amplitude $\Lambda(\omega)$ so that dominant optimal response and simulation agree at two points



Dominant singular value



Spectrum reproduced!



Relationship to Quasi Linear (QL) and Generalized Quasi Linear (GQL)

These divide modes into two categories:
small set of low (mean) modes and large set of high modes.

Retain low-low interactions.

Retain low-high interactions, leading to linear equations for the high modes.

Retain only high-high interactions which generate low modes

RZIF is neither, since it is not a closed system (uses full $\overline{U}$ in linear equations)

SCM is neither, since it includes all interactions between included modes

	linearize about base flow	linearize about mean flow RZIF	linearize about approximate base flow SCM
cylinder wake	X	√	√
thermosolutal TW	X	√	X
thermosolutal SW	X	X	X
counter-rotating Taylor-Couette	X	√	
driven cavity	√	√	

Conclusions / Questions

RZIF: prediction of nonlinear frequency from mean flow

SCM: frequency without DNS by using inexact mean flow

When?

Why?

Stay tuned …

Thank you!