Non-dimensionalization of linear single-layer Faraday problem

Set $\tilde{z} \equiv kz$, $\tilde{t} = \omega t$. Bulk equation:

$$\begin{aligned}
0 &= \left(\frac{\partial}{\partial t} - \nu\left(\frac{\partial^2}{\partial z^2} - k^2\right)\right)\left(\frac{\partial^2}{\partial z^2} - k^2\right) w \\
0 &= \left(\omega\frac{\partial}{\partial \tilde{t}} - \nu\left(k^2\frac{\partial^2}{\partial \tilde{z}^2} - k^2\right)\right)\left(k^2\frac{\partial^2}{\partial \tilde{z}^2} - k^2\right)\frac{\omega}{k}\tilde{w} \\
0 &= \left(\frac{\partial}{\partial \tilde{t}} - \frac{\nu k^2}{\omega}\left(\frac{\partial^2}{\partial \tilde{z}^2} - 1\right)\right)k^2\left(\frac{\partial^2}{\partial \tilde{z}^2} - 1\right)\frac{\omega}{k}\tilde{w} \\
0 &= \left(\frac{\partial}{\partial \tilde{t}} - \left[\frac{\nu k^2}{\omega}\right]\left(\frac{\partial^2}{\partial \tilde{z}^2} - 1\right)\right)\left(\frac{\partial^2}{\partial \tilde{z}^2} - 1\right)\tilde{w}
\end{aligned}$$

Normal stress condition at interface:

$$\begin{aligned}
\left(\rho\frac{\partial}{\partial t} - \eta\left(\frac{\partial^2}{\partial z^2} - 3k^2\right)\right)\left.\frac{\partial w}{\partial z}\right|_{z=0} &= -\left(\rho(g - a\cos(\omega t)) - \sigma k^2\right)k^2\zeta \\
\left(\rho\omega\frac{\partial}{\partial \tilde{t}} - \eta\left(k^2\frac{\partial^2}{\partial \tilde{z}^2} - 3k^2\right)\right)\frac{\omega}{k}k\left.\frac{\partial \tilde{w}}{\partial \tilde{z}}\right|_{\tilde{z}=0} &= -\left(\rho(g - a\cos\tilde{t}) - \sigma k^2\right)k^2\frac{1}{k}\tilde{\zeta} \\
\left(\frac{\partial}{\partial \tilde{t}} - \frac{\eta k^2}{\rho\omega}\left(\frac{\partial^2}{\partial \tilde{z}^2} - 3\right)\right)\left.\frac{\partial \tilde{w}}{\partial \tilde{z}}\right|_{\tilde{z}=0} &= -\frac{k}{\omega^2}\left(g - \frac{\sigma}{\rho}k^2 - a\cos\tilde{t}\right)\tilde{\zeta} \\
\left(\frac{\partial}{\partial \tilde{t}} - \left[\frac{\nu k^2}{\omega}\right]\left(\frac{\partial^2}{\partial \tilde{z}^2} - 3\right)\right)\left.\frac{\partial \tilde{w}}{\partial \tilde{z}}\right|_{\tilde{z}=0} &= -\left(\left[\frac{1}{\omega^2}\left(gk + \frac{\sigma k^3}{\rho}\right)\right] - \left[\frac{ak}{\omega^2}\right]\cos\tilde{t}\right)\tilde{\zeta}
\end{aligned}$$

The other equations to be satisfied at the interface do not introduce any new constants.

Using simple dimensional analysis on the 7 variables $\{k, \omega, \nu, g, \sigma, \rho, a\}$, i.e. choosing scales for mass, length, time would usually lead to $7 - 3 = 4$ nondimensional variables. But here we get only 3 variables because g, σ, and ρ participate only via the single expression $gk + \sigma k^3/\rho \equiv \omega_0^2$. In particular, the Bond number $\rho g/\sigma k^2$ comparing the relative size of the gravitational and capillary terms does not play a role in the linear problem.

If the layer is not infinite but of depth h, then the boundary conditions at the bottom yield kh as a fourth nondimensional parameter.