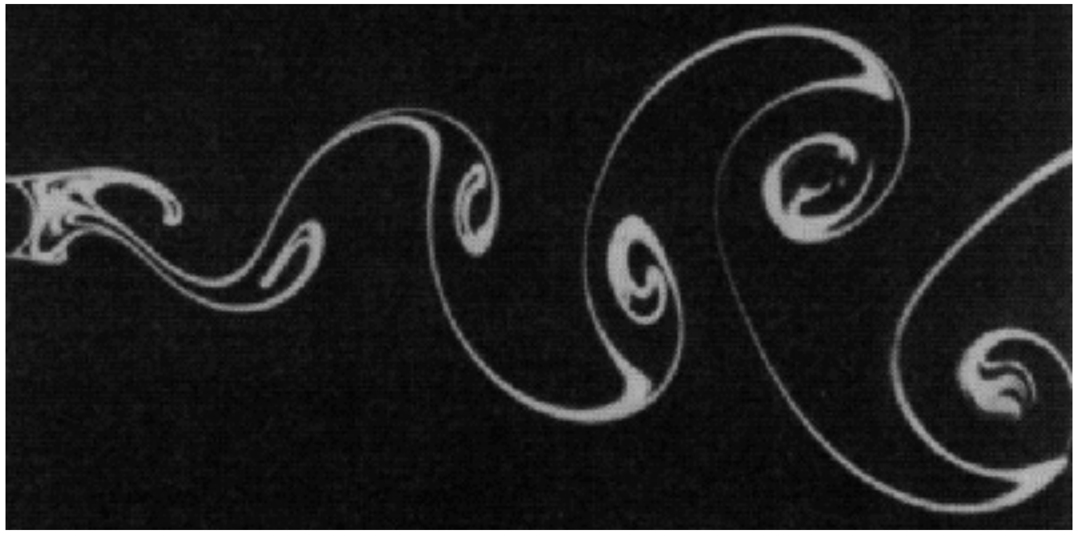
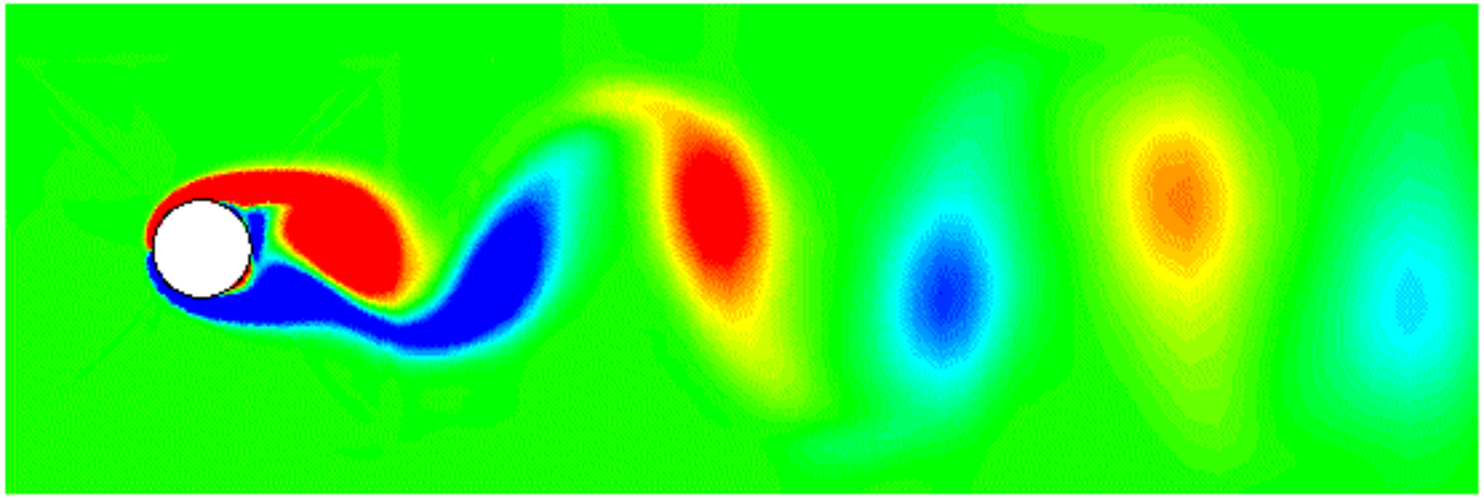
Mean flows and frequency prediction

Sam Turton (Cambridge, Part II/III)
Laurette Tuckerman (PMMH-CNRS-ESPCI)
Dwight Barkley (University of Warwick)

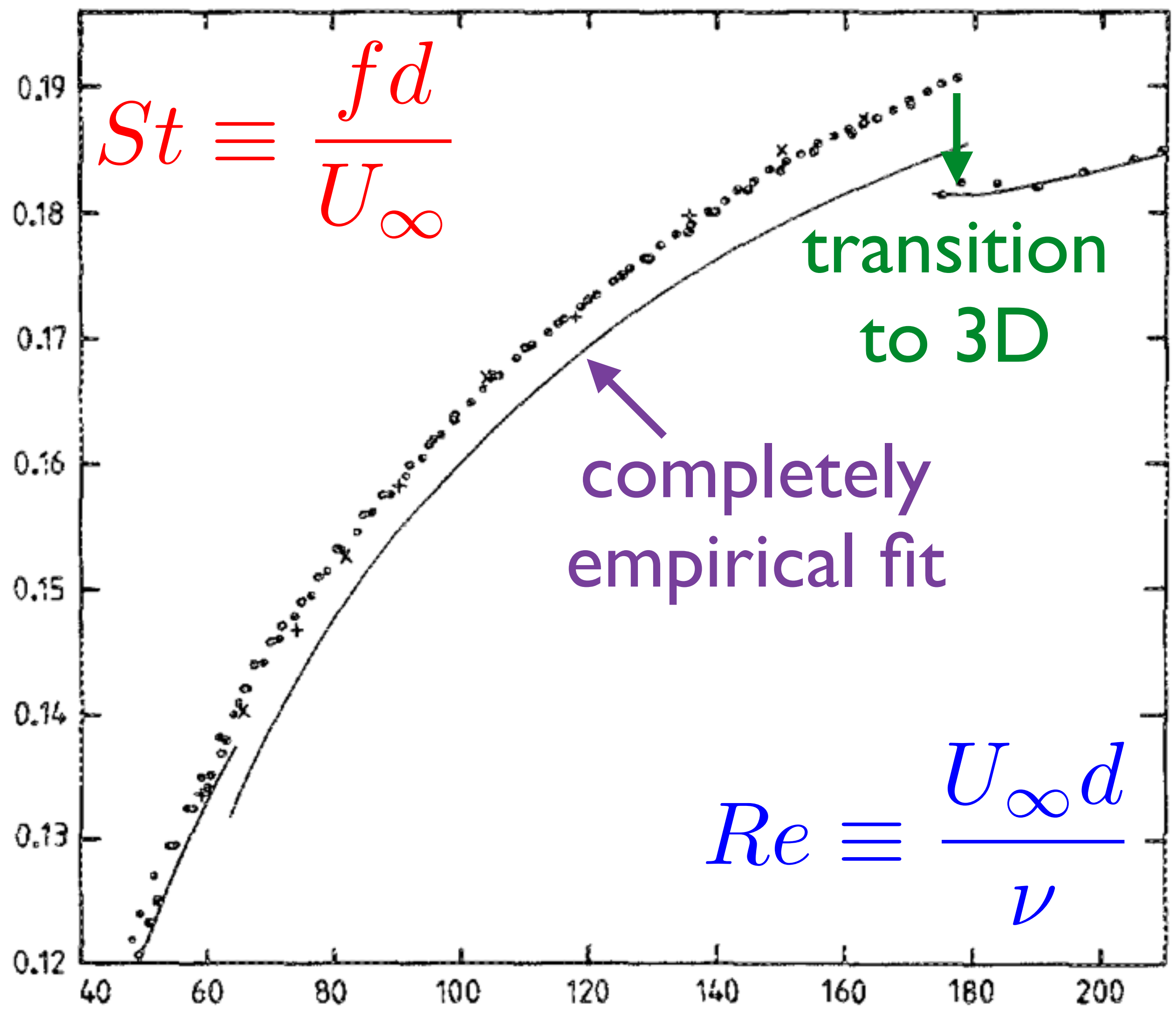
Vortex-shedding frequency of cylinder wake





Defining a universal and continuous Strouhal–Reynolds number relationship for the laminar vortex shedding of a circular cylinder

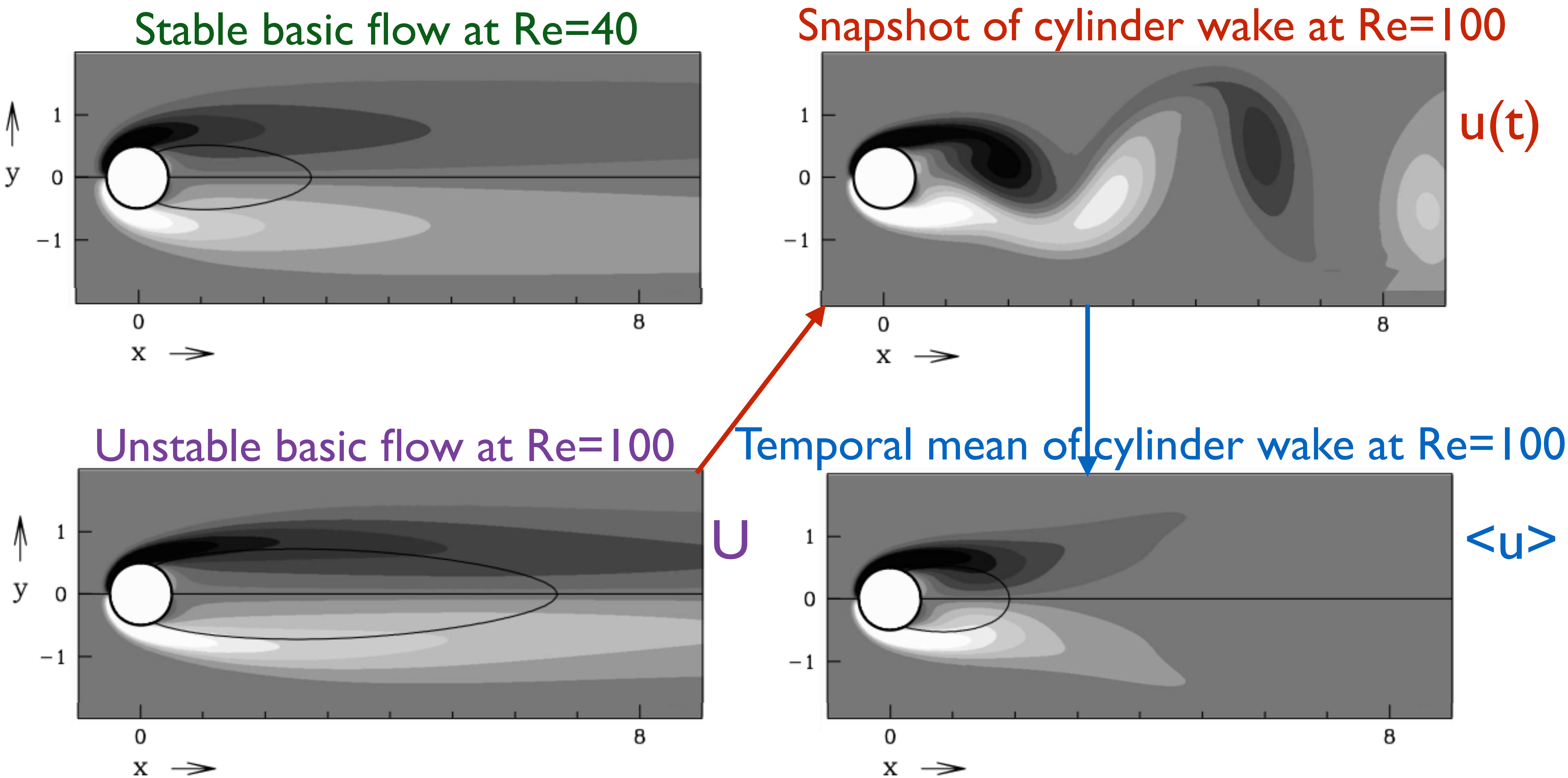
C. H. K. Williamson Physics of Fluids **31**, 2742 (1988)



Seek prediction from equations/physics

Linear analysis of the cylinder wake mean flow

D. BARKLEY(*) *Europhys. Lett.*, **75** (5), pp. 750–756 (2006)



Basic flow

$$0 = -(U \cdot \nabla)U - \nabla P + \frac{1}{Re}\nabla^2 U$$

Temporally periodic wake flow

$$\partial_t u = -(u \cdot \nabla)u - \nabla p + \frac{1}{Re}\nabla^2 u$$

Temporal mean

$$\langle u \rangle = \int_0^T u(t)dt$$

Linearise about steady base flow

$$\partial_t u = -(U \cdot \nabla)u - (u \cdot \nabla)U - \nabla p + \frac{1}{Re}\nabla^2 u$$

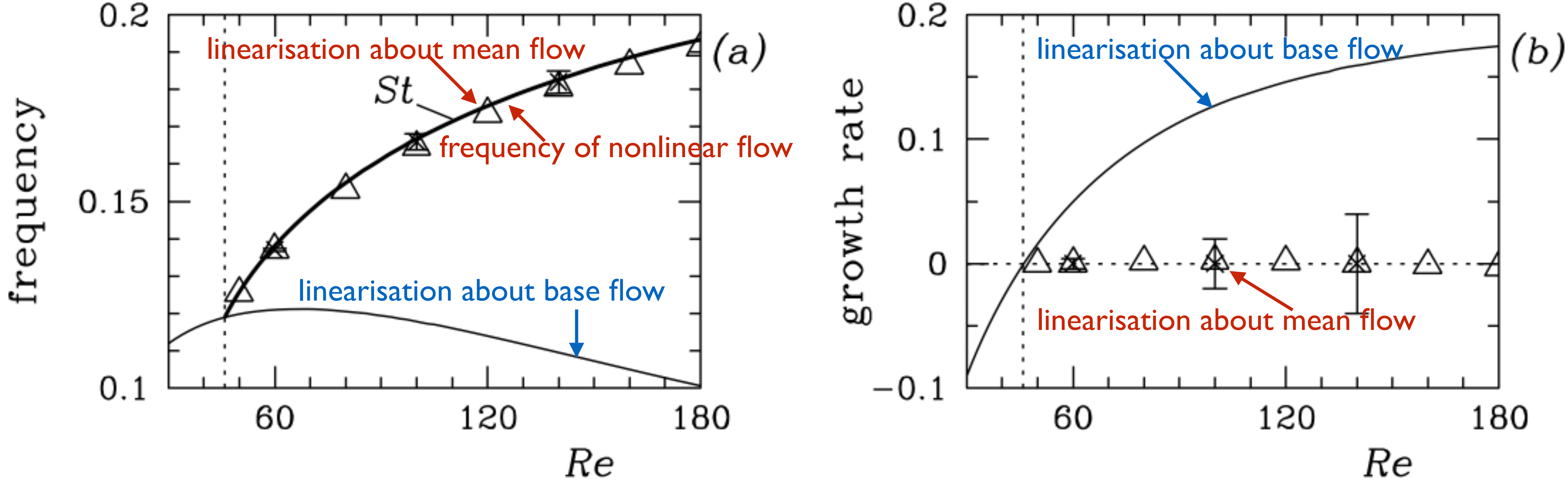
Linearise about temporal mean

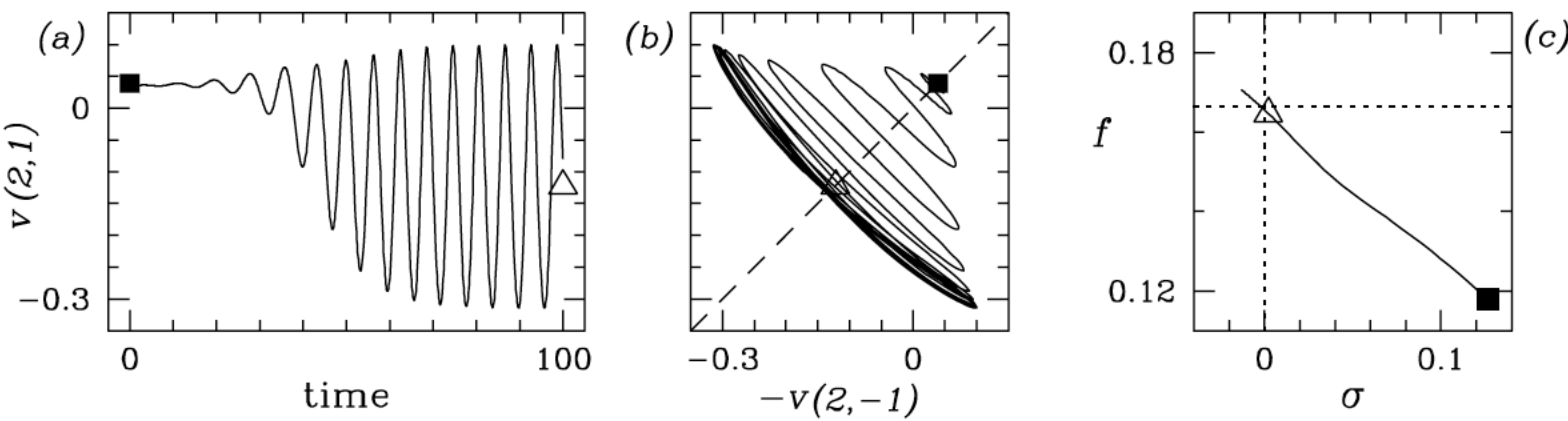
$$\partial_t u = -(\langle u \rangle \cdot \nabla)u - (u \cdot \nabla)\langle u \rangle - \nabla p + \frac{1}{Re}\nabla^2 u$$

Strange and unjustified procedure, but quite successful

Linear analysis of the cylinder wake mean flow

D. BARKLEY(*) *Europhys. Lett.*, **75** (5), pp. 750–756 (2006)



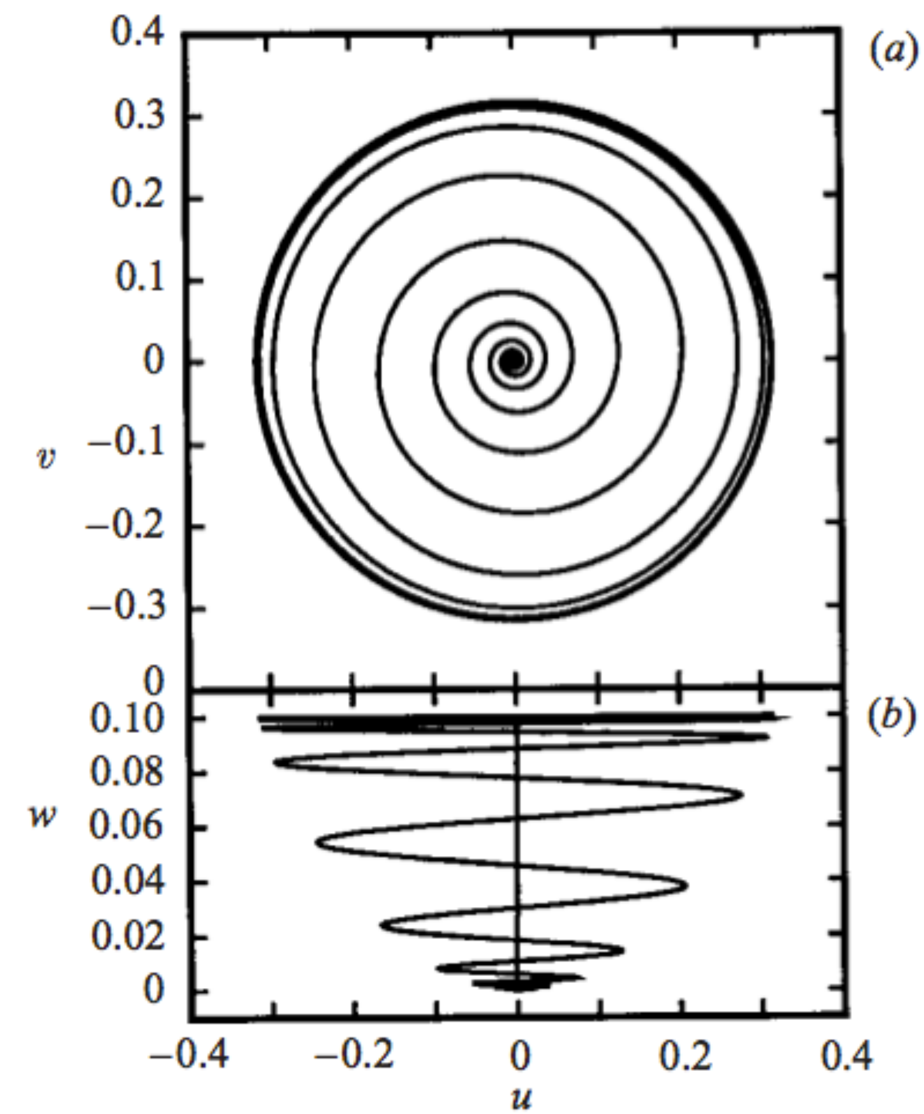


J. Fluid Mech. (2003), *vol.* 497, *pp.* 335–363.
DOI: 10.1017/S0022112003006694 Printed in the United Kingdom

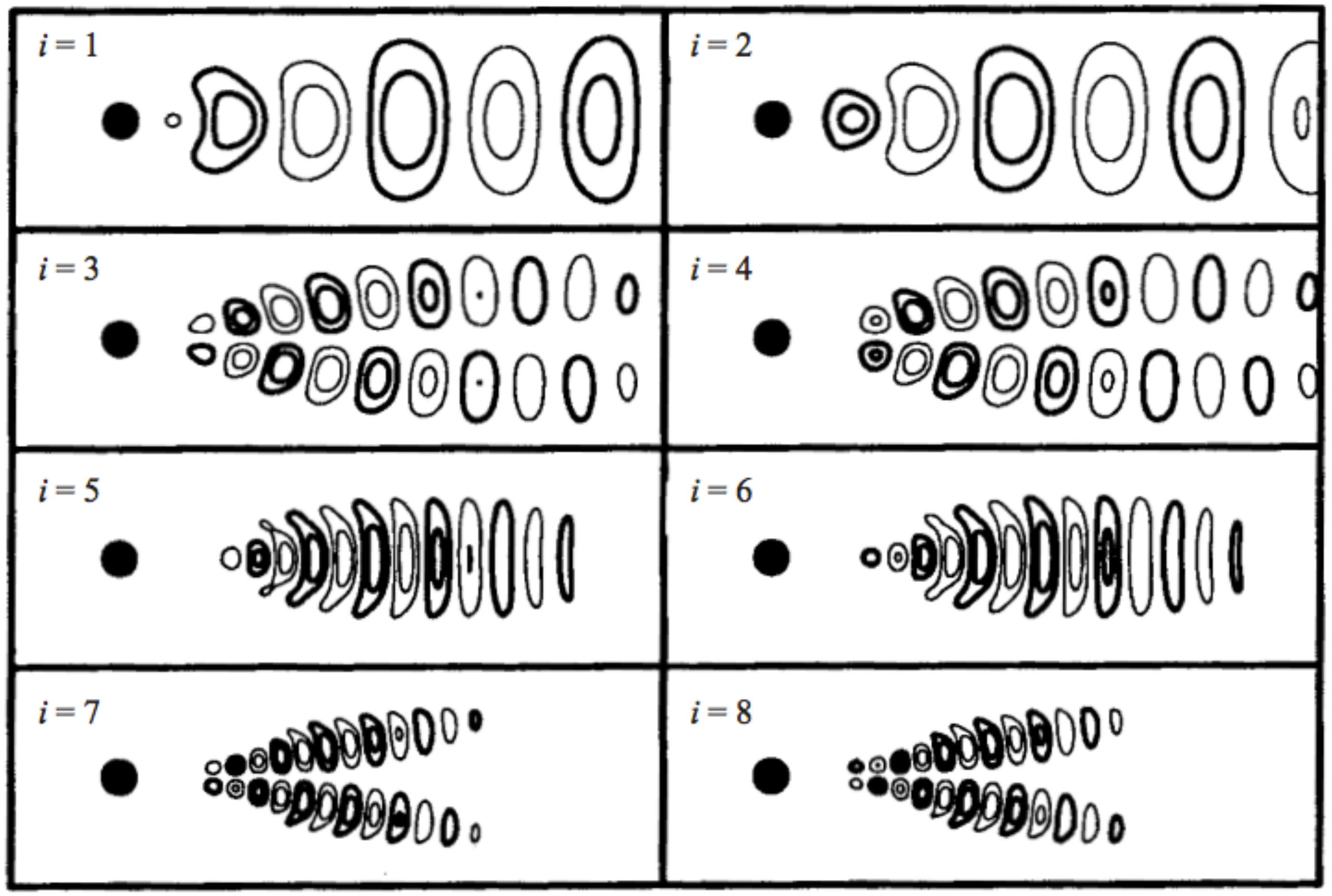
A hierarchy of low-dimensional models for the transient and post-transient cylinder wake

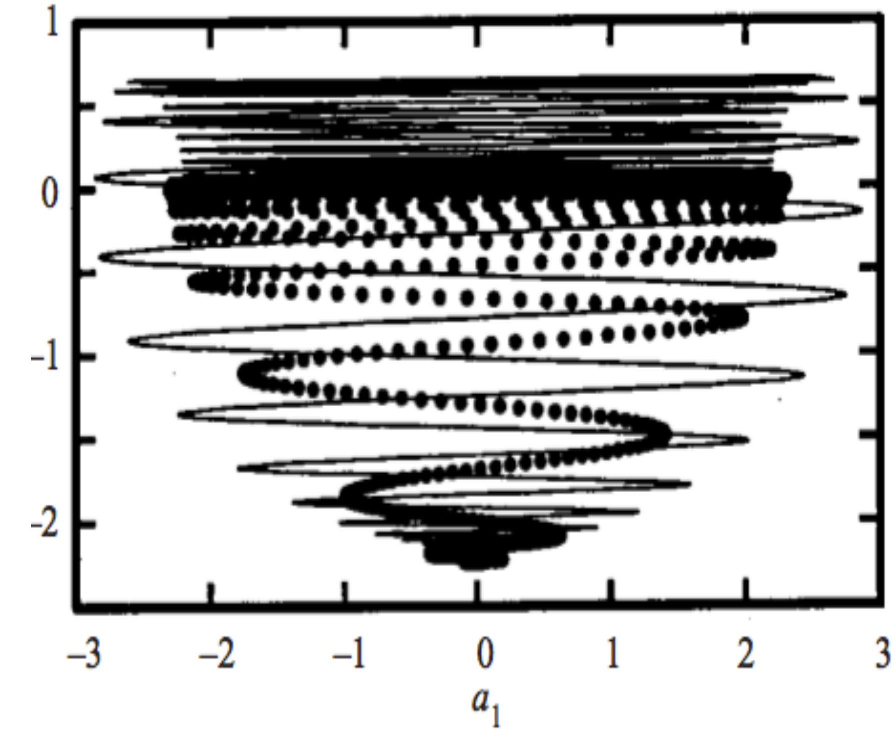
By BERND R. NOACK[1]†, KONSTANTIN AFANASIEV[2], MAREK MORZYŃSKI[3], GILEAD TADMOR[4] AND FRANK THIELE[1]

$$\frac{\mathrm{d}}{\mathrm{d}t}u = \mu u - v - uw,$$
$$\frac{\mathrm{d}}{\mathrm{d}t}v = \mu v + u - vw,$$
$$\frac{\mathrm{d}}{\mathrm{d}t}w = -w + u^2 + v^2.$$

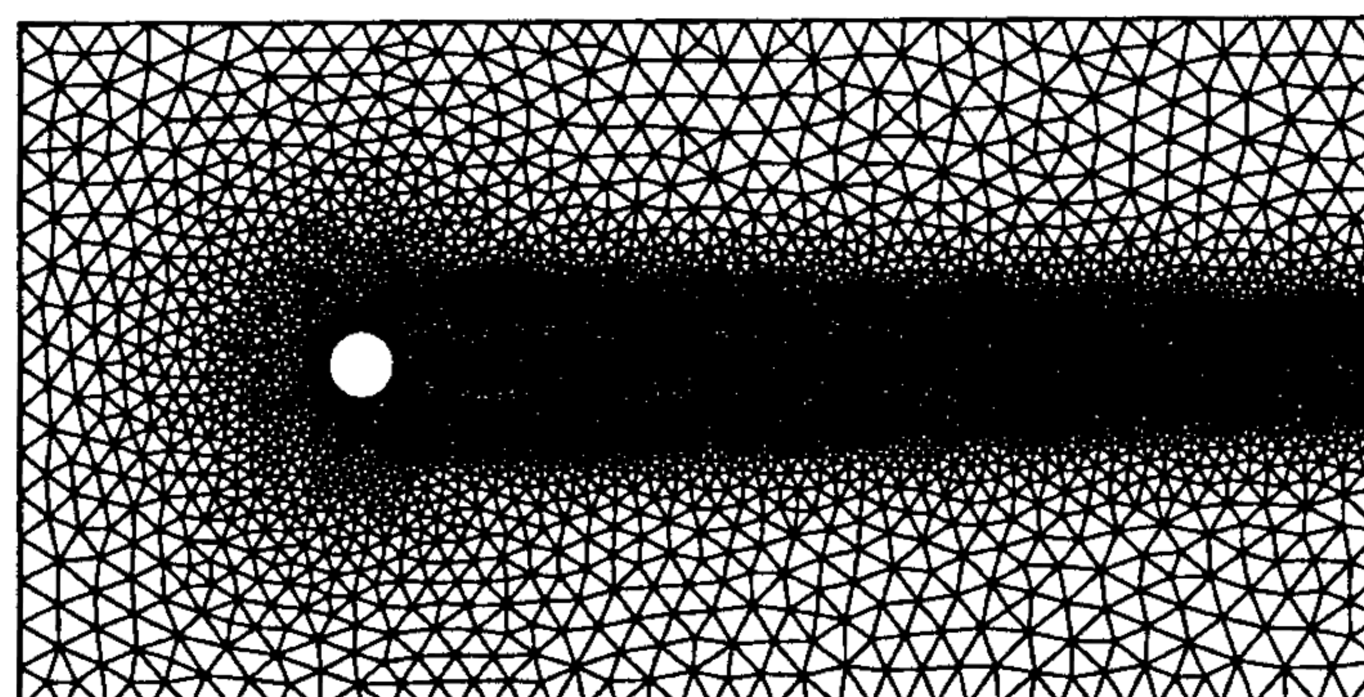


A hierarchy of low-dimensional models for the cylinder wake





Galerkin / POD / Karhunen-Loeve basis set



J. Fluid Mech. (2007), *vol.* 593, *pp.* 333–358.
doi:10.1017/S0022112007008907 Printed in the United Kingdom

Global stability of base and mean flows: a general approach and its applications to cylinder and open cavity flows

DENIS SIPP AND ANTON LEBEDEV
ONERA, 8 rue des Vertugadins, 92190 Meudon, France

Multiple scale expansion near Hopf threshold

$$Re^{-1} = Re_c^{-1} - \epsilon$$

$$\boldsymbol{U}(t) = \boldsymbol{U}_0 + \sqrt{\epsilon}\boldsymbol{U}_1(t, t_1) + \epsilon\boldsymbol{U}_2(t, t_1) + \epsilon\sqrt{\epsilon}\boldsymbol{U}_3(t, t_1) + \cdots$$

Asymptotic/numerical calculation of
mean flow, limit cycle, eigenvectors, …

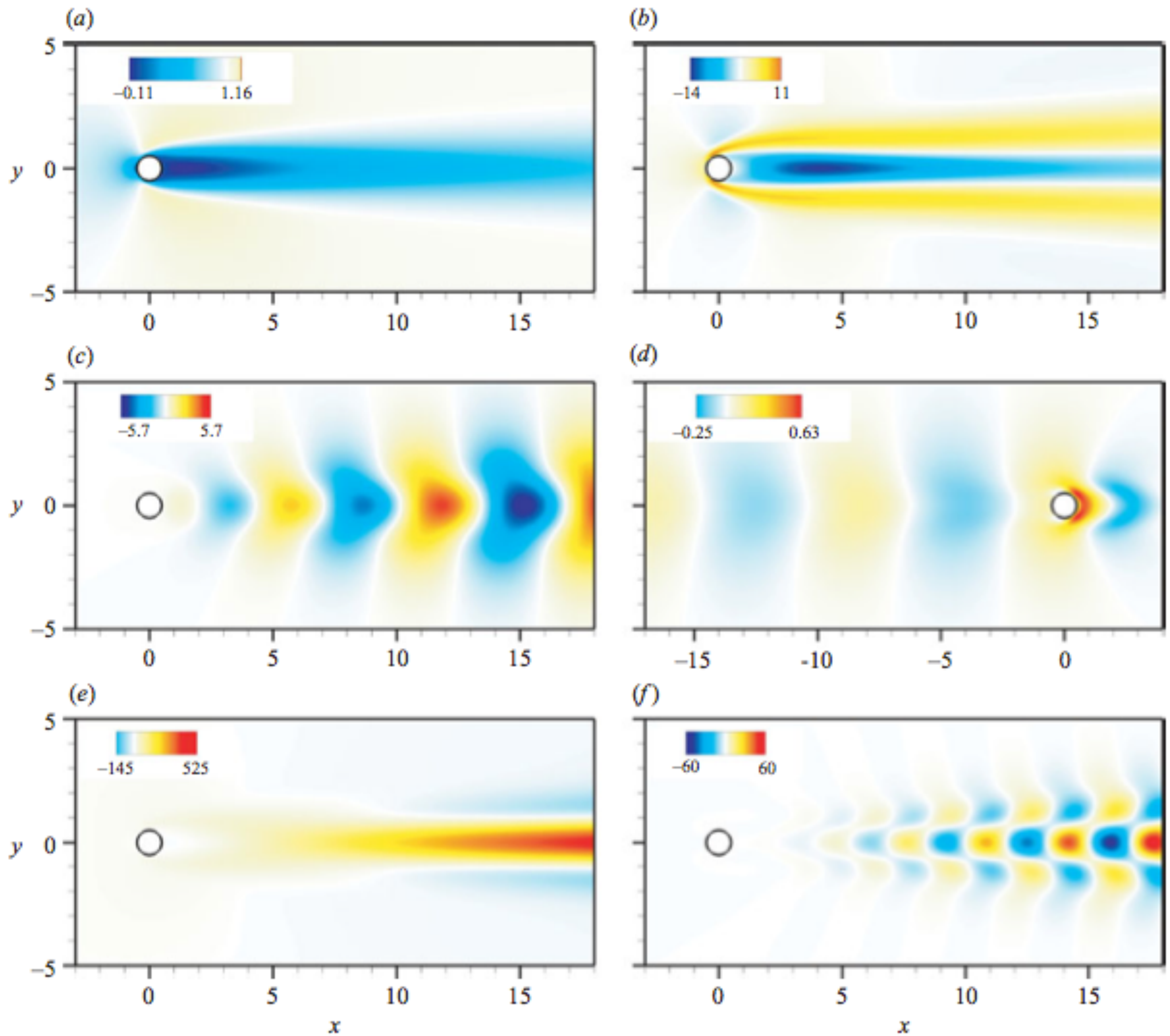


FIGURE 3. Cylinder flow at $Re_c = 46.6$. Representation of the various flow fields appearing at each order in the weakly nonlinear analysis. Mesh C1. Only a small portion of the full computational domain is shown, (*a*) base flow u_0, (*b*) base flow modification u_2^1 due to an ϵ Reynolds increase, (*c*) first harmonic $\mathrm{Re}(v_1^A)$, (*d*) corresponding adjoint eigenmode $\mathrm{Re}(\hat{v}_1^A)$, (*e*) zeroth (mean flow) harmonic $u_2^{|A|^2}$, (*f*) second harmonic $\mathrm{Re}(u_2^{A^2})$.

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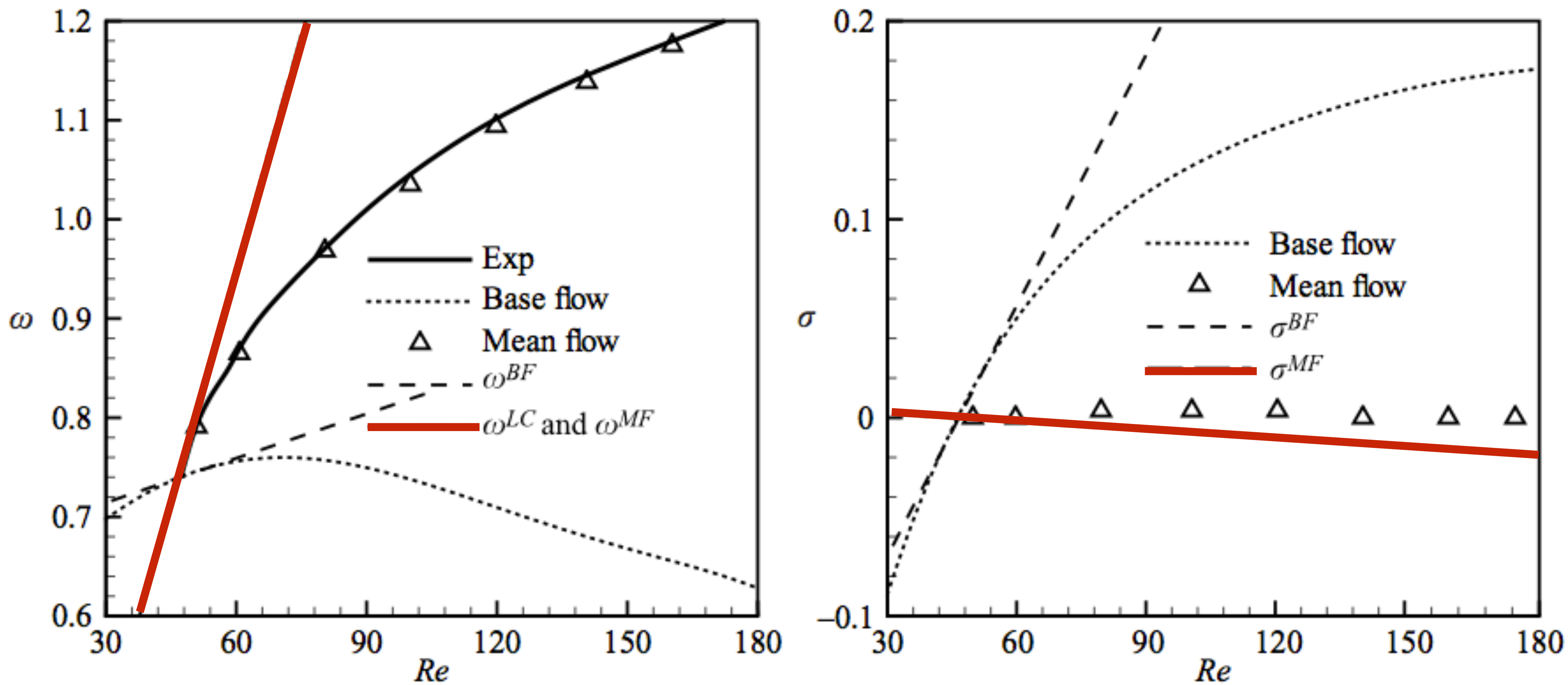

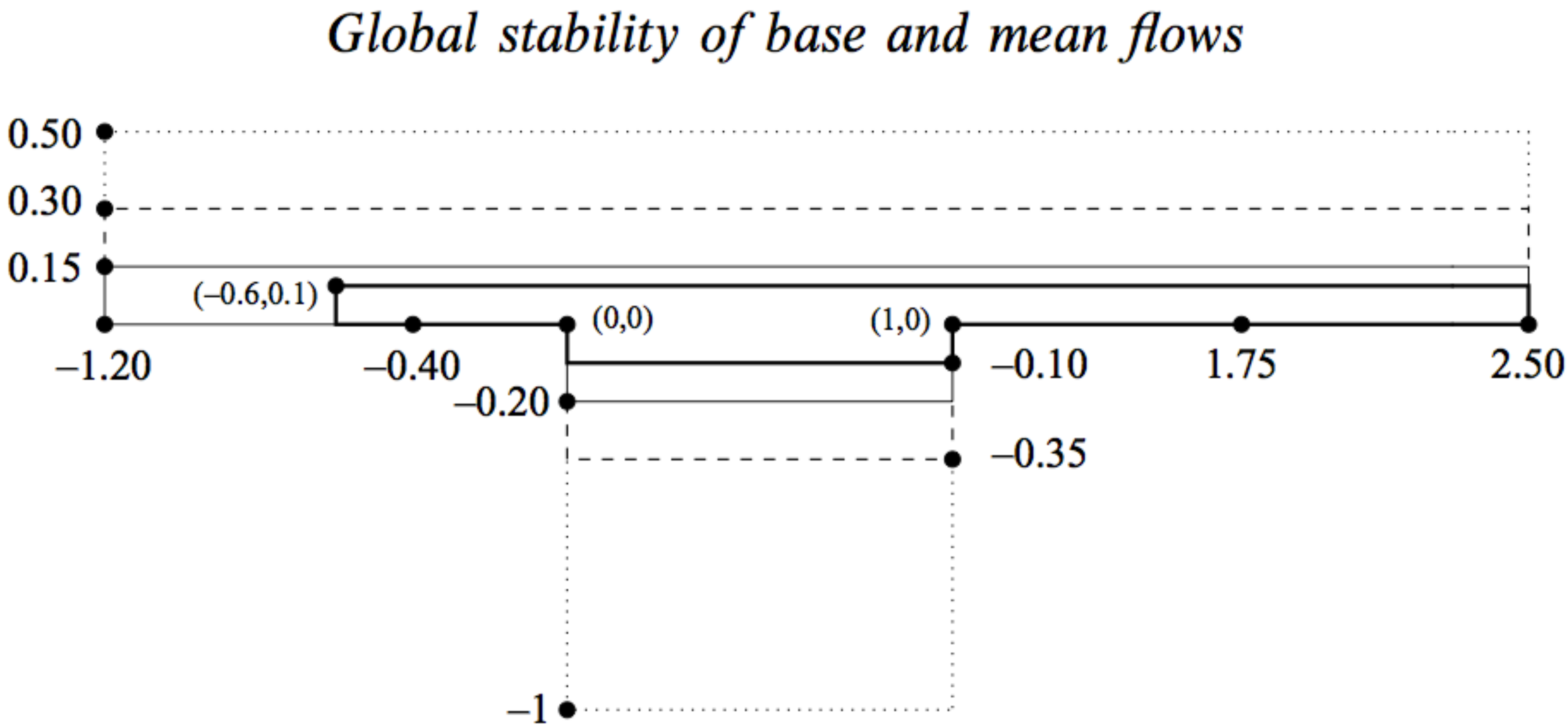

Counter-example of open-cavity flow:
eigenvalues of mean flow of limit cycle
do NOT predict the frequency.
What is the difference? not of traveling wave type?

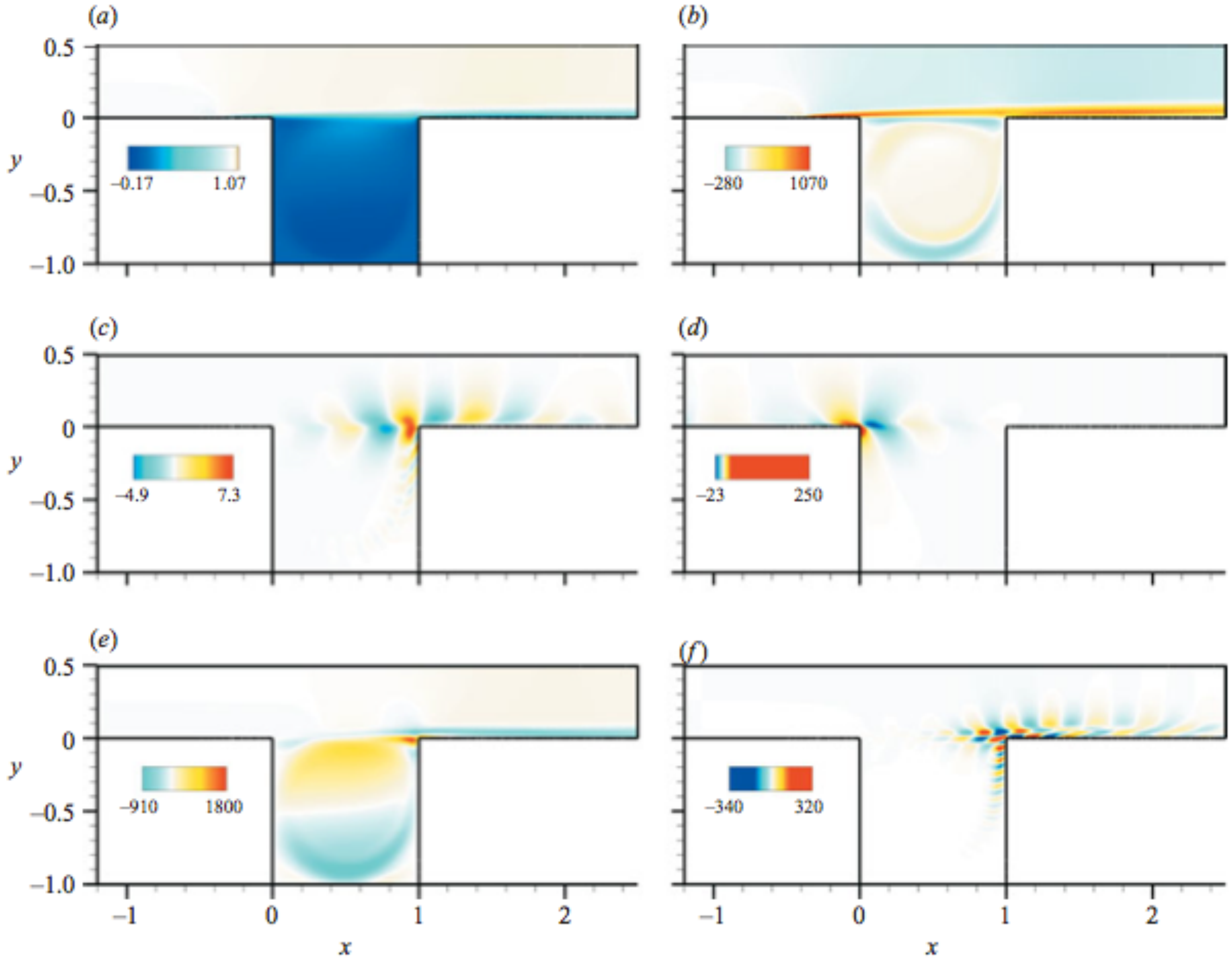

FIGURE 8. Cavity flow at $Re_c = 4140$. Mesh D1. Representation of the various flow fields appearing at each order in the weakly nonlinear analysis. (*a*) Base flow u_0, (*b*) base flow modification u_2^1 due to an increase of the Reynolds number, (*c*) first harmonic $\mathrm{Re}(v_1^A)$, (*d*) corresponding adjoint eigenmode $\mathrm{Re}(\hat{v}_1^A)$, (*e*) zeroth (mean flow) harmonic $u_2^{|A|^2}$, (*f*) second harmonic $\mathrm{Re}(u_2^{A^2})$.

J. Fluid Mech. (2002), *vol.* 458, *pp.* 407–417.
DOI: 10.1017/S0022112002008054 Printed in the United Kingdom

On the frequency selection of finite-amplitude vortex shedding in the cylinder wake

By BENOÎT PIER

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VOLUME 79, NUMBER 20 PHYSICAL REVIEW LETTERS 17 NOVEMBER 1997

Strongly Nonlinear Effect in Unstable Wakes

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(Received 19 December 1996; revised manuscript received 28 May 1997)

We perform experiments and numerical simulations on the mean streamwise velocity field of an unstable cylinder wake. We show that the mean velocity exhibits two weak secondary minima as the consequence of nonlinear interactions, resulting in a strong mean flow correction of the unstable mode. This correction, dominating the basic flow, governs the decrease in the length of the recirculation region ΔL_r in the supercritical regime. This explains the early classical observations of M. Nishioka and H. Sato [J. Fluid Mech. **89**, 49 (1978)]. [S0031-9007(97)04455-4]

PRL **113,** 084501 (2014) PHYSICAL REVIEW LETTERS week ending 22 AUGUST 2014

Self-Consistent Mean Flow Description of the Nonlinear Saturation of the Vortex Shedding in the Cylinder Wake

Vladislav Mantič-Lugo,* Cristóbal Arratia,† and François Gallaire
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(Received 19 December 2013; published 20 August 2014)

$$\partial_t U + \partial_t u = -(U\cdot\nabla)U - (U\cdot\nabla)u - (u\cdot\nabla)U - (u\cdot\nabla)u - \nabla P - \nabla p + \frac{1}{R}\nabla^2 U + \frac{1}{R}\nabla^2 u$$

Temporal mean:

$$0 = -(U\cdot\nabla)U - \langle (u\cdot\nabla)u\rangle - \nabla P + \frac{1}{R}\nabla^2 U$$

Subtract:

$$\partial_t u = -(U\cdot\nabla)u - (u\cdot\nabla)U - (u\cdot\nabla)u + \langle (u\cdot\nabla)u\rangle - \nabla p + \frac{1}{R}\nabla^2 u$$

$$\partial_t U + \partial_t u = -(U \cdot \nabla)U - (U \cdot \nabla)u - (u \cdot \nabla)U - (u \cdot \nabla)u - \nabla P - \nabla p + \frac{1}{R}\nabla^2 U + \frac{1}{R}\nabla^2 u$$

Temporal mean:

$$0 = -(U \cdot \nabla)U - \langle (u \cdot \nabla)u \rangle - \nabla P + \frac{1}{R}\nabla^2 U$$

Subtract:

$$\partial_t u = -(U \cdot \nabla)u - (u \cdot \nabla)U - (u \cdot \nabla)u + \langle (u \cdot \nabla)u \rangle - \nabla p + \frac{1}{R}\nabla^2 u$$

Neglecting nonlinear terms leads to eigenvalue problem for (λ, u):

$$\lambda u = -(U \cdot \nabla)u - (u \cdot \nabla) - \nabla p + \frac{1}{R}\nabla^2 u$$

Reynolds-stress term $A^2 \langle (u \cdot \nabla)u \rangle$ drives U:

$$-(U \cdot \nabla)U - \nabla P + \frac{1}{R}\nabla^2 U = A^2 \langle (u \cdot \nabla)u \rangle$$

Find $A, U, (\lambda, u)$ such that $\lambda = i\omega$

PRL 113, 084501 (2014) PHYSICAL REVIEW LETTERS week ending 22 AUGUST 2014

Self-Consistent Mean Flow Description of the Nonlinear Saturation of the Vortex Shedding in the Cylinder Wake

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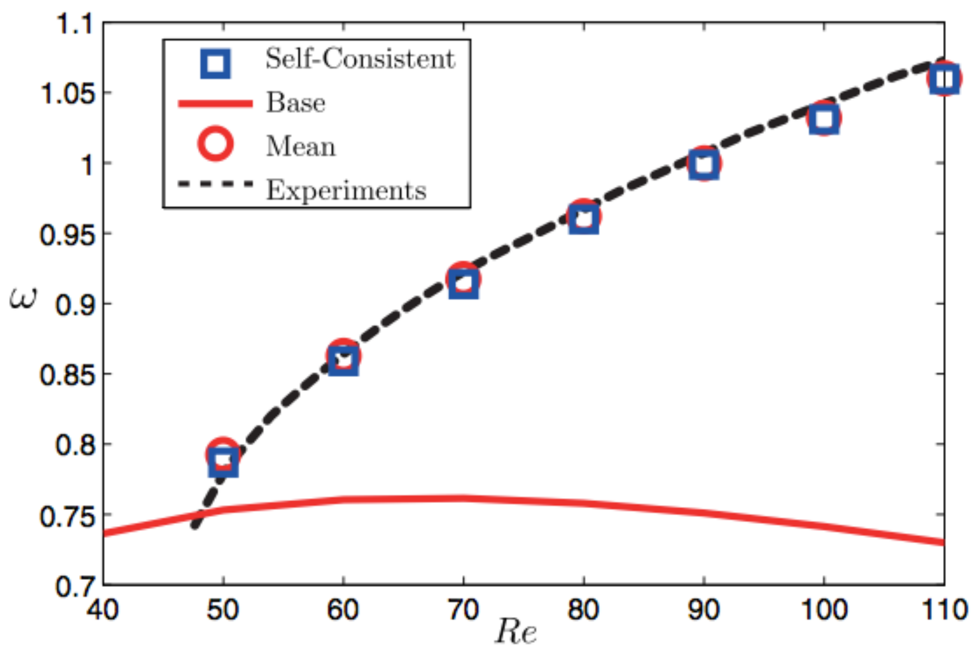
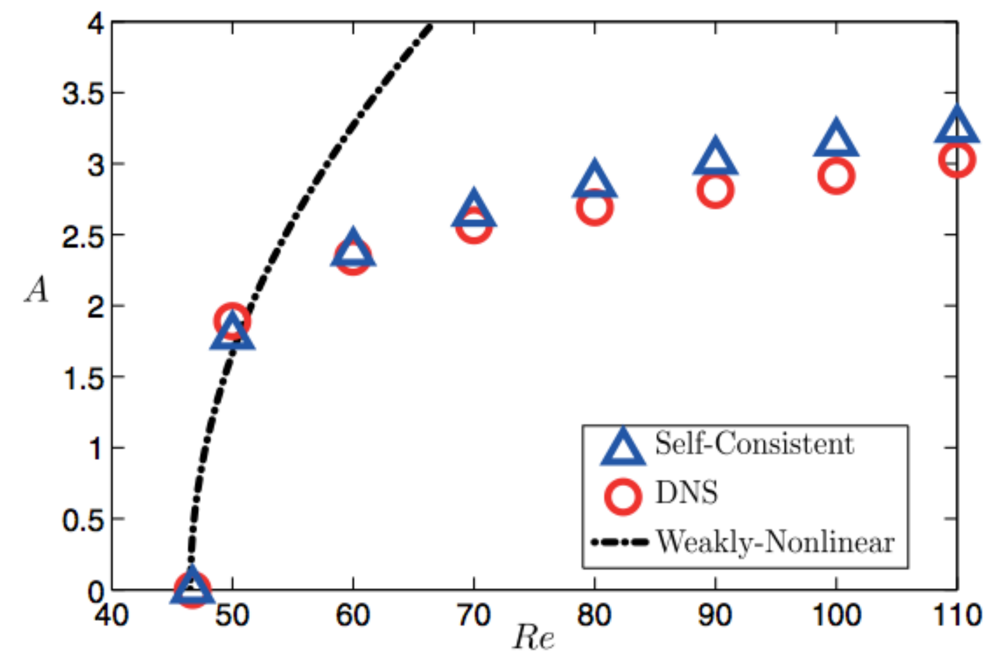
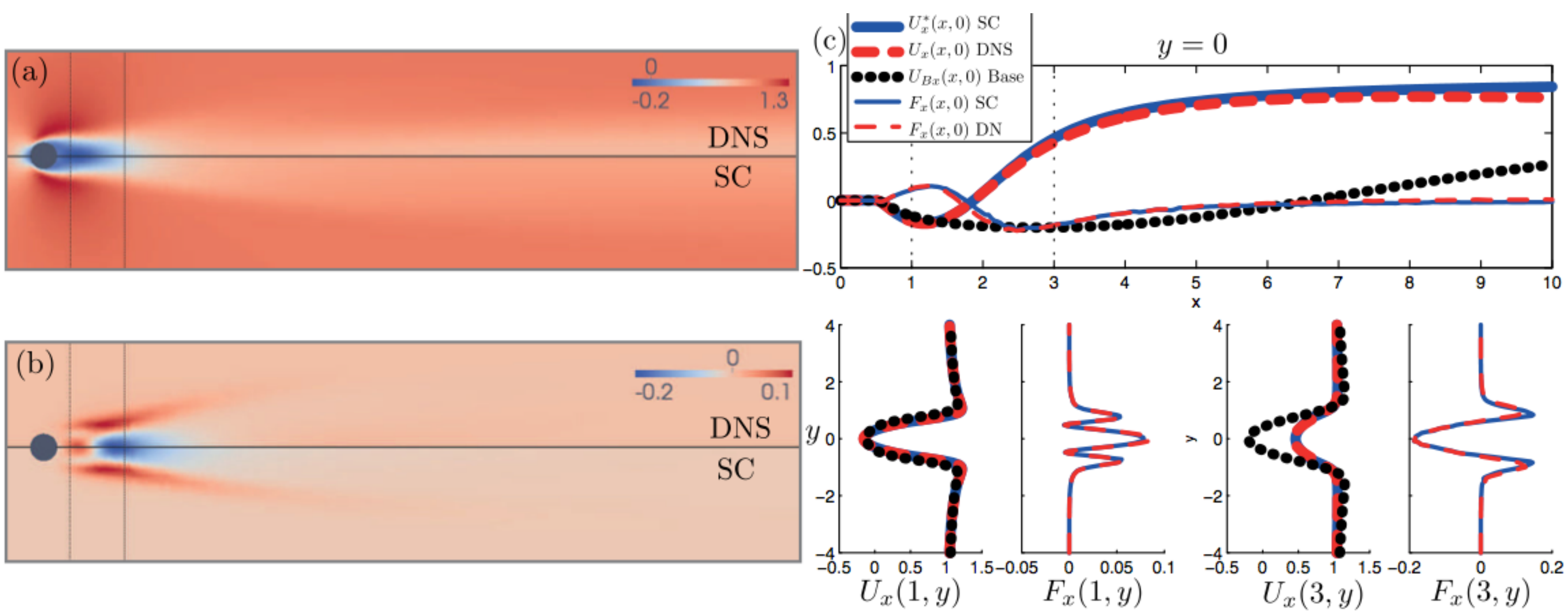




U

$\langle (u\cdot\nabla)u \rangle$



1) Why should $A, U, (\lambda, u)$ be such that $\lambda = i\omega$?

Marginal stability criterion of Malkus (hypothesis)

Outline of a theory of turbulent shear flow
W. V. R. Malkus
Journal of Fluid Mechanics / Volume 1 / Issue 05 / November 1956, pp 521 - 539
1956 Cambridge University Press

2) Why should solution u to linear equation

$$\partial_t u = -(U \cdot \nabla)u - (u \cdot \nabla)U - \nabla p + \frac{1}{R}\nabla^2 u$$

consist of a single eigenmode, instead of a superposition?

Certainly not true in every case.
Temporal spectrum of cylinder wake has a single large peak.

3) How can we justify neglecting nonlinear terms?

$-(u \cdot \nabla)u$ usual linearisation procedure

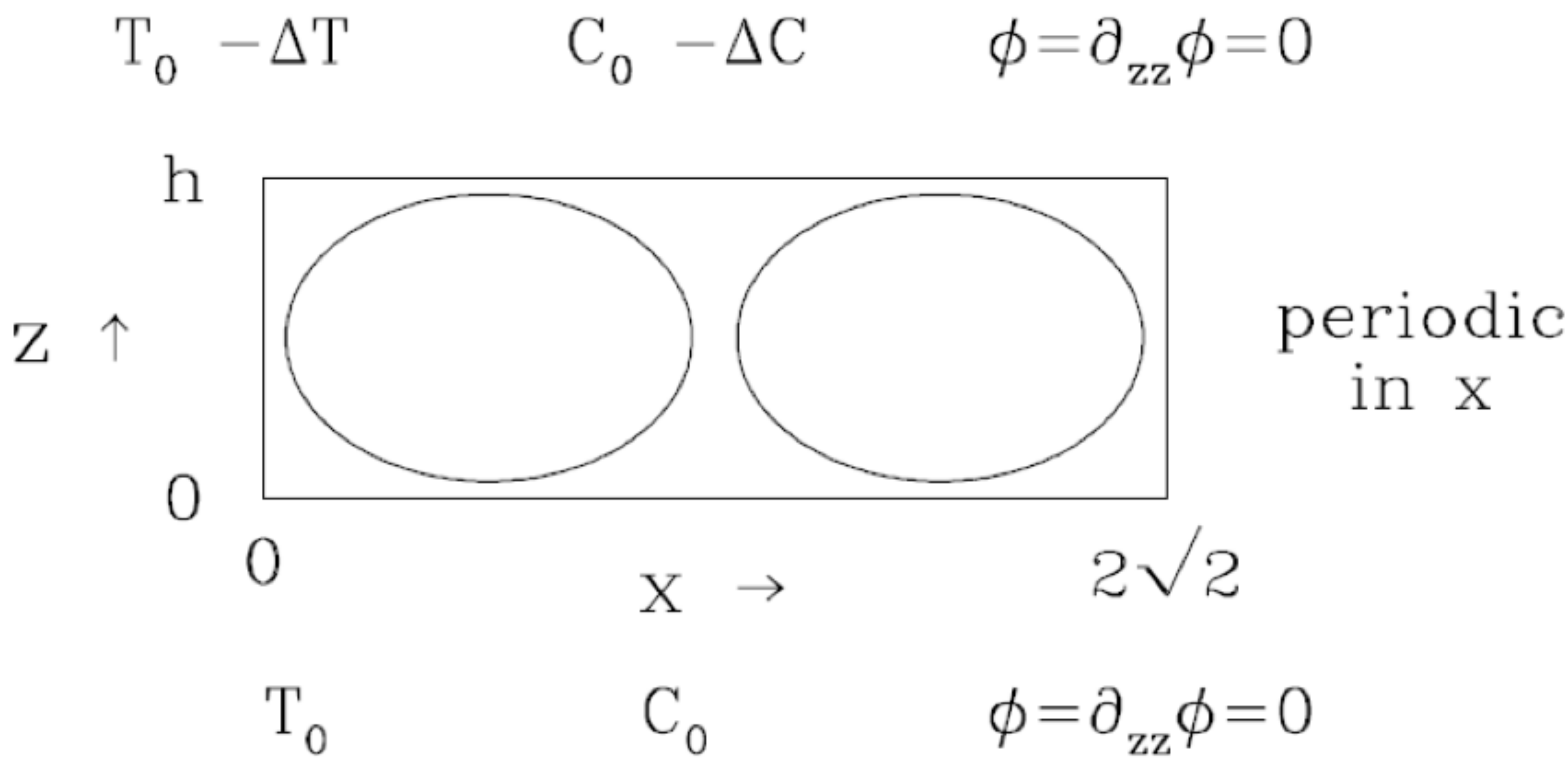
$\langle (u \cdot \nabla)u \rangle$ usual hypothesis used when linearising about something other than a steady state (“force” which maintains the non-steady state)

But what if their SUM were zero or small?

i.e.

$$(u \cdot \nabla)u \approx \langle (u \cdot \nabla)u \rangle$$

Simple Model: 2D Thermosolutal Problem



Vertical thermal and solutal gradients imposed at $z = 0, 1$

Boundary conditions: free-slip at $z = 0, 1$; periodic in x with length $2\sqrt{2}$

Streamfunction $\mathbf{U} = \boldsymbol{\nabla} \times \phi(x, z)\mathbf{e}_y$

Density: $\rho(T,C) = \rho_0 + \rho_T(T - T_0) + \rho_C(C - C_0)$

Diffusivities: κ_T (thermal), κ_C (solutal), ν (momentum)

Conductive solution:

$T = T_0 - z\Delta T/h,\qquad C = C_0 - z\Delta C/h,\qquad \mathrm{U} = \nabla \times \phi \mathrm{e}_y = 0$

Four nondimensional parameters:

Fix: Lewis number $L \equiv \frac{\kappa_C}{\kappa_T} \ll 1$ Prandtl number $P \equiv \frac{\nu}{\kappa_T} \gg 1.$

Vary: Rayleigh number $R \equiv \frac{g\rho_T \Delta T h^3}{\nu \kappa_T}$ Separation ratio $S \equiv \frac{\rho_C \Delta C}{\rho_T \Delta T}$

Subtract conductive solution and nondimensionalize.

Governing Equations:

$$\begin{aligned}
\partial_t \tilde{T} &= \partial_x \tilde{\phi} + \mathrm{e_y} \cdot (\nabla \tilde{\phi} \times \nabla \tilde{T}) + \nabla^2 \tilde{T} \\
\partial_t \tilde{C} &= \partial_x \tilde{\phi} + \mathrm{e_y} \cdot (\nabla \tilde{\phi} \times \nabla \tilde{C}) + L \nabla^2 \tilde{C} \\
\partial_t \nabla^2 \tilde{\phi} &= PR\partial_x(\tilde{T} + S\tilde{C}) + \mathrm{e_y} \cdot (\nabla \tilde{\phi} \times \nabla \nabla^2 \tilde{\phi}) + P\nabla^4 \tilde{\phi}
\end{aligned}$$

Linear Analysis:

$$\left\{ \begin{array}{c} \tilde{T} \\ \tilde{C} \\ \tilde{\phi} \end{array} \right\} (x, z, t) = \left\{ \begin{array}{c} T\cos(kx) \\ C\cos(kx) \\ \phi\sin(kx) \end{array} \right\} \sin(\pi z) e^{(k^2+\pi^2)\sigma t}$$

Nonlinear interaction of these eigenmodes of the basic state

$$\nabla\phi \times \nabla\nabla^2\phi = \nabla\phi \times \nabla(-k^2 - \pi^2)\phi = 0$$

$$\nabla\phi \times \nabla T = \phi T \frac{k\pi}{2} \sin(2\pi z)$$

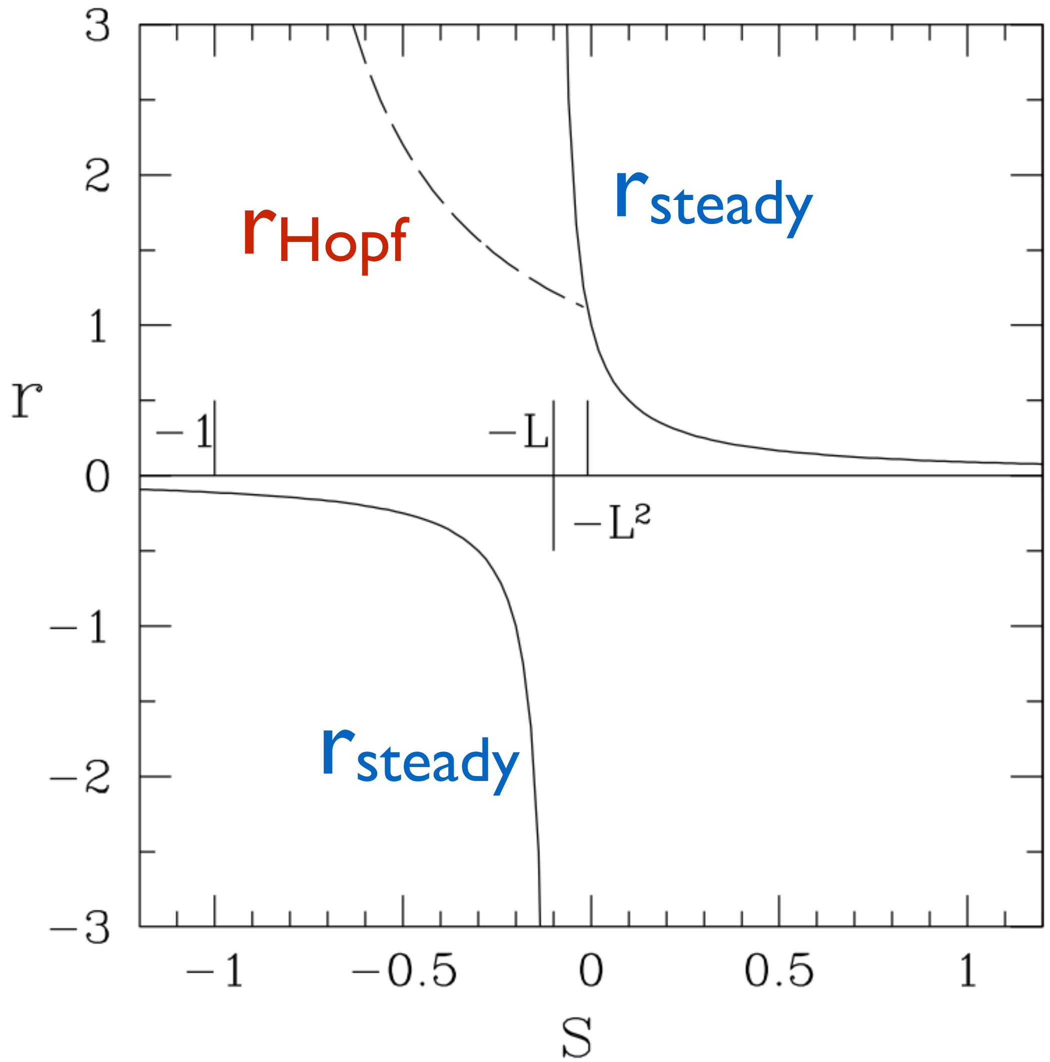
$$\nabla\phi \times \nabla C = \phi C \frac{k\pi}{2} \sin(2\pi z)$$

At lowest order, mean “flow” has $u = 0, T \neq 0, C \neq 0$

Hopf bifurcation to standing or traveling waves if separation ratio $S=Ra_C/Ra_T<0$

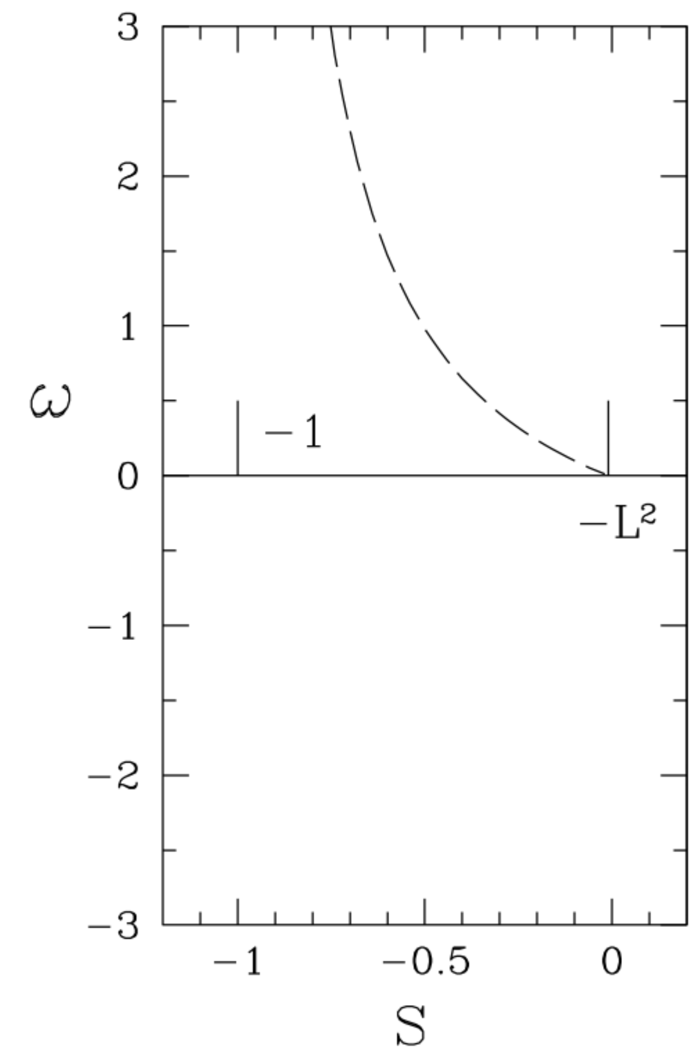
Temperature and concentration gradients are in opposite directions

convection threshold
(reduced Rayleigh number)

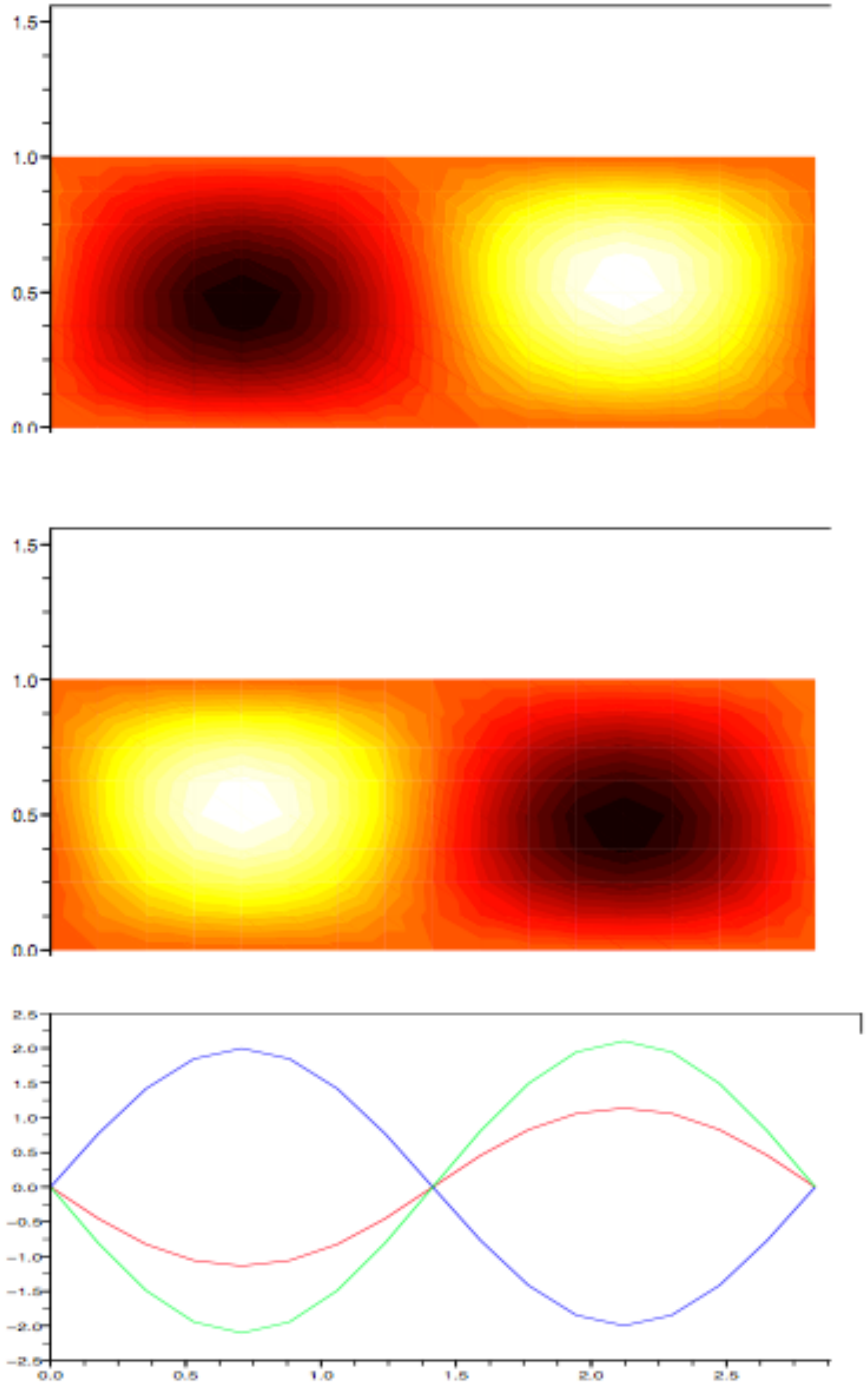


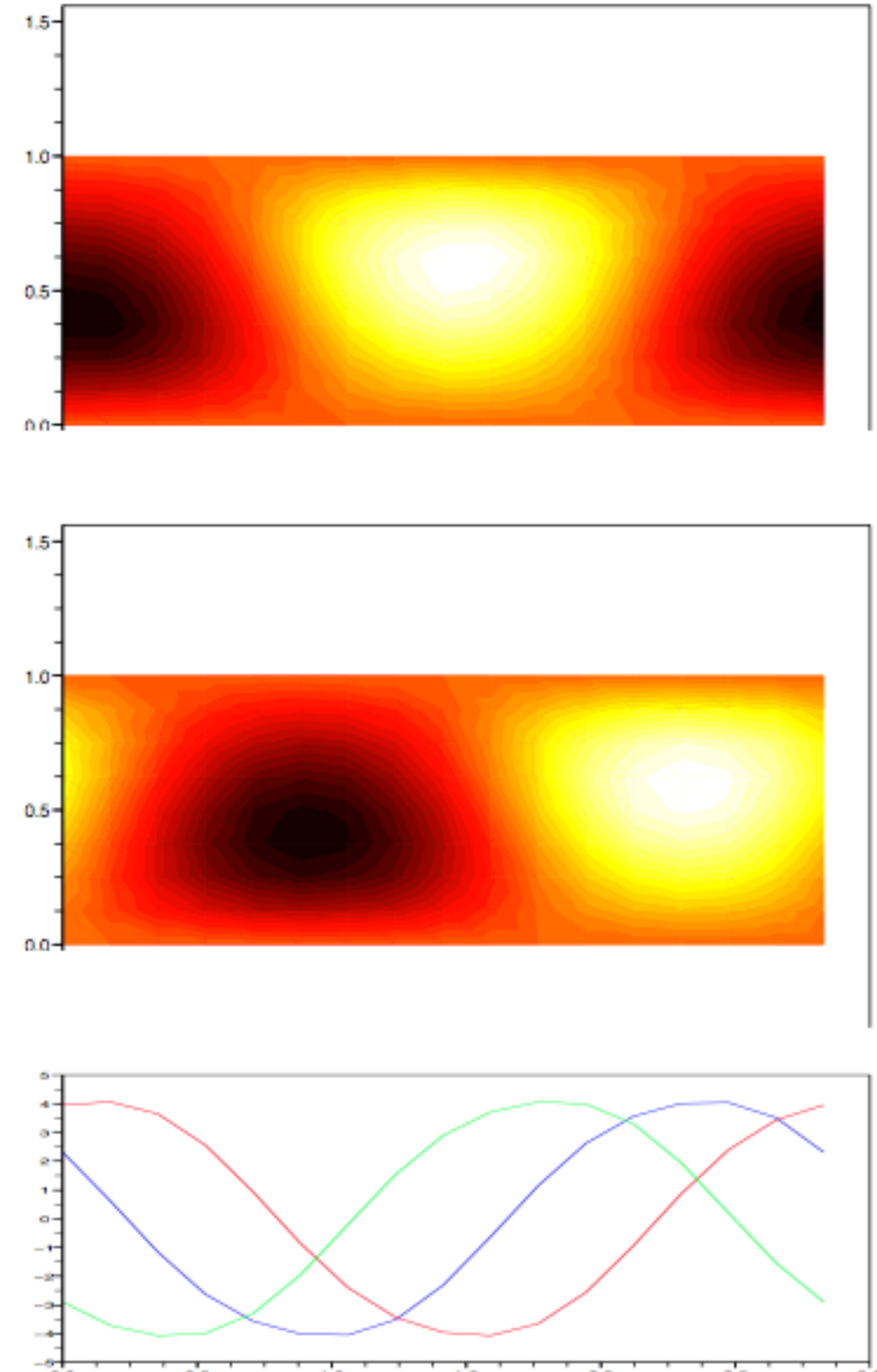
Separation ratio

Hopf frequency



Separation ratio

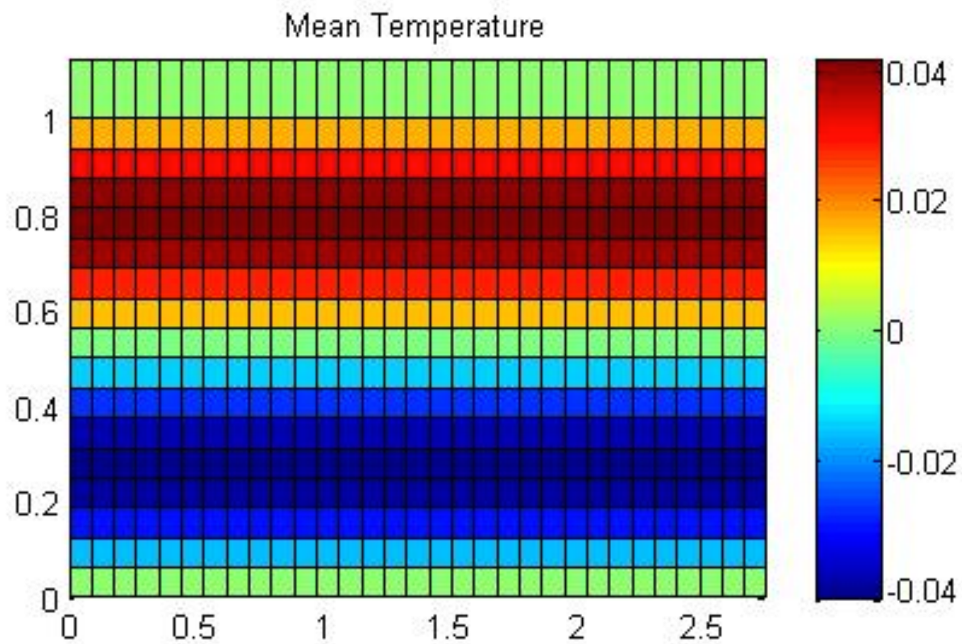


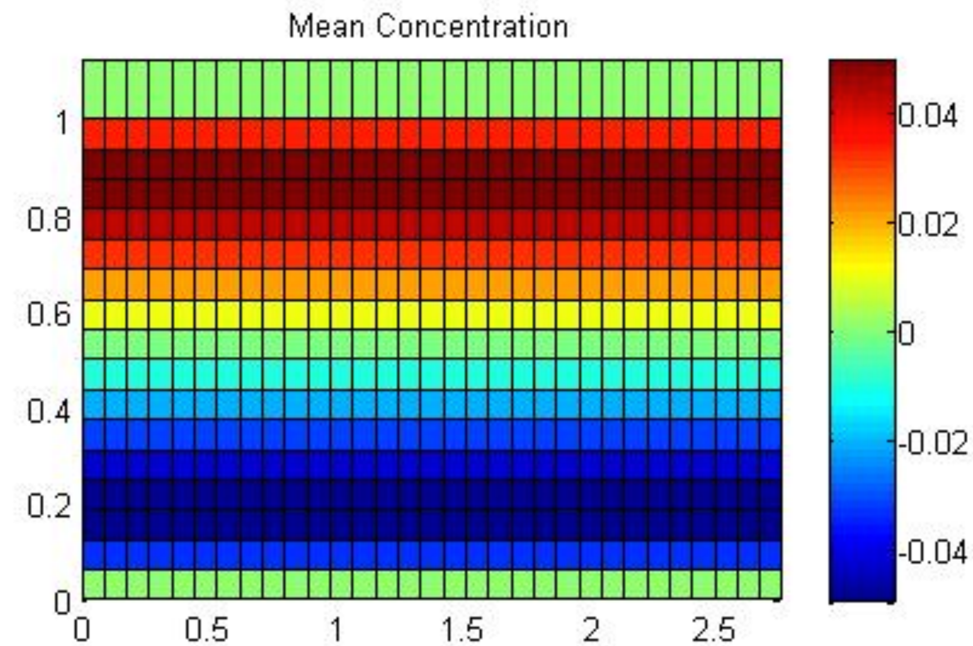


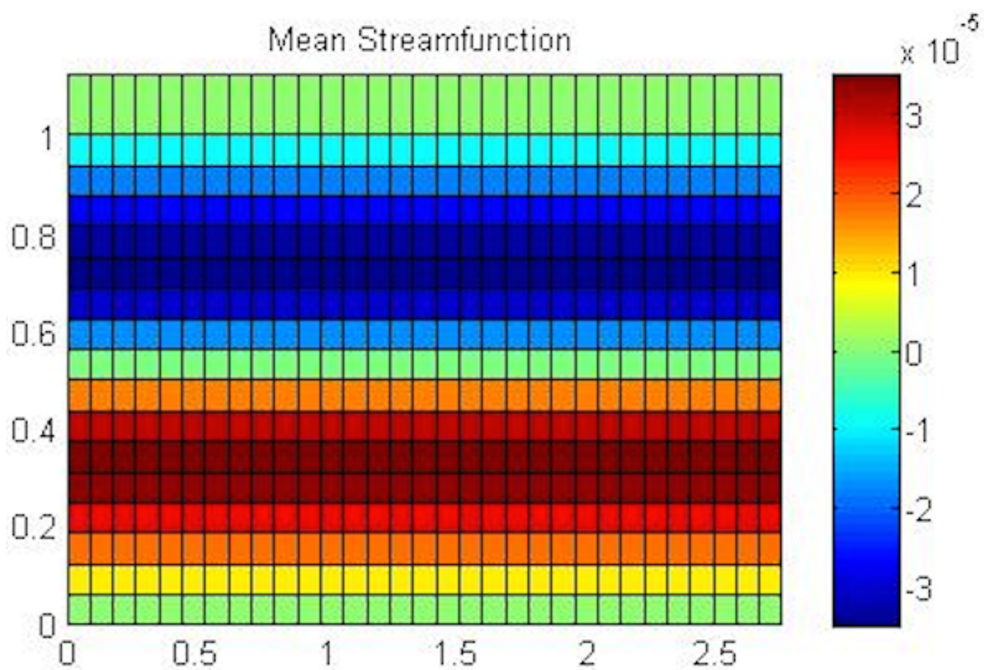
Standing waves **Travelling waves**

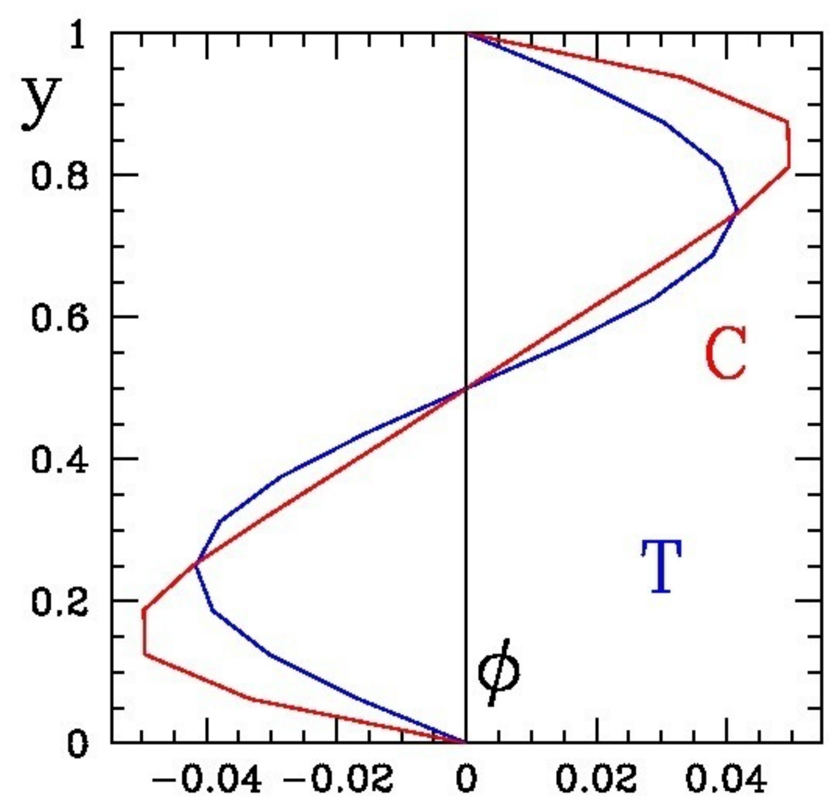
Thermosolutal convection with $S = -0.1, L = 0.1, Pr = 10, r \equiv Ra/Ra_c = 1.3.$

Mean field $\sim \sin(2\pi z)$



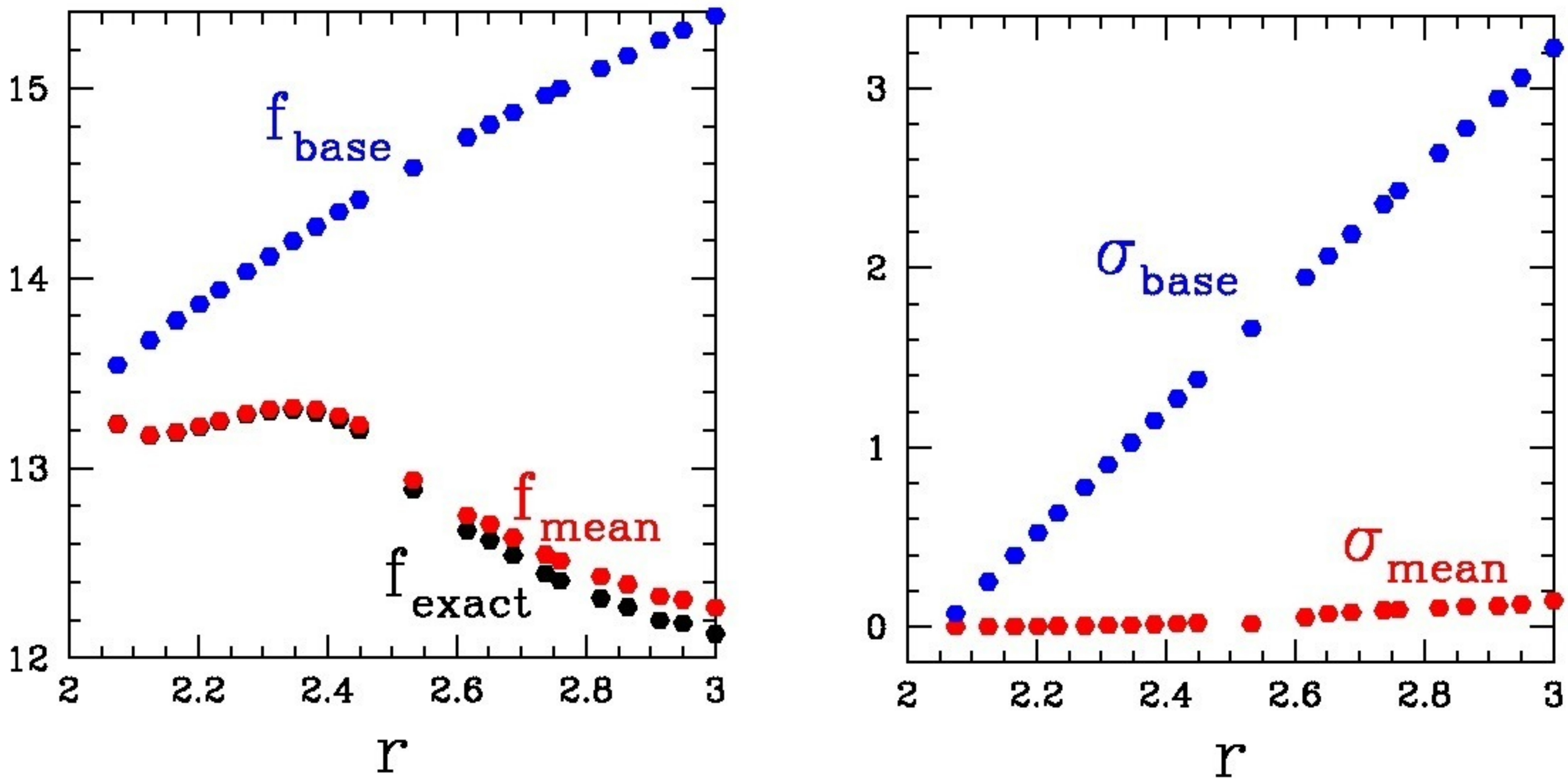






r/r_c = 2.333/2.0539=1.14

Thermosolutal convection
Eigenvalues of base flow and of mean field



P=10, L=0.1, S=-0.5

Justification of self-consistent theory
(temperature/concentration fields)

For rightgoing linear traveling waves in thermosolutal convection, we can write

$$\mathbf{u}(x,z,t) = \begin{pmatrix} \tau_c(z) \\ c_c(z) \\ \phi_c(z) \end{pmatrix} \cos(kx-\omega t) + \begin{pmatrix} \tau_s(z) \\ c_s(z) \\ \phi_s(z) \end{pmatrix} \sin(kx-\omega t)$$

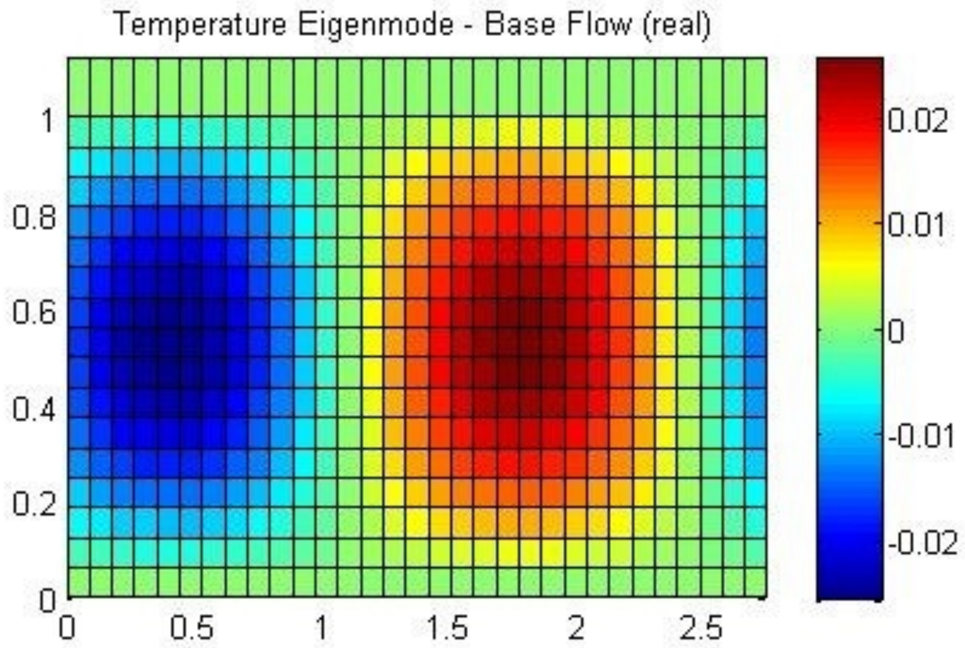
The nonlinear interaction terms are $\begin{pmatrix} J[\phi,\tau] \\ J[\phi,c] \\ J[\phi,\nabla^2\phi] \end{pmatrix}$ where $J[f,g] \equiv \partial_z f \partial_x g - \partial_x f \partial_z g$

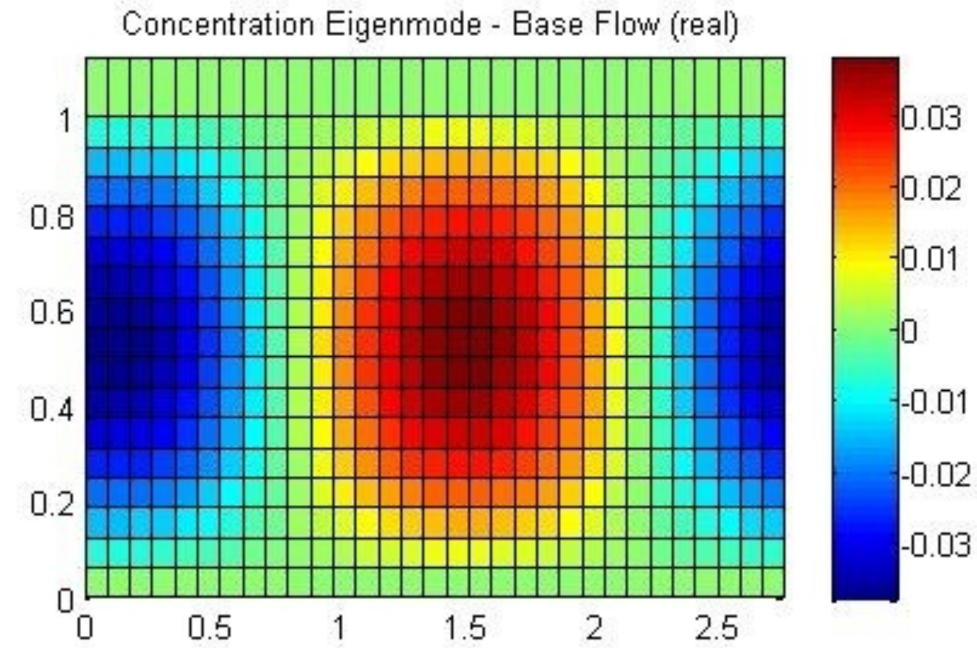
$$\begin{aligned}
J[\phi,\tau] &= k\left[(-{\phi_c}'\tau_c + {\phi_s}'\tau_s + \phi_c{\tau_c}' - \phi_s{\tau_s}')\cos()\sin()\right. \\
&+ \left.({\phi_c}'\tau_s - \phi_s{\tau_c}')\cos()\cos() + (-{\phi_s}'\tau_c + \phi_c{\tau_s}')\sin()\sin()\right] \\
&= \frac{k}{2}\left[({\phi_c}'\tau_c - {\phi_s}'\tau_s - \phi_c{\tau_c}' + \phi_s{\tau_s}')\sin(2(kx-\omega t))\right. \\
&+ ({\phi_c}'\tau_s - \phi_s{\tau_c}' - {\phi_s}'\tau_c + \phi_c{\tau_s}') \\
&+ \left.({\phi_c}'\tau_s - \phi_s{\tau_c}' + {\phi_s}'\tau_c - \phi_c{\tau_s}')\cos(2(kx-\omega t))\right] \\
&= \frac{k}{2}\left[\left((\tau_s/\phi_s)'\,\phi_s^2 - (\tau_c/\phi_c)'\,\phi_c^2\right)\sin(2(kx-\omega t))\right. \\
&+ (\phi_c\tau_s)' - (\phi_s\tau_c)' \\
&- \left.\left((\tau_c/\phi_s)'\,\phi_s^2 + (\tau_s/\phi_c)'\,\phi_c^2\right)\cos(2(kx-\omega t))\right] \\
\langle J[\phi,\tau]\rangle &= \frac{k}{2}\left[(\phi_c\tau_s)' - (\phi_s\tau_c)'\right]
\end{aligned}$$

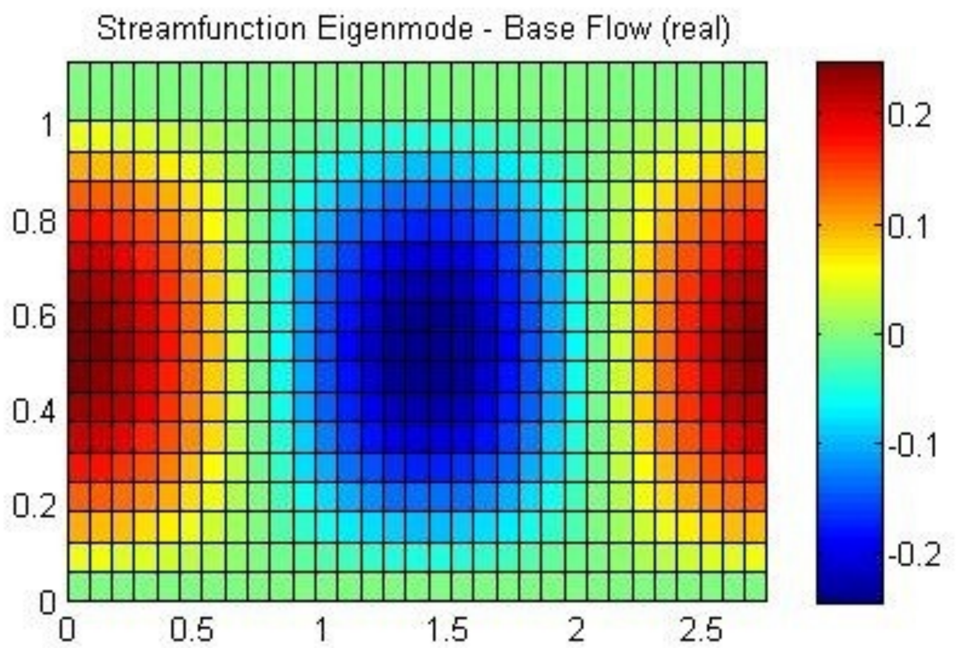
If $\phi_c, \tau_c, \phi_s, \tau_s$ all have the same z dependence, then the terms multiplying the trigonometric functions are zero, and so:

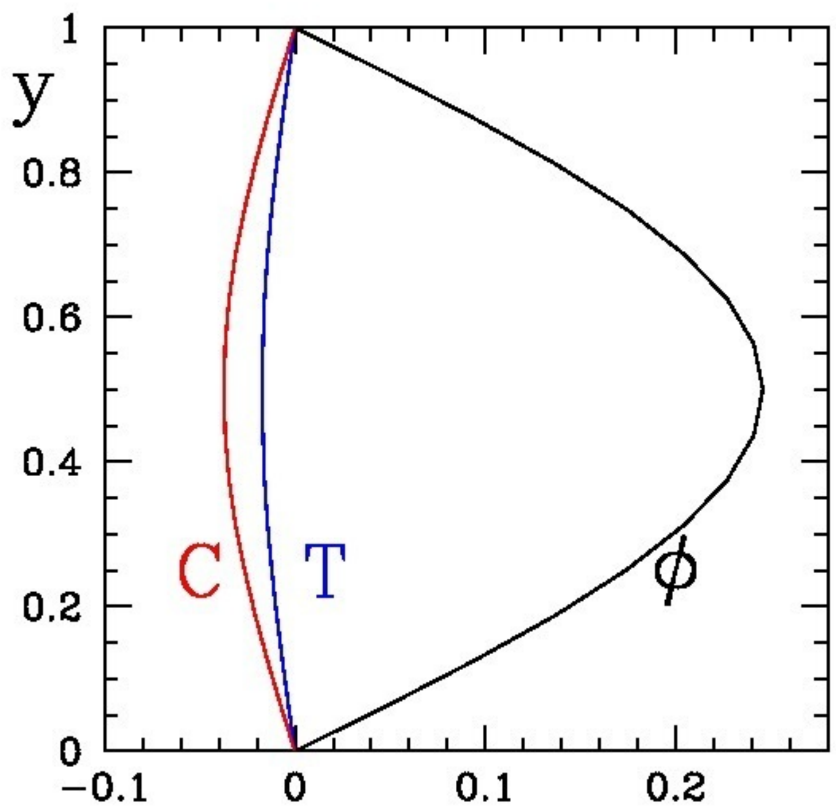
$$J[\phi,\tau] = \langle J[\phi,\tau]\rangle$$

Eigenmode of base flow (real part)

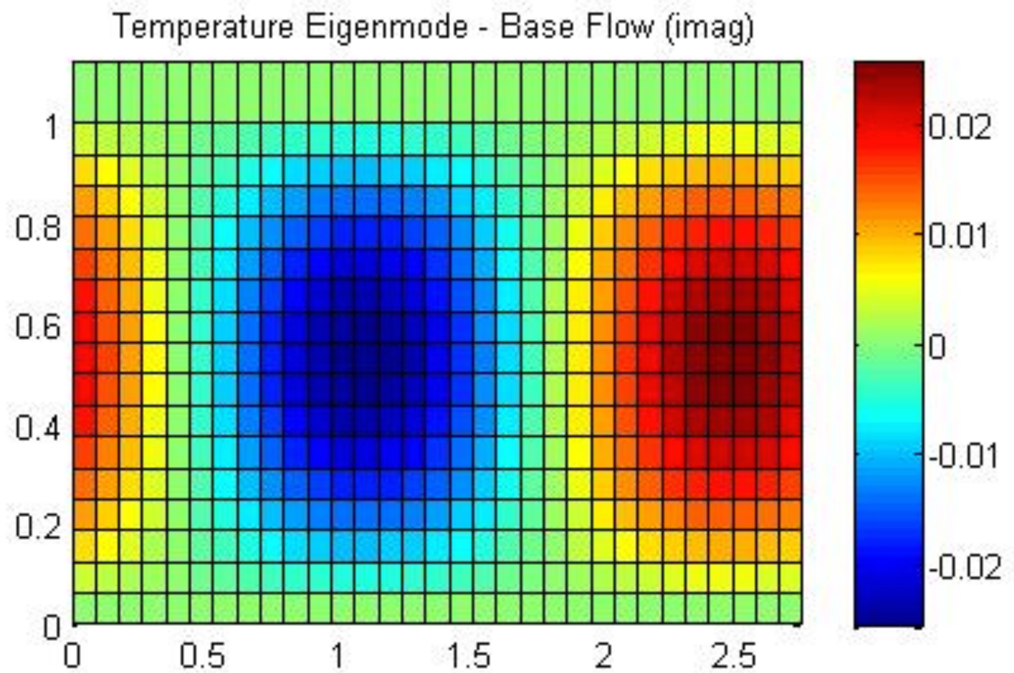


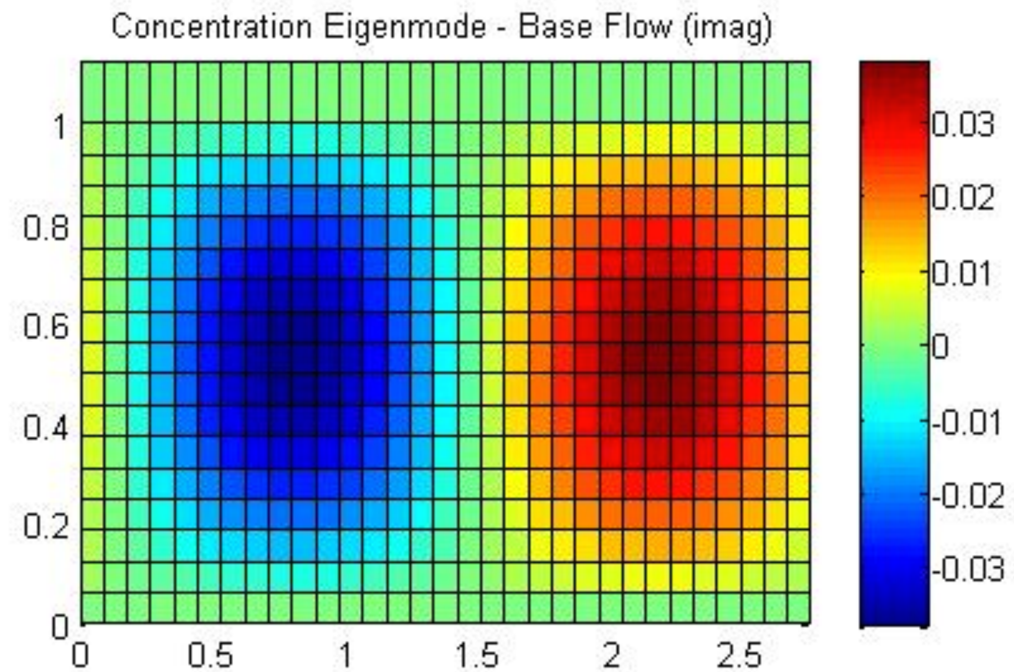


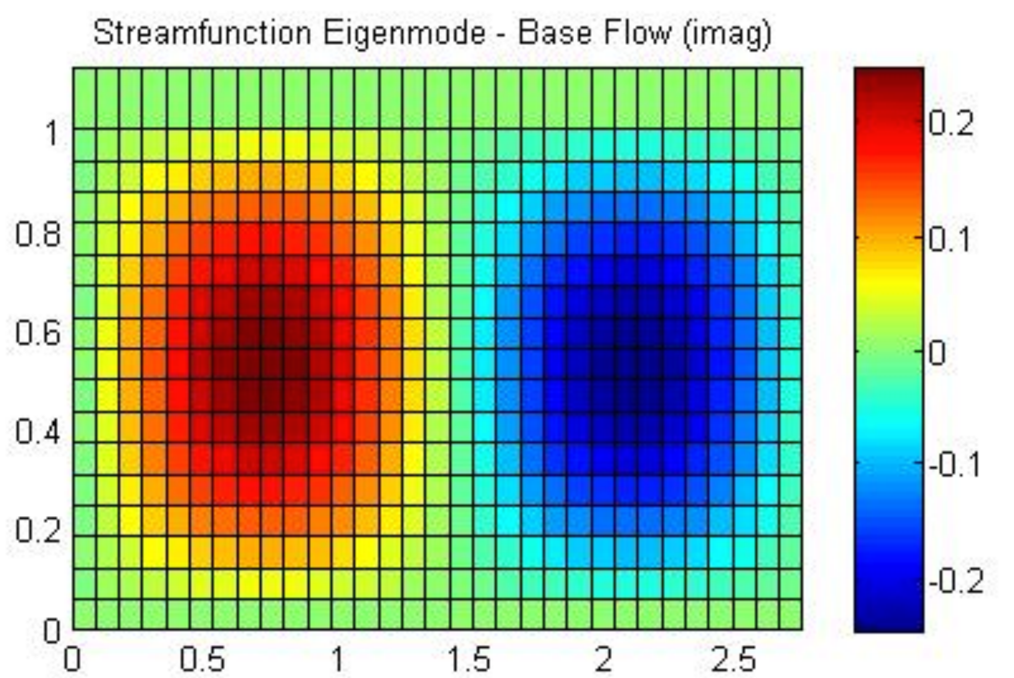


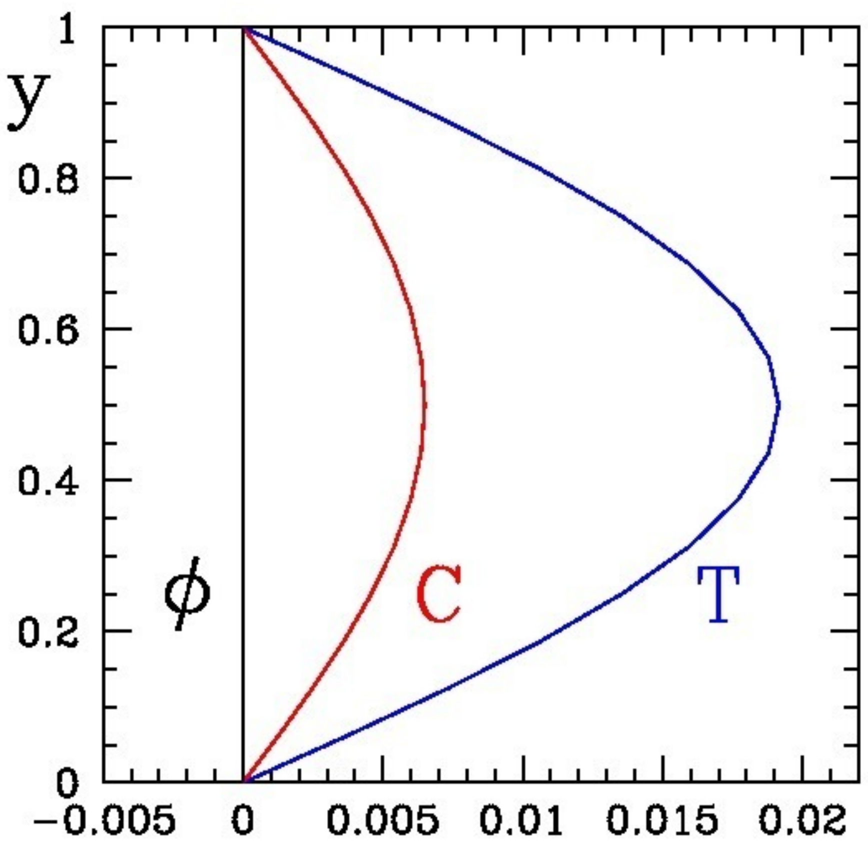


Eigenmode of base flow (imag part)

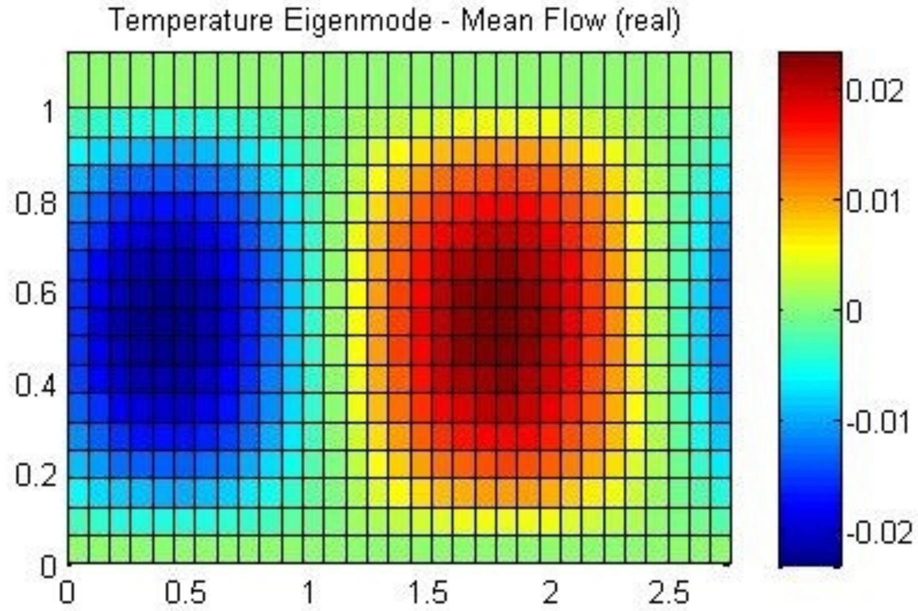


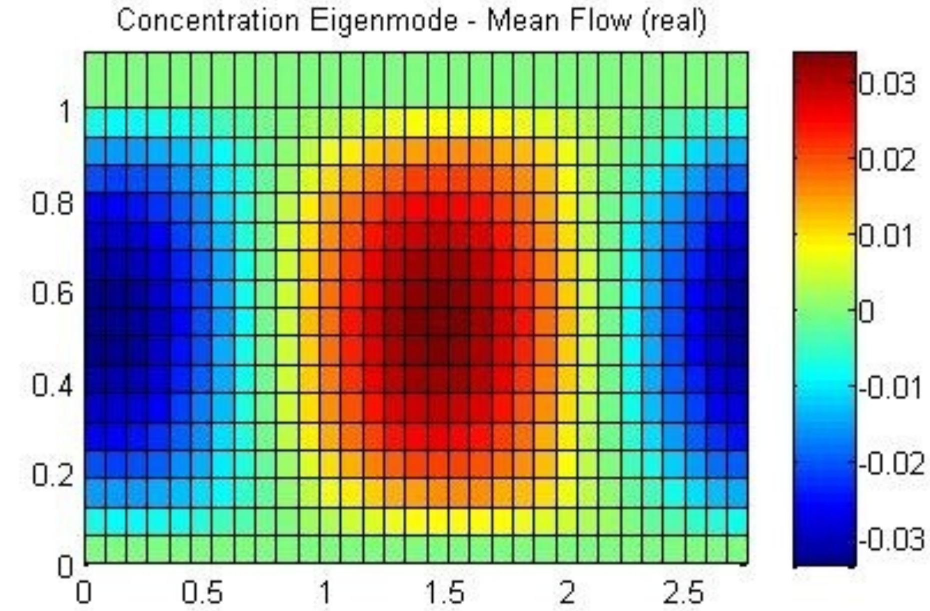


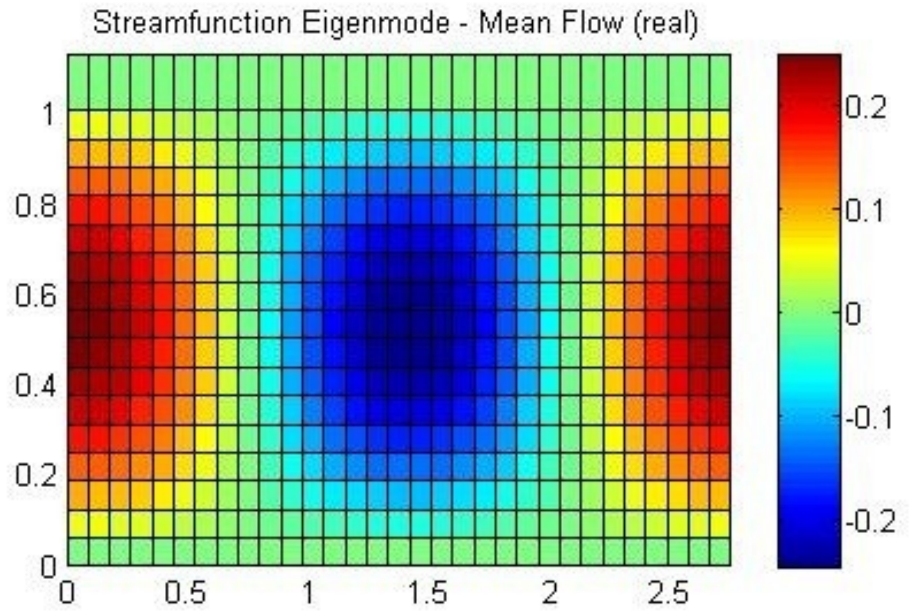


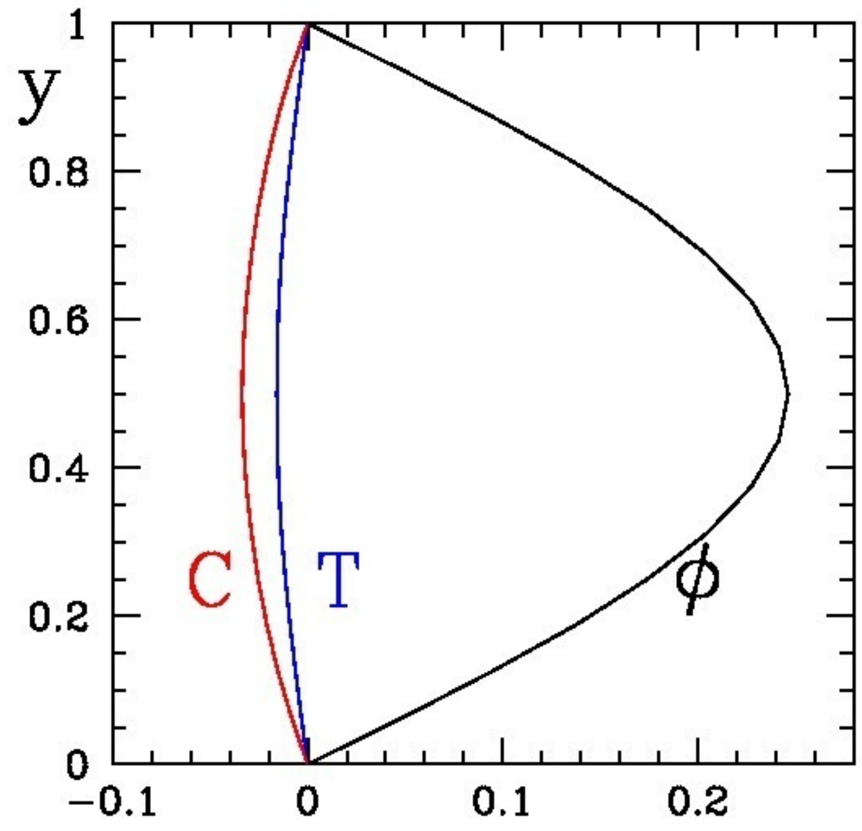


Eigenmode of mean flow (real part)

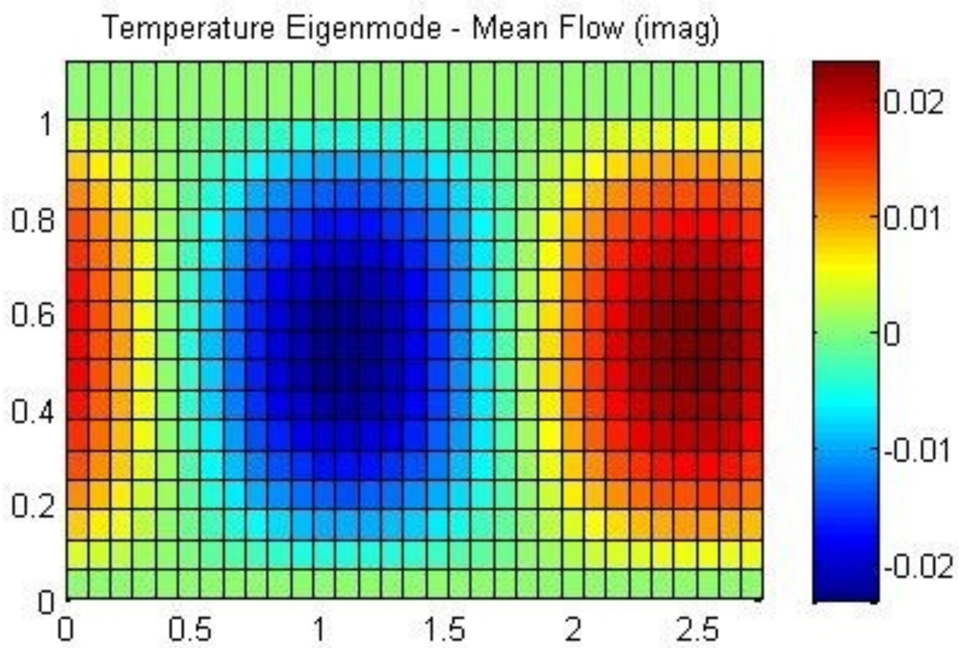


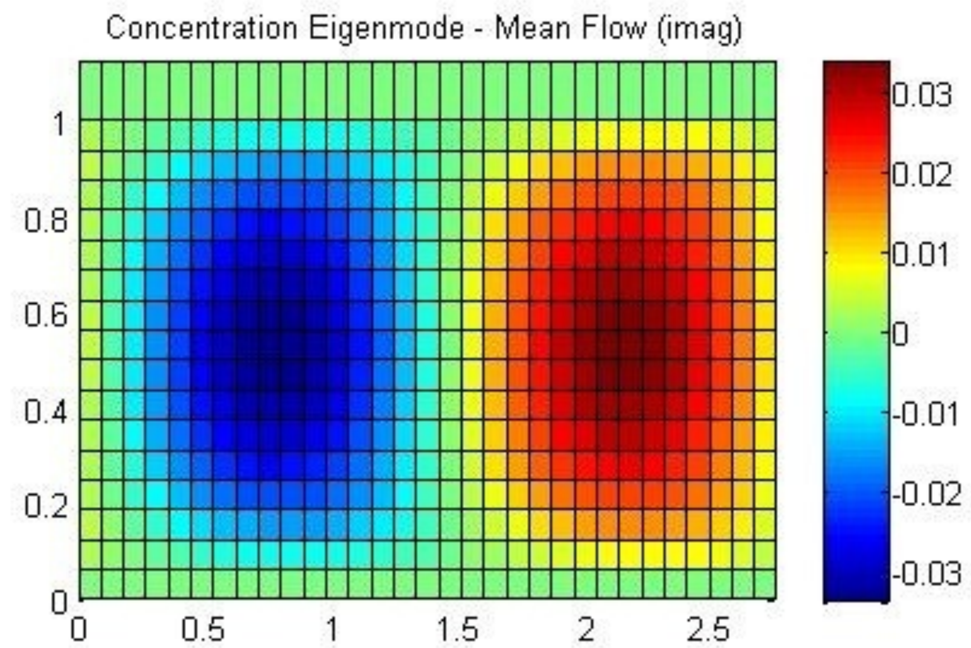


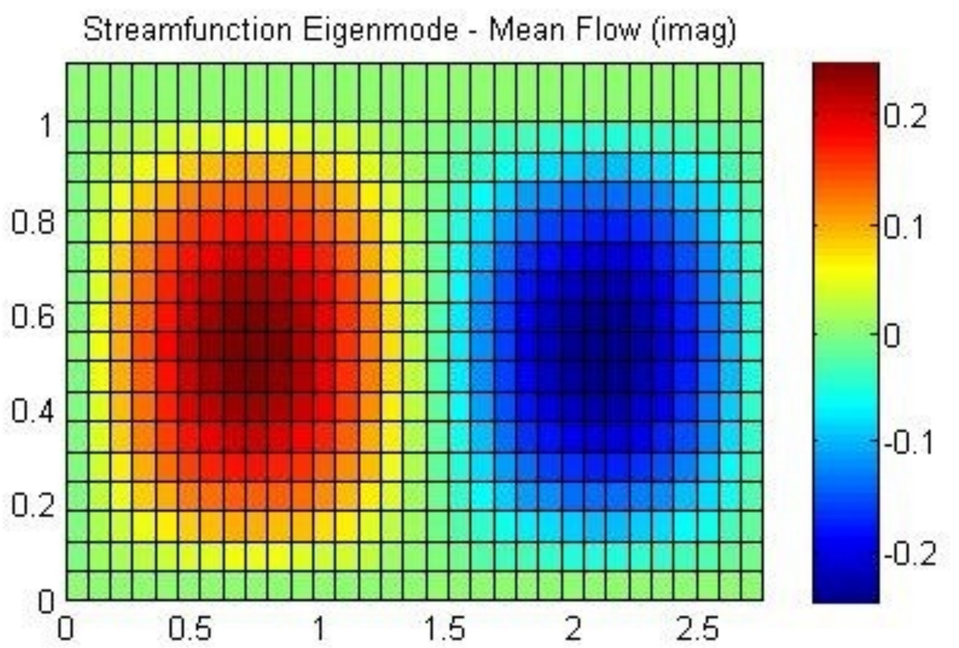


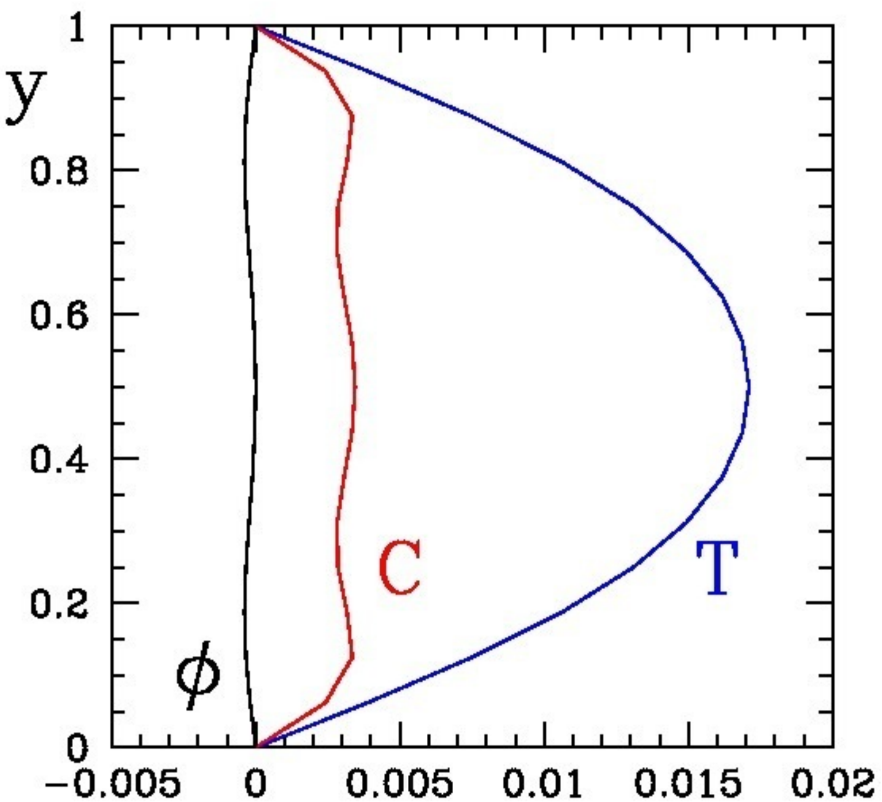


Eigenmode of mean flow (imag part)









Justification of self-consistent theory (velocity field)

$$u(x,z,t) = u_c(x,z)\cos(\omega t) + u_s(x,z)\sin(\omega t)$$

The nonlinear interaction is:

$$\begin{aligned}
J &= ((u_s\cdot\nabla)u_c + (u_c\cdot\nabla)u_s)\ \cos(\omega t)\sin(\omega t) \\
&+ (u_c\cdot\nabla)u_c\ \cos^2(\omega t) + (u_s\cdot\nabla)u_s\ \sin^2(\omega t) \\
&= ((u_s\cdot\nabla)u_c + (u_c\cdot\nabla)u_s)\ \sin(2\omega t)/2 \\
&+ (u_c\cdot\nabla)u_c\ (1+\cos(2\omega t))/2 + (u_s\cdot\nabla)u_s\ (1-\cos(2\omega t))/2 \\
&= ((u_s\cdot\nabla)u_c + (u_c\cdot\nabla)u_s)\ \sin(2\omega t)/2 \\
&+ (u_c\cdot\nabla)u_c + (u_s\cdot\nabla)u_s \\
&+ ((u_c\cdot\nabla)u_c - (u_s\cdot\nabla)u_s)\ \cos(2\omega t)/2
\end{aligned}$$

The condition $J = \langle J\rangle$ requires that the terms multiplying $\sin(2\omega t)$ and $\cos(2\omega t)$ vanish, i.e.

$$\begin{aligned}
(u_s\cdot\nabla)u_c + (u_c\cdot\nabla)u_s &= 0 \\
(u_c\cdot\nabla)u_c - (u_s\cdot\nabla)u_s &= 0
\end{aligned}$$

The mean flow is generated by

$$(u_c\cdot\nabla)u_c + (u_s\cdot\nabla)u_s$$

Stay tuned …

Thank you!