Numerical simulation of banded turbulence in plane Couette flow

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ABSTRACT

An analogy exists between Taylor-Couette and plane Couette flow. Both are driven by differential motion of their boundaries in the azimuthal (θ) or streamwise (x) direction, and contain another homogeneous direction (z), termed axial or spanwise, for the respective flows, whose extent is usually taken as infinite in theoretical analyses and large in experiments. Spiral turbulence, an intriguing state in which a region of turbulent flow spirals around a laminar region, was observed in Taylor-Couette flow by Andereck et al. [1] and Hegseth et al. [2]. By using an experimental apparatus with a very narrow gap ($r_{\text{out}} - r_{\text{in}}$), Prigent et al. [3] were able to produce a spiral turbulent state containing several regular repetitions of the pattern and to measure the dependence of azimuthal and axial wavelength on Reynolds number. They also discovered that such a state exists in plane Couette flow as well, where it takes the form of oblique bands, oriented at an angle to the streamwise direction.

We have carried out numerical simulations of plane Couette flow using Prism, a spectral element code written by Henderson [4]. An appealing feature of the banded turbulent flow is that the Reynolds numbers at which it occurs are relatively low. The major numerical challenge is the large size of the domain, necessitated by the fact that the streamwise and spanwise wavelengths of the turbulent banded pattern are extremely large when compared to the distance Δy between the bounding plates. The conventional nondimensionalization in plane Couette flow uses $\Delta y = y_{\text{top}} - y_{\text{bot}} = 2$ and $\Delta U = U(y_{\text{top}}) - U(y_{\text{bot}}) = 2$. In these units, the streamwise and spanwise wavelengths of the experimentally observed turbulent banded pattern [3] are $\lambda_x \approx 110$ and $\lambda_z \approx 50 - 80$, and the banded turbulent pattern exists in the Reynolds number range $340 \lesssim Re \lesssim 415$.

We have carried out simulations in two geometries. In the first, the streamwise and spanwise directions have extents $L_x = 110$ and $L_z = 50$, and the bounding plates have velocities $\mathbf{U} = \pm \mathbf{e}_x$. The second geometry is "tilted" with respect to the motion of the bounding plates: boundary conditions $\mathbf{U} = \pm \mathbf{e}_{||} \equiv \pm(\cos(\phi)\mathbf{e}_x + \sin(\phi)\mathbf{e}_z)$ are imposed, where the angle $\phi = 0.4266$ has been chosen such that $\tan(\phi) = \lambda_z/\lambda_x = 50/110$. In this tilted geometry, the turbulent bands should be oriented parallel to the x direction, permitting a very small L_x to be used, and should have a wavelength in the z direction of $\lambda_x \lambda_z / \sqrt{\lambda_x^2 + \lambda_z^2} = 45.51$. We use $L_x = 4$ and $L_z = 240$.

We follow the experimental protocol of generating a homogeneous turbulent flow at $Re = 500$ and gradually lowering the Reynolds number. As a measure of the turbulent intensity, we use the energy along a line at mid-height $(x, y) = (0, 0)$ of the flow components perpendicular to the motion of the bounding plates, i.e. $||\mathbf{U} - \mathbf{U}_{||}||^2$, with $\mathbf{U}_{||} \equiv (\mathbf{U} \cdot \mathbf{e}_{||})\mathbf{e}_{||}$. (In the usual geometry, this is $v^2 + w^2$.) Figure 1 shows a time-space plot of the turbulent intensity during a series of simulations in the tilted geometry during which the Reynolds number is gradually decreased through 450, 425, 400, and then 380. We indeed find that a portion of the domain becomes and remains laminar, and another portion remains turbulent, although the width of the turbulent region is smaller than that expected from the experiments.

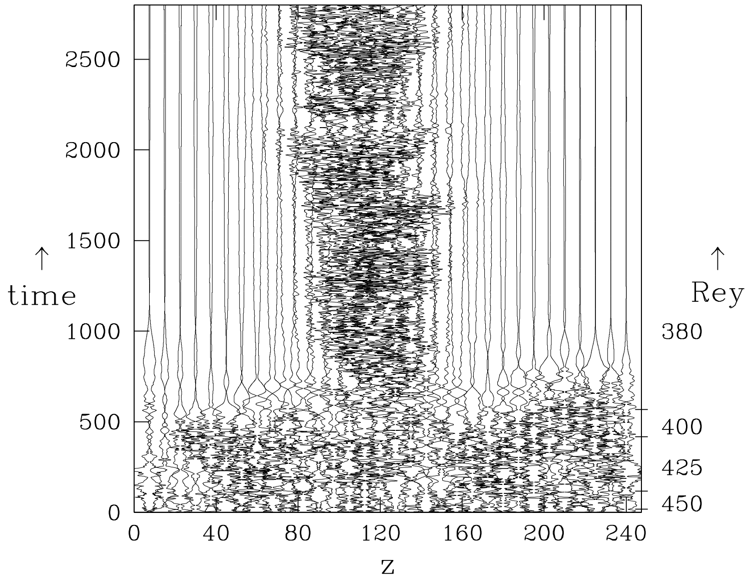

Figure 1: Space-time plot of turbulent intensity $||\mathbf{U} - \mathbf{U}_{||}||^2$ of flow perpendicular to motion of bounding plates along the line $(x, y) = (0, 0)$ as the Reynolds number is decreased.

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