Mean flow and modeling of turbulent-laminar patterns in plane Couette flow

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The fields and phenomena of pattern formation and of turbulence have generally remained well-separated, the first restricted to regular periodic patterns observed at low Reynolds numbers and the second to statistical behavior observed at high Reynolds numbers. In early 2000s, an overlap betwen these two fields appeared, when experimentalists at GIT-Saclay [1, 2, 3] discovered that the coexisting laminar and turbulent regions observed in Taylor-Couette flow in the 1960s and 1980s [4, 5, 6] were a single wavelength of a statistically regular, spatially periodic pattern. The same group discovered that these turbulent-laminar patterns of comparable wavelengths existed in plane Couette flow at the Reynolds numbers at which transition is observed.

The primary new feature of the pioneering experiments at GIT-Saclay that led to the discovery of turbulent-laminar patterns is very simple: size. The Taylor-Couette and plane Couette experiments were carried out in apparati whose lateral dimensions (radius and height for TC; streamwise and spanwise for PC) are very large compared to the gap between the confining cylinders or plates. turbulent-laminar patterns have wavelengths that are on the order of 30–60 times the half-gap. The turbulent and laminar bands form at an angle of 20°–40° to the streamwise direction, at Reynolds numbers (based on half the imposed velocity difference and half the gap) between 300 and 400.

We have computed these turbulent-laminar patterns in plane Couette flow via direct numerical simulation of the 3D time-dependent Navier-Stokes equations. Our simulation uses a spectral element/Fourier code [7] in a rectangular periodic domain which is *tilted* in the direction of the expected pattern as shown in figure 1 (left), economizing on the length in the direction in which the pattern is homogeneous. More specifically, we impose periodic boundary conditions in the lateral directions x, z and rigid boundary conditions $\mathbf{u}(y = \pm 1) = \pm(\mathbf{e}_x \cos\theta + \mathbf{e}_z \sin\theta)$ in the cross-channel direction y. Our domain is of size $L_x \times L_y \times L_z = 10 \times 2 \times 40$ or $10 \times 2 \times 120$, with $O(10^6)$ modes or gridpoints. Our computations show a rich variety of turbulent-laminar patterns as the angle θ and Reynolds number are varied, including spatio-temporal intermittency, branching and travelling states, and localized states analogous to spots [8, 9] some of which are shown in figure 1 (right).

We have conducted a quantitative analysis [10] of the mean flow $\mathbf{U}(y,z) \equiv \langle \mathbf{u} \rangle$ averaged over $T = 2000$ and $L_x = 10$ (the direction in which the pattern is statistically homogeneous), which obeys the averaged Navier-Stokes equations

$$0 = -\left(\mathbf{U} \cdot \nabla\right)\mathbf{U} - \mathbf{f} - \nabla P + \frac{1}{Re}\boldsymbol{\Delta}\mathbf{U} \tag{1}$$

where $\mathbf{f} = (f^U, f^V, f^W) \equiv -\langle(\tilde{\mathbf{u}} \cdot \nabla)\,\tilde{\mathbf{u}}\rangle$ is the Reynolds stress force. We find that the mean flow in the quasi-laminar region of the pattern is not the linear profile of laminar plane Couette flow, but instead represents a non-trivial balance between the viscous and advective forces. Surprisingly, both $\mathbf{U}$ and $\mathbf{f}$ are almost exactly centrosymmetric, i.e. they obey $\mathbf{g}(y,z) = -\mathbf{g}(-y,-z)$, as shown in figure 2. Even more surprisingly, we find that the z-dependent components of $\mathbf{U}$ and $\mathbf{f}$ are almost exactly trigonometric, i.e. that both can be represented by only three functions of y as:

$$\mathbf{g}(x,y,z) = \mathbf{g}_0(y) + \mathbf{g}_c(y)\cos(kz) + \mathbf{g}_s(y)\sin(kz) \tag{2}$$

Substituting the form (2) for $\mathbf{U}$ and $\mathbf{f}$ into (1) leads to a system of 6 ODEs in the 12 functions of y representing the mean flow and the Reynolds stress force. Turbulence modelling would close this system by providing a relation between $\mathbf{f}$ and $\mathbf{U}$. The primary difficulty encountered is that turbulence models are usually formulated and calibrated for high Reynolds numbers, not those near transition, and also present difficulties near walls [11]. Work on this is currently underway, based on the $k - \omega$ model [12].

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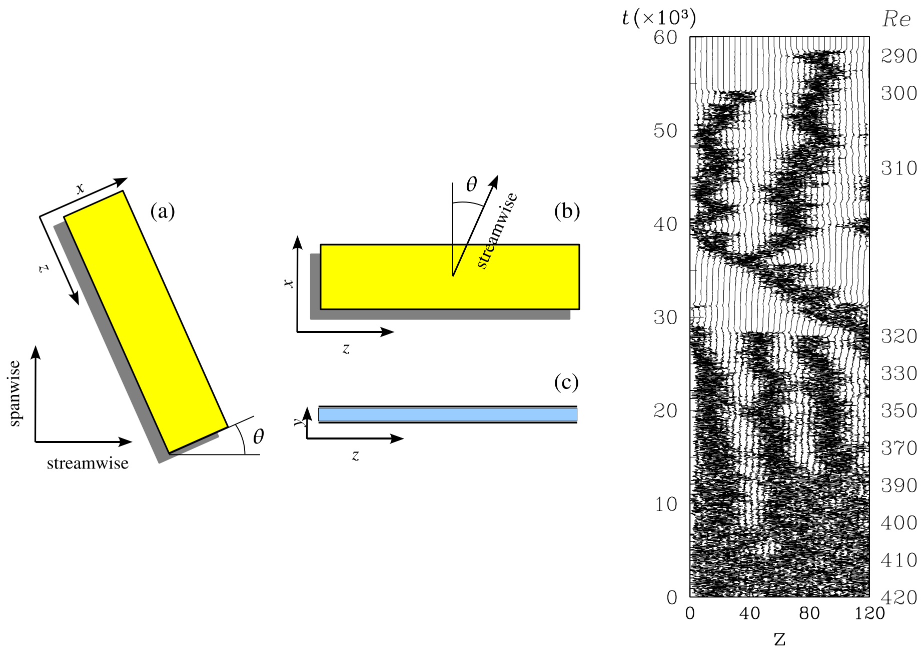

Fig. 1. Left: Computational domain oriented at angle θ to the streamwise-spanwise directions. The z direction is aligned to the pattern wavevector while the x direction is perpendicular to the pattern wavevector. (a) Domain oriented with streamwise velocity horizontal. (b) Domain oriented with z horizontal. (c) View between plates. Right: Timeseries of $w(x = 0, y = 0, z = z_i)$, $z_i = L_z i/N_z$ for $L_z = 120$ and $\theta = 24°$. The Reynolds number (indicated on the right) is lowered in steps over a time $0 \leq t \leq 60,000$. Uniform turbulence at $Re = 420$ is succeeded by formation of three bands at $Re \approx 390$. Two bands disappear almost simultaneously at $Re = 320$. The remaining band moves left, periodically emitting turbulent spurs, of which one finally becomes a second turbulent band. Single band at $Re = 300$ is a localized state, succeeded by laminar Couette flow at $Re = 290$.

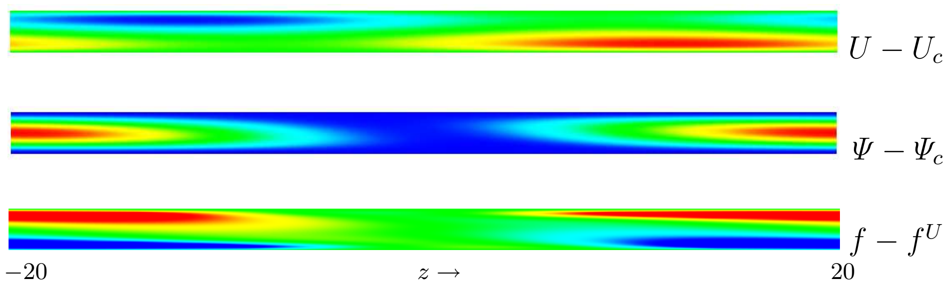

Fig. 2. Mean flow, averaged over x and t, of turbulent-laminar pattern is centro-symmetric in the (y, z) plane. Turbulent region centered around $z = 0$, laminar region around $z = \pm 20$. Top: $U - U_c$ is deviation from laminar Couette profile, here $y \cos\theta$. Middle: $\Psi - \Psi_c$ is deviation of (V, W) streamfunction from that of Couette profile. Bottom: Reynolds stress force.