Dynamic behavior of superconducting microbridges

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Measurements of the I-V curves of superconducting thin-film tin microbridges are reported. A vortex flux-flow model is developed to explain, close to T_c, the regime where a plateau in current is observed. This regime is active when the coherence length ξ is smaller that half the width $w/2$ of the bridge. At temperatures much closer to T_c, where $\xi \geq w/2$, the phase-slip mechanism is effective.

I. INTRODUCTION

Interest in Josephson structures[1] is sustained by their potential applications, but also by the difficulties in correctly describing the different physical mechanisms acting in the various types of structures.

In this paper we give an interpretation for the dynamic behavior of the so-called Dayem microbridges.[2] In such a weak link a superconductor of very small cross section and length connects two more massive superconductors made out of the same material, tin in our case. These microbridges have been intensively studied, especially the dependence of the critical current with temperature, the regime of paraconductivity above T_c, and the microwave response.[3–5]

Our interest will be focused more on the transport properties. We will mainly try to explain the shape of the current-voltage characteristics, and its temperature dependence near the critical temperature T_c (Fig. 1).

Two different regimes seem to be operating: a first one close to T_c is well described by the one-dimensional formulation of the Ginzburg-Landau theory applied to the so-called phase-slip process.[6–8] A second one appears at lower temperature, where the current-voltage characteristic shows a "plateau" in current. We propose here a model based on the dynamics of vortices crossing the bridge which explains the plateau in current, the number of vortices per unit time following the Josephson relation. In this model a vortex moves at constant velocity V_L in the bridge of width w, the transit time being w/V_L.

As the voltage increases, more and more vortices cross the bridge to maintain the Josephson relation. At a limiting voltage, the distance between vortices becomes of the order of the coherence length and this regime can no longer exist; we have thus a maximum voltage defining the end of the current plateau. For larger voltages the phase-slip process, where the whole bridge relaxes from the normal to the superconducting state[8] at the Josephson frequency, reappears.

The I-V characteristic curves in Fig. 1 show a striking similarity with the well-known pressure-volume diagram of a liquid-gas transition: within our model the transition is between two homogeneous states in space, the first being a dissipative state near zero voltage with probably no vortex motion, the second being a time-fluctuating normal-superconductivity state, the flat coexistence line representing the regime with vortices in the bridge playing a role similar to the liquid drops in the plateau of the liquid-gas transition. We should, however, note that this transition occurs between two dissipative states, and not between two phases in thermodynamic equilibrium.

II. DEVELOPMENT OF THE MODEL

We suppose that, for a critical current I_c, magnetic flux enters the bridge by single-fluxoid vortices. This assumption is sustained by various experimental and theoretical results on thin films of type-I superconductors. It was shown that, below a critical thickness, the film behaves as a type-II superconductor in a magnetic field, i.e., a single-fluxoid vortex state exists. This mixed state was observed directly, in thin films of Pb, Sn, and In, by decorating the samples with fine ferromagnetic

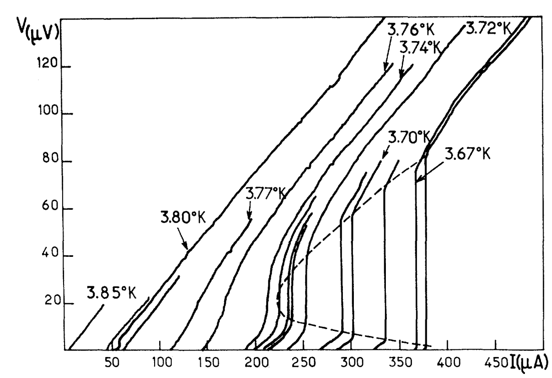

FIG. 1. Experimental I–V curves at different temperatures for sample 19 022 of thickness $d = 1000$ Å, width $w = 1$ μ, $l = 6000$ Å.

particles.[9] In our experiments, the Sn films are always below the critical thickness. The current I_c flowing through the microbridge in the mixed state causes a motion of the flux lines in a direction perpendicular to the current flow. We are in the flux-flow regime and a dc voltage V appears across the bridge.[10] This voltage is given by

$$V=\Phi_0\nu \ , \tag{1}$$

where Φ_0 is the flux quantum and ν the number of vortices crossing the bridge per unit time. In this regime, as the voltage increases, the current remains constant and a plateau in current appears. Since the experiments are done with a constant current supply this sort of plateau can arise from a jump between two states, each of which is metastable further along. However, the experiments were done with both directions of current sweep and no significant hysteresis was observed in our temperature range.

The geometry of the magnetic field lines generated by the current through the bridge is such that probably two vortices of opposite sign enter on both edges, move towards the center and annihilate there. If v_L is the average vortex-line velocity, the transit time of these lines is $\tau_T=w/2v_L$ and the instantaneous voltage is $V_i=\Phi_0/\tau_T$. If the vortices are created at the frequency ν, we measure a mean dc potential difference

$$V=V_i\,\tau_T\,\nu=\Phi_0\nu \ . \tag{2}$$

In addition, we have an ac voltage at the Josephson frequency ν. This model is valid only if $1/\nu>\tau_T$, i.e., at sufficiently low voltages. If we increase the voltage, more and more vortices are created per unit time. ν has a maximum value corresponding to the fact that a single vortex covers a surface of transverse dimension ξ, the coherence length. Vortices are distinct entities if the cores of two adjacent flux lines are at a distance greater than 2ξ, if not they collapse. That means that the flux-flow regime can only exist if $v_L(1/\nu)>\xi$. The maximum frequency ν_M is given by

$$\nu_M=v_L/\xi \ , \tag{3}$$

and the maximum dc voltage by

$$V_M=\Phi_0\,\nu_M \ . \tag{4}$$

For $V>V_M$, the whole bridge transits to the normal state and we switch to the phase slip process where there is a relaxation regime from the superconducting to the normal state. In this regime a linear slope is observed in the I-V characteristic. The value of this slope is independent of temperature and equal to the resistance of the bridge at T_c (fluctuation regime).

What are the conditions for observing the flux-flow regime? In our interpretation we suppose that a single fluxoid vortex can enter the bridge; this is only possible if half the width $\frac{1}{2}w$ is greater than ξ, the minimum size of a vortex. Very close to T_c, ξ is greater than w and the phase slip regime is immediately reached. The "plateau" is observed only for temperatures below a critical value T_0. If our model is correct, T_0 is such that $\xi(T_0)$ is of the order of $w/2$. This point will be discussed in the next chapter.

Let us now calculate the maximum voltage V_M defining the end of the current "plateau." To do this we use the viscous model of vortex motion developed by Bardeen and Stephen.[11]

The single fluxoid vortex is submitted to a Lorentz force $F_L=J\,\Phi_0$ per unit length, where J is the current density. The vortex, moving at a velocity v_L, is submitted to a viscous force due to the friction of the normal core against the crystal lattice. This force is given by ηv_L. The lines move at constant velocity when both forces are equal, i.e.,

$$v_L=J\,\Phi_0/\eta \ . \tag{5}$$

The viscosity coefficient η was calculated by several authors and is given by[11]

$$\eta=\sigma\,\Phi_0^2/\pi\,\xi^2 \ , \tag{6}$$

where σ is the normal conductivity of the core. In fact J varies from the edge to the center of the bridge, and so does v_L. If $J(x)$ is the current density at a distance x from the edge, we can write approximately

$$J=J_0\,e^{-x/\lambda_{\rm eff}} \ . \tag{7}$$

$\lambda_{\rm eff}$ is not the usual penetration depth of bulk tin, but an effective penetration depth due to the fact that we are dealing with a thin film. $\lambda_{\rm eff}$ is given by de Gennes.[12] For $d\ll\lambda$ it is given by

$$\lambda_{\rm eff}=\lambda^2/d \ , \tag{8}$$

where d is the thickness of the film. J_0 is determined by the total current I flowing through the film:

$$I=2\int_0^{w/2} dJ_0\,e^{-x/\lambda_{\rm eff}}\,dx \tag{9}$$

$$=2d\,J_0\,\lambda_{\rm eff}(1-e^{-w/2\lambda_{\rm eff}})=2d\,\lambda_{\rm eff}\,J_0\,g(w) \ . \tag{10}$$

As v_L is a function of x, the maximum frequency ν_M is given by

$$\frac{1}{\nu_M}=\int_0^{\xi}\frac{dx}{v_L(x)} \ , \tag{11}$$

$$=\int_0^{\xi}\frac{\eta}{\Phi_0 J_0}\,e^{x/\lambda_{\rm eff}}\,dx \ , \tag{12}$$

$$=(\eta/\Phi_0\,J_0)\,\lambda_{\rm eff}\,(e^{\xi/\lambda_{\rm eff}}-1) . \tag{13}$$

We give in Table I some characteristic values for

TABLE I. Characteristic dimensions, critical temperature, coherence length and effective penetration depth for two of the samples.

Sample	d (Å)	w (μm)	l (Å)	T_c (°K)	T_0 (°K)	ξ at T_0 (Å)	λ_{eff} at T_0 (Å)
19022	1000	1	6000	3.88	3.74	4800	6600
25061	1000	3,5	8000	3.80	3.75	8000	20000

our samples, l being the length of the bridge (Fig. 2).

These values are obtained from formulas (18) and (19) below. For these samples we took a mean free path $l_0 = 500$ Å. For both samples at $T < T_0$ the condition $\frac{1}{2}w < \xi$ is fulfilled. J_0 is given by

$$J_0 = I/2d\lambda_{eff}\, g(w) \tag{14}$$

and

$$1/\nu_M = (2d\eta/\Phi_0 I)\, \xi\, \lambda_{eff}\, g(w)\,, \tag{15}$$

and

$$V_M = I\, \Phi_0^2/2d\,\eta\, \xi\, \lambda_{eff}\, g(w)\,, \tag{16}$$

$$= I\,(\pi/2\sigma)\, \xi/\lambda^2 g(w)\,. \tag{17}$$

Let us now consider the variation of V_M/I with temperature. We must take the expression[13] of ξ and λ for a "dirty metal" with mean free path l_0:

$$\xi(T) = 0.85\,(\xi_0 l_0)^{1/2}[T_c/(T_c - T)]^{1/2}\,, \tag{18}$$

$$\lambda(T) = 0.64\,\lambda_L(0)\,(\xi_0/l_0)^{1/2}[T_c/(T_c - T)]^{1/2}\,, \tag{19}$$

where $\lambda_L(0)$ is the London penetration depth. We thus can predict, as $g(w)$ is practically constant with T, that

$$V_M/I \propto (T_c - T)^{1/2}\,. \tag{20}$$

III. EXPERIMENTAL METHODS AND RESULTS

For the preparation of microbridges we use the technique[14] of scratching a groove on a glass substrate before the evaporation of a thin film of tin. After the evaporation we cross-cut perpendicularly, thus leaving a small tin bridge at the intersection of the grooves (Fig. 2).

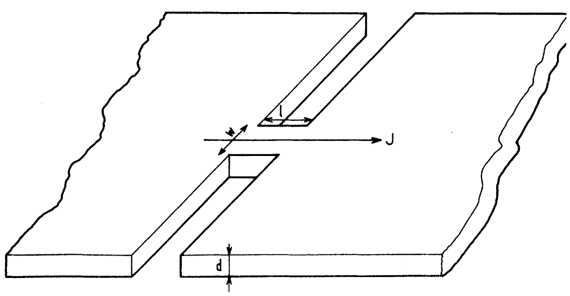

FIG. 2. Geometry of the samples, w width, l length, and d thickness.

The scratchings are done mechanically, using a small moving machine which allows one to displace the glass substrate at a constant velocity, 0.3 mm/sec, under a fixed knife holder. The knife used for the glass substrate cut is a commercially available diamond pickup with a 6-μm radius of curvature. The vertical cutting force is about 1 g. The glass substrate is then rapidly etched (30 sec) in 100 g/l HF solution. It is then ultrasonically cleaned, first in an alkali solution, then in deionized water and finally rinsed in isopropylic alcohol. We then evaporate 1000 Å of tin at a rate of 100 Å/sec. To increase the adhesion of the tin film we first evaporate a thin layer[15] of tin oxide (40 Å of SnO). The evaporated glass plate is then placed again on the cutting machine. The instrument used is now a razor blade, the cutting force about 0.5 g, the cutting angle about 2°, the cut being perpendicular to the first one. For the electrical connections we used silver leads, soldered with an alloy, 50% In–50% Sn.

The smallest bridges we obtain are about 5.000 Å wide by 2.000 Å long. In order to avoid burn out, the bridges are short circuited until we start the experiments at helium temperature. The samples are immersed in a helium bath contained in a cryostat electrically shielded and magnetically screened by three concentric μ-metal cylinders. The bath temperature is regulated by both a variable pumping valve and an electronic bolometer-heater feedback loop, with better than 1-mK stability.

The current versus voltage characteristics are measured using a dc constant current supply. The voltage is measured using a Keithley 150B microvoltmeter.

Let us now analyze our data, and focus on two of a large number of samples, No. 19022 and 25061, whose dimensions are given in Table I. In our theoretical model we interpret the plateau in current as due to the motion of vortices, then the ratio of the maximum voltage reached on the plateau divided by the current should follow a $(T_c - T)^{1/2}$ law. In Fig. 3 we have plotted $(V_M/I)^2$ versus temperature. A linear behavior is observed, confirming formula (20), the transverse dimension of the bridge w entering only in the factor $g(w)$ in formula (17). Thus the quantity $K=(V_M/I)[g(w)/(1-T/T_c)^{1/2}]$ should be a constant independent of the sample size. We find at $T_c - T = 0.170\,^\circ\mathrm{K}$, $K=0.273$ for the 25 061 sample and $K=0.207$ for sample 19 022.

Another confirmation of the model comes from the study of the critical current. In Fig. 4 we plot the critical current I_c vs $T_c - T$ in logarithmic coordinates for the same samples. The transition temperature T_c is experimentally determined by the disappearance of the dc supercurrent within a 1-μA resolution. Let us remark that the temperature dependence of I_c cannot be fitted to the power law $I_c \propto (T_c - T)^\alpha$ with α constant, but instead there is a transition from a power law $\alpha = \frac{3}{2}$ near T_c to another with $\alpha = 1$. This effect is similar to that observed by Song and Rochlin[16,17] and interpreted as a smooth transition, from near ideal Josephson junctionlike behavior to bulk superconductorlike behavior as the temperature decreases.

In the light of our model, one can reformulate this interpretation. Close to T_c, the one-dimensional model for the bridge can be applied and there the critical current follows a $(T_c - T)^{3/2}$ law, i.e., $\alpha = \frac{3}{2}$.

Below T_c, when the plateau in current appears, the one-dimensional model does not apply and the vortex regime appears, similar in fact to a bulk-like behavior where α is closer to unity.

In Fig. 4 we indicate with an arrow the temperature where the current plateau appears, and it is effectively between the two limiting regions $\alpha = \frac{3}{2}$ and $\alpha = 1$.

We have estimated the value of the coherence

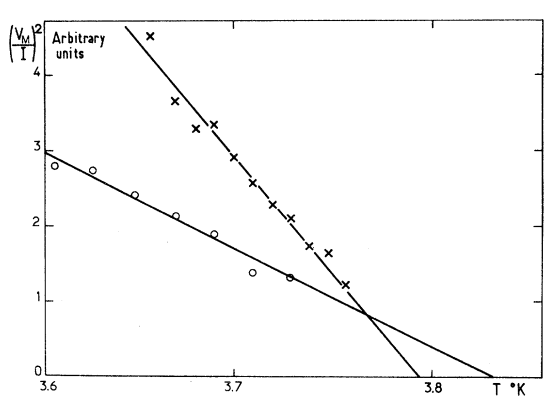

FIG. 3. Curves showing $(V_M/I)^2$ vs temperature for two samples: ×××, sample 25 061: $d=1000$ Å, $w=3.5\ \mu$, $l=8000$ Å. ○○○, sample 19 022: $d=1000$ Å, $w=1\ \mu$, $l=6000$ Å.

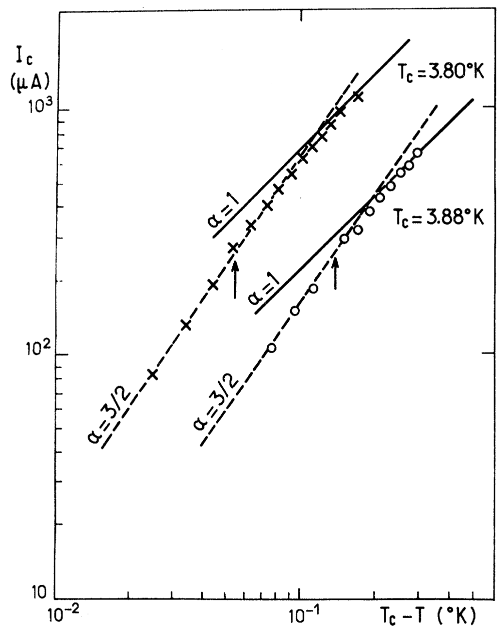

FIG. 4. Critical current vs $T_c - T$ for two samples. ×××: 25 061, ○○○: 19 022.

length ξ at temperature T_0 (Table I) and indeed one obtains ξ smaller than the width of the bridge but comparable to it, which is the basic assumption of the model. It in fact implies that at temperature T_0, $\xi(T_0)$ should be less than half the width of the bridge.

We have two determinations for the critical temperature T_c, one comes from the onset of a supercurrent, the other from the dependence of $(V_M/I)^2$ with temperature (Fig. 3). For the large samples the agreement is very good (typically 3.80 and 3.79 °K). For the small ones fair (3.88 and 3.83 °K).

We must point out one interesting feature of our I-V curves (Fig. 1); close to T_0 when the current plateau appears, it does not start at zero voltage. The plateau current I_p is greater than the critical current I_c (maximum current at zero voltage). In this range the voltage increases with current so that the system is not in a purely superconducting state. Within the framework of our model this would imply that we have there, for a small range of voltage, the phase slip regime before reaching that of the vortex. We have not found a convincing explanation for this effect.

Also, one must realize that in our experiment we work with a constant current source. If we worked with a constant voltage source, the I-V curves might be different in the plateau region (in analogy with the liquid-gas transition).

A final, very important, problem is related to

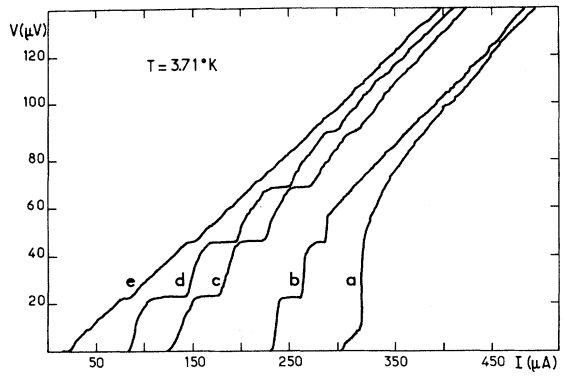

FIG. 5. Experimental I-V characteristic curves. Effect of microwaves. Temperature $T=3.71\,°K$. Microwave frequency = 11.8 GHz. (a) Without microwave. (b) Arbitrary microwave power applied. (c) Microwave power, 11 dB above that of curve b. (d) Microwave power, 14 dB above that of curve b. (e) Microwave power, 17 dB above that of curve b. Sample 5012: $d=1000$ Å, $l=1\ \mu m$, $w=1.4\ \mu m$.

the power dissipation. The power, typically less than 0.1 μW, is dissipated in the very small volume of the bridge, $10^{-10}\ mm^3$. With such high-power densities hot spots can develop. Skocpol *et al.*[18] have studied this problem. To avoid heating effects, one must work with bridges of small length, so that the two heat sinks represented by the large-size superconducting regions can be effective. Also, one should not work at a temperature much smaller than T_c. We have observed that if the temperature is more than 200 mK below T_c, then hysteresis effects appear and the I-V curves show instabilities which can be related to the dynamics of hot spots. In our range of temperatures, and for our bridges of small lengths, no such effects are observed. A clear verification of the absence of hot spots is the presence of the ac Josephson effect, as demonstrated in Fig. 5 by the appearance of steps in the curves when microwaves are applied.

IV. CONCLUSION

The model of a vortex flux-flow regime seems to be in fairly good agreement with our experiments.

Three regimes seem to be present in our microbridges. Close to T_c, within let us say 100 mK, one observes the one-dimensional phase-slip regime. At much lower temperatures, more than 200 mK from T_c, hot spots develop in the microbridges and control the dynamics of the bridges. In the intermediate region, one observes the regime with a characteristic current "plateau" that we have analyzed here and interpreted by a vortex flux-flow model. Good Josephson behavior is present only at a temperature close to T_c where heating effects are not important, and this evidently limits the possible applications of the bridges to a small temperature range.

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