Variation of the electron-phonon mobility with carrier density for highly doped and highly excited semiconductors

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We study the response, to a small electric field, of a doped semiconductor or an optically excited electron-hole plasma, as a function of carrier density and temperature. We limit the discussion to the case of nonpolar semiconductors such as Ge and Si and include the screening of the electron-phonon interaction by the carriers, which are considered as classical as well as degenerate. We show that the behavior of the relaxation time $\tau(n)$ is much more complex than a simple increase due to electron-phonon interaction screening. For optical phonons, τ goes through a maximum, which is reached in the degenerate limit, and finally decreases when the density n continues to increase. For acoustical phonons we predict that, with increasing n, τ goes first to a maximum, then decreases to a minimum, reached for $n \sim 10^{19}$ cm^3 in the case of Ge at 20 K, and finally increases again for very large densities and low temperatures.

I. INTRODUCTION

The development of extremely short and intense laser pulses[1,2] has motivated various studies on the energy transfer from the electromagnetic field to the lattice via the dense electron-hole plasma created by the laser pulse. In particular, the carrier density dependence of the energy relaxation time[1] via phonon emission has been investigated when the carriers are not in thermal equilibrium with the lattice. The variation of this energy relaxation time has also been studied in doped semiconductors[2-5] when the carriers are heated by a large electrical field.

In this paper we study the carrier-density dependence of the momentum relaxation time when the carriers are in thermal equilibrium with the lattice. The carriers will be considered in the classical as well as the degenerate limits. Although the Boltzmann-equation approach[6] is quite standard for the calculation of the electric current, we will recall it briefly in order to trace the origin of the density dependence in the degenerate limit: This limit has been essentially studied for metals when the number of free electrons is a few times more the number of atoms while these two numbers do not correlate in semiconductors. We will include the screening of the electron-phonon interaction which has a different form[7] for doped semiconductors and optically excited electron-hole plasma.

The main results obtained in this paper are the following. If one first neglects the screening, the relaxation time τ calculated for nondegenerate electrons does not change with the increasing density of carriers; in the degenerate limit it decreases essentially because of the increase of the carrier velocity. At very high density, τ finally increases and tends to infinity because of quantum phase-space restriction in the case of scattering with acoustical phonons, while such restrictions do not appear for nonpolar optical phonons. This variation of the relaxation time with density qualitatively remains the same when the screening of the electron-phonon interaction is included: It gives of course an overall increase of τ in the nondegenerate as well as in the degenerate case, but this increase is smaller than the decrease due to the carriers velocity, in the quantum limit. Moreover, as seen in a previous work,[7] the screening of the electron-phonon interaction gives a sizable effect only for doped semiconductors with intermediate densities: it remains of the order of 1 for a very dilute or very dense electron gas as well as for an electron-hole plasma at any density. These behaviors of $\tau(n)$ are rather different from the common belief that the electron-phonon collision time should simply decrease when the carrier density increases.

In Sec. II, we recall briefly the Boltzmann formalism and write an integral equation valid in the classical limit as well as in the degenerate case. In Sec. III, we solve it exactly for nonpolar optical phonon, when the screening of the electron-phonon interaction is not included. A similar calculation for acoustical phonon is done in Sec. IV, while in Sec. V we discuss the modifications which appear if one takes into account the screening of the electron-phonon interaction. Section VI is a discussion of the previous results using the numerical parameters of germanium.

II. BOLTZMANN EQUATION

The Boltzmann equation[6] has been extensively studied in the classical limit for semiconductors and in the quantum limit for metals. We will recall briefly the main steps of the formalism.

The linearized Boltzmann equation for the distribution function f of electrons in presence of an electric field $\mathscr{E}$ reads

$$\frac{\partial f_k^0}{\partial \epsilon_k} \frac{\hbar \mathbf{k}}{m} \cdot (-e)\mathscr{E} = \frac{V}{(2\pi)^3} \int d^3k' \{ [-|M_{k \to k'}|^2 f_k(1-f_{k'}) + |M_{k' \to k}|^2 f_{k'}(1-f_k)] \} . \tag{1}$$

For electron-phonon scattering, the matrix element $M_{k \to k'}$ is composed of an absorption and an emission term[8]

$$| M_{kk'} |^2 = A_q [n_q \delta(E_k + \hbar\omega_q - E_{k'})$$
$$+ (n_q + 1)\delta(E_{k'} + \hbar\omega_q - E_k)] .$$ (2)

n_q is the usual phonon occupation number $n_q = n_-(\hbar\omega_q / k_B T)$ with

$$n_\pm(x) = (e^x \pm 1)^{-1} ,$$ (3)

$\hbar\omega_q$ is the phonon energy, T is the common lattice and electron temperature, E_k is the electron energy, and $E_k = \hbar^2 k^2 / 2m$ where m is an effective electron mass.[9] $\mathbf{k} = \mathbf{k}' \pm \mathbf{q}$. The coefficient A_q has the form

$$A_q = \frac{2\pi}{\hbar} \frac{\hbar}{2N_A M \omega_q} \times \begin{cases} \tilde{E}_1^2 q^2 \text{ for acoustical phonon ,} \\ \tilde{D}_1^2 \text{ for nonpolar optical phonon .} \end{cases}$$ (4)

N_A is the number of atoms per unit volume and M their mass, $\tilde{E}_1$ or $\tilde{D}_1$ are the screened deformation potentials. In a previous work,[7] we showed that $\tilde{E}_1$ or $\tilde{D}_1$ differs from the bare deformation potential by a screening factor

$$S(q^2) = \frac{q^2 + sq_{sc}^2}{q^2 + q_{sc}^2} ,$$ (5)

q_{sc} is the screening wave vector, which is the sum of the contributions of electrons and the holes in the case of an electron-hole plasma: $q_{sc}^2 = q_e^2 + q_h^2$ with

$$q_e^2 = (4\pi e^2 / \epsilon)(n / E_e^*) ,$$

E_e^* being $k_B T$ or $\frac{2}{3} E_{Fe}$, the electron Fermi energy depending on whether the electron gas is degenerate or not. In the case of acoustical phonons, the parameter s equals E_e^* / E_1 for n-doped semiconductor and is usually very small as the bare deformation potential E_1 is of the order of 10 eV, while $s = (E_1 - E_2)/E_1$ is of the order of 1 for an optically excited electron-hole plasma, E_2 being the hole bare deformation potential. For nonpolar optical phonons, $s = (D_1 - D_2)/D_1$ for an electron-hole plasma, while $s = 0$ for a doped semiconductor (the term sq_{sc}^2 comes from the Coulomb potential due to the change of charged impurities, and in optical modes, the density of the ions does not change).

$f_\mathbf{k}$ appearing in Eq. (1) is the unknown distribution function, f_k^0 is the equilibrium one. One can first check that, in the limit of small electric field $\mathscr{E}$, the angular dependence of $f_\mathbf{k}$ is exactly $\mathbf{k} \cdot \mathscr{E}$; so that, to lower order in $\mathscr{E}$, the Boltzmann equation (1) transforms into an integral Eq. (7) for the unknown function $\tau(E_k)$ defined as

$$f_\mathbf{k} = f_k^0 + f_k^0(1 - f_k^0)(-e)\mathscr{E} \cdot \frac{\hbar\mathbf{k}}{m} \frac{\tau(E_k)}{k_B T} ,$$ (6)

$$1 - f_k^0 = \frac{V}{(2\pi)^3} \int d^3 k' (1 - f_{k'}^0) | M_{k \to k'} |^2$$
$$\times [\tau(E_k) - \frac{\mathbf{k} \cdot \mathbf{k}'}{k^2} \tau(E_{k'})] .$$ (7)

Although the function $\tau(E_k)$ introduced in Eq. (6) has the

dimension of a time, we want to stress that this is not a relaxation-time approximation.

The integration of Eq. (7) is particularly easy as $\mathbf{k} = \mathbf{k}' \pm \mathbf{q}$ is constant in the integration; one can then transform the three-dimensional (3D) variable $\mathbf{k}'$ into scalar variables k' and q with $d^3 k' = (2\pi/k)k' dk' q dq$ and the condition $| k - q | < k' < k + q$; the first integration, over k', is then trivial due to the δ function appearing in $M_{kk'}$.

Before going further, it is useful at this stage to introduce dimensionless variables and appropriate temperature-like quantities in order to see clearly the dependence of the resulting conductivity relaxation time in the various parameters. Let us use K, K', and Q instead of k, k', and q defined as

$$K^2 = \frac{\hbar^2 k^2}{2m k_B T} \equiv x + \frac{\mu}{k_B T} ,$$ (8)

m is the electron mass and μ its chemical potential. The phonon energy will be written as $\hbar\omega_q = k_B T_Q$; for optical phonons, ω_q is essentially constant, i.e., $T_Q \sim T_{op}$, while for acoustical phonons, $\omega_q \sim q V_S$, so that $T_Q = 2Q(TT_S)^{1/2}$, T_S being related to the sound velocity V_S as

$$\tfrac{1}{2} m V_S^2 = k_B T_S .$$ (9)

A typical value of T_S is 0.1 K, i.e., T_S is extremely small compared with the other temperaturelike parameters appearing in the problem.

One can rewrite the function $\tau(E_k)$ appearing in Eq. (7) in terms of a dimensionless function φ, and a constant τ_0 having the dimension of a time

$$\tau(E_k) = \tau_0 \varphi(K^2) .$$ (10)

From Eq. (4) and (7), τ_0 is given by

$$\frac{\hbar}{\tau_0} = \frac{3\pi}{8} k_B (T_{dp}^2 / T_A)(T/T_A)^{1/2}(T/T_S)^\alpha ,$$ (11)

$\alpha = 0$ for optical phonons and $\frac{1}{2}$ for acoustical phonons. T_A is related to the density of the atoms n_A by

$$k_B T_A = (\hbar^2 / 2m)(3\pi^2 n_A)^{2/3} .$$ (12a)

We define similarly a temperature T_n related to the electronic density n by

$$k_B T_n = (\hbar^2 / 2m)(3\pi^2 n)^{2/3} .$$ (12b)

Note that, in semiconductors, T_n is always much smaller than T_A, while for metals they are similar. T_{dp} is related to the deformation potentials. For acoustical phonons, we have set $k_B T_{dp} = E_1(m/M)^{1/2}$, while for optical phonons, T_{dp} is defined as $k_B T_{dp} = D_1(\hbar/M\omega_0)^{1/2}$.

The function φ is the solution of an integral equation deduced from Eqs. (7) and (3):

$n_+(-x) = \Phi_+ - \Phi_-$,

$$\Phi_\pm = \frac{1}{K} \int_\Delta dQ\, \hat{S}^2(Q) Q^{2\alpha+1} \theta \left[K^2 \pm \frac{T_Q}{T} \right]$$

$$\times n_+ \left[-x \mp \frac{T_Q}{T} \right] n_- \left[\pm \frac{T_Q}{T} \right]$$

$$\times \left[\varphi(K^2) - \left[1 - \frac{Q^2 \mp (T_Q/T)}{2K^2} \right] \right.$$

$$\left. \times \varphi \left[K^2 \pm \frac{T_Q}{T} \right] \right] . \tag{13}$$

Φ_+ corresponds to phonon absorption and Φ_- to phonon emission. The integration over Q is such that K, Q, and $(K^2 \pm T_Q/T)^{1/2}$ form a triangle. The screening factor (5) reads, with dimensionless variables, $\hat{S}(Q) = (Q^2 + sQ_{sc}^2) / (Q^2 + Q_{sc}^2)$ where the screening wave vector gives

$$Q_{sc}^2 = \frac{T_0^{1/2} T_n^{3/2}}{TT^*} , \tag{14}$$

T^* being $\frac{2}{3} T_n$ or T for a degenerate or a nondegenerate electron gas, respectively; and T_0 is the effective rydberg $(k_B T_0 = me^4/2\hbar^2\epsilon^2)$.

Once the integral equation (13) is solved, as it will be done in Secs. II and III, the electric current is simply obtained from $J = Ne^2\bar{\tau}E/m$ with

$$\bar{\tau} = \frac{2}{3}\tau_0 \frac{\int_0^\infty dK\, K^4 \varphi(K^2) n_+(x) n_+(-x)}{\int_0^\infty dK\, K^2 n_+(x)} . \tag{15}$$

In the classical limit, $n_+(-x) \sim 1$, and the leading contribution in the integral comes from $K \sim 1$, so that

$$\bar{\tau} \simeq \frac{8}{3\pi^{1/2}} \tau_0 \int_0^\infty K^4 \varphi(K^2) e^{-K^2} dK . \tag{16}$$

In the quantum limit $(T \ll T_n)$, the leading part in the integral (15) comes from $x \sim 0$ and the effective collision time reads

$$\bar{\tau} \simeq \tau_0 \int_{-\infty}^{+\infty} \varphi(x + T_n/T) n_+(x) n_+(-x) dx . \tag{17}$$

III. NONPOLAR OPTICAL PHONONS

The resolution of Eq. (13) is extremely simple in the case of nonpolar optical phonon $(T_Q = T_{op}, \alpha = 0)$ when the screening is neglected. Using the variable $V = 1 - (Q^2 \pm T_{op}/T)/2K^2$, one finds that the second term of the bracket of Eq. (13) gives exactly zero as the integral over V is odd [this can also be seen directly on Eq. (7): as A_q, n_q, and $E_{k'}$ are constant, the integration of $\mathbf{k} \cdot \mathbf{k'}$ over $\mathbf{k'}$ produces zero]. The remaining unknown function $\varphi(K^2)$ can be taken out from the integral equation (13) and the *exact* solution of the Boltzmann equation is simply in that case:

$$\frac{1}{2\varphi(K^2)} = \frac{(K^2 + T_{op}/T)^{1/2}}{e^{T_{op}/T} - 1} \frac{n_+(-x - T_{op}/T)}{n_+(-x)}$$

$$+ \frac{(K^2 - T_{op}/T)^{1/2} \theta(K^2 - T_{op}/T)}{1 - e^{-T_{op}/T}}$$

$$\times \frac{n_+(-x + T_{op}/T)}{n_+(-x)} . \tag{18}$$

The first term of Eq. (18) corresponds to phonon absorption and the second term to phonon emission. We want to emphasize that Eq. (18) is also the exact solution for a degenerate plasma and inelastic scattering.

In the classical limit, $n_+(-x) \simeq 1$; Eqs. (15) and (18) give for the effective collision time,

$$\bar{\tau} = \frac{4}{3\pi^{1/2}} \tau_0 (e^{T_{op}/T} - 1) \left[\int_0^{T_{op}/T} \frac{K^4 e^{-K^2}}{(K^2 + T_{op}/T)^{1/2}} dk + \int_{T_{op}/T}^\infty \frac{K^4 e^{-K^2}}{(K^2 + T_{op}/T)^{1/2} + e^{T_{op}/T}(K^2 - T_{opp}/T)^{1/2}} dk \right] \tag{19}$$

The first integral comes from electrons with energy that is smaller than the phonon one and which can only absorb phonons. For $T \ll T_{op}$ they are the ones which contribute to the collision processes, as the electrons with energy larger than the phonons are negligible; one then finds that the first integral of Eq. (19) is of the order of $(T/T_{op})^{1/2}$ while the second one is exponentially small. For $T \gg T_{op}$, the electrons can equally emit and absorb phonons; the second integral is then of the order of one while the first one is negligible.

Using Eqs. (11) and (19), one finds for the asymptotic behavior of the effective collision time in the classical limit

$$\frac{\hbar}{\tau} \sim k_B \frac{T_{dp}^2}{T_A^{3/2}} \frac{\sup\{T^{1/2}, T_{op}^{1/2}\}}{e^{T_{op}/T} - 1} . \tag{20}$$

In the quantum limit, the effective collision time results from the behavior of $\varphi(x + T_n/T)$ for $x \simeq 0$. Using Eqs. (17) and (18), one finds

$$\bar{\tau} \simeq \tau_0 (e^{T_{op}/T} - 1)[T/(T_n + T_{op})]^{1/2}$$

$$\times \int_{-\infty}^{+\infty} dx \frac{n_+^2(-x) n_+(x)}{n_+(-x - T_{op}/T) + \theta(T_n - T_{op}) e^{T_{op}/T} n_+(-x + T_{op}/T)[(T_n - T_{op})/(T_n + T_{op})]^{1/2}} . \tag{21}$$

For $T_{\rm op} \ll T$, the denominator in the integral (21) is of the order of $n_+(-x)$ so that the whole integral is of the order of 1. For $T \ll T_{\rm op}$, the denominator is of the order of 1 or $1 + e^x$ depending if $T_{\rm op}$ is larger or smaller than T_n, but in both cases the integral is still of the order of 1. So that finally the collision time in the quantum limit is

$$\frac{\hbar}{\bar{\tau}} \sim k_B \frac{T_{dp}^2}{T_A^{3/2}} \frac{(T_n + T_{\rm op})^{1/2}}{e^{T_{\rm op}/T} - 1} \ . \tag{22}$$

The asymptotic behaviors of $\bar{\tau}$ in the classical and quantum limits, Eq. (20) and (22), can be written in a condensed form

$$\frac{\hbar}{\bar{\tau}_{\rm op}} \sim k_B \frac{T_{dp}^2}{T_A^{3/2}} \frac{(T_{\rm op} + T + T_n)^{1/2}}{e^{T_{\rm op}/T} - 1} \ , \tag{23}$$

which very simply shows that the effective relaxation time for electron—optical-phonon collisions contains a phonon distribution term and an effective electron velocity $T^{1/2}$, $T_n^{1/2}$, or $T_{\rm op}^{1/2}$ resulting from the electron-phonon collision. Without the screening of the electron-phonon interaction, $\bar{\tau}_{\rm op}$ stays constant when the electron density increases, up to the degenerate limit; for $T_n \sim n^{2/3} > T_{\rm op}$ and T, the collision time starts to decrease as $T_n^{-1/2} \sim n^{-1/3}$, due to the increase of the electron velocity.

IV. ACOUSTICAL PHONONS

As in a usual experimental situation, the temperature is much larger than $T_S \sim 0.1$ K, the acoustical-phonon energy $k_B T_Q = k_B 2Q(TT_S)^{1/2}$ is always much smaller than the electron one and can be neglected in front of it $[T_S \ll \sup(T, T_n) \Rightarrow T_Q/T \ll K^2$ or $Q^2]$. But the phonon energy might be larger than $k_B T$ in the extreme quantum limit [9] $[T_Q = 2Q(TT_S)^{1/2} \gg T \Rightarrow T_n \gg T^2/T_S$ as $Q \sim (T_n/T)^{1/2}$ in the quantum limit], so that in this domain T_Q/T will have to be kept in front of $x = K^2 - T_n/T$.

It is useful, at this stage to introduce a new variable $z = T_Q/T$ which allows one to rewrite the integral equation (13) in a closed form

$$n_+(-x) = \frac{1}{8K} \left[\frac{T}{T_S} \right]^{3/2}$$
$$\times \int_{-\lambda}^{+\lambda} dz \frac{z^2 n_+(-x-z)}{|e^z - 1|}$$
$$\times \left[\varphi(K^2) - \left[1 - \frac{2z^2}{\lambda^2} \right] \varphi(K^2 + z) \right] \tag{24}$$

with $\lambda = 4K(T_S/T)^{1/2}$. We have not set $\varphi(K^2 + z) \sim \varphi(K^2)$ although $z \ll K^2$ as φ depends a priori on K^2 and x, and z can be larger than x (as for a very dense elec-

tron gas). We have to consider the following two cases.

(1) For $T^2 \gg T_S \sup\{T, T_n\}$, λ is small and z will also be smaller than $x \sim 1$ in the quantum limit so that one can set $\varphi(K^2 + z) \sim \varphi(K^2)$. The integral equation (25) is in that case immediately solved, and one finds

$$\frac{1}{\varphi(K^2)} = \frac{1}{8K} \left[\frac{T}{T_S} \right]^{3/2} \int_{-\lambda}^{+\lambda} dz \frac{2z^4}{\lambda^2} \frac{1}{|e^z - 1|}$$
$$\simeq 2K(T/T_S)^{1/2} \ . \tag{25}$$

As $K \sim 1$ or $(T_n/T)^{1/2}$ for a nondegenerate or a degenerate electron gas, respectively, this gives for the asymptotic behavior of the average collision time, using Eq. (11),

$$\frac{\hbar}{\bar{\tau}} \sim k_B \frac{T_{dp}^2}{T_A^{3/2}} \frac{T}{T_S} \sup\{T^{1/2}, T_n^{1/2}\} \ . \tag{26}$$

(2) The preceding solution assumes that the average phonon energy stays smaller than $k_B T$. This is no more the case in the extreme quantum limit, when $T_n \gg T^2/T_S$; z cannot then be neglected in front of x, and one has, for the first time, to really solve an integral equation in order to get the function φ. The problem is the same as the one solved by Bloch for metals,[10] and an exact solution can also be found in this case to leading order in $T_n T_S/T^2$, although most text books use in the very low temperature limit the unnecessary trick of an expansion close to the impurity collision-time value.

In this limit, λ is very large. As the integral (25) is convergent, one can extend its bounds to infinity. Moreover one is tempted to drop z^2/λ^2 in front of one as the distribution functions for electron and phonon ensure $z \sim 1$. If one does that, λ disappears from the integral equation, but φ seems to be defined within an arbitrary additive constant which would lead to a highly unphysical arbitrary collision time. In fact, there is no finite solution to the integral equation (24) for λ strictly infinite: this is easily checked if Eq. (24) is multiplied by $n_+(x)$ and integrated over x. The left-hand side produces one, while the right-hand side

$$\int_{-\infty}^{+\infty} dz \frac{z^2}{|e^z - 1|} \int_{-\infty}^{+\infty} dx \, n_+(x) n_+(-x-z)$$
$$\times [\varphi(x) - \varphi(x+z)] \tag{27}$$

equals zero [to see this, one changes $x + z$ into x' and z into $-z$ in the part with $\varphi(x+z)$ and one notes that $|e^{-z} - 1| = e^{-z}|e^z - 1|$].

The above discussion shows, in fact, simply that the only possible solution for φ when λ goes to infinity is an infinite constant, for the determination of which the term in z^2/λ^2 has to be kept. Using a similar procedure as the one used for Eq. (27) and setting $\varphi \simeq \alpha \lambda^2$ to the leading order in λ, Eq. (24) gives

$$1 = \frac{1}{8K} \left[\frac{T}{T_S} \right]^{3/2} \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dz \frac{n_+(x) n_+(-x-z)}{|e^z - 1|} \frac{2z^4}{\lambda^2} \alpha \lambda^2 \ , \tag{28}$$
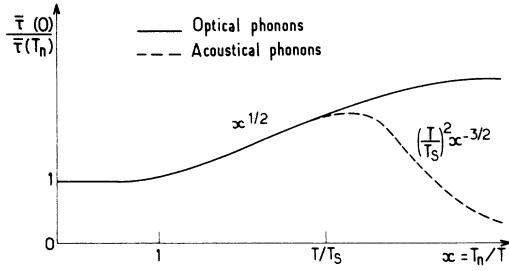
FIG. 1. Variation of the momentum relaxation time $\bar{\tau}$ with density $x = T_n/T$ (x varies as $n^{2/3}$). The variation with x is given in various parts of the curve.

i.e, $\alpha \sim (T_n/T)^{1/2}(T_S/T)^{3/2}$ as $K = (T_n/T)^{1/2}$. One finally obtains for the effective collision time in the extreme quantum limit.

$$\frac{\hbar}{\bar{\tau}} \sim k_B(T_{dp}^2/T_A^{3/2})(T/T_S)T_n^{1/2}(T^2/T_n T_S)^2 . \qquad (29)$$

The asymptotic behaviors for $T \ll T_n$, $T_n \ll T \ll (T_n T_S)^{1/2}$, and $(T_n T_S)^{1/2} \ll T$ given in Eqs. (27) and (30) can be condensed in order to give for the electron-acoustical phonon collision time a single expression

$$\frac{\hbar}{\bar{\tau}_{ac}} \sim k_B \frac{T_{dp}^2}{T_A^{3/2}} \frac{T}{T_S}(T + T_n)^{1/2} \frac{1}{\left[1 + \dfrac{T_S T_n}{T^2}\right]^2} . \qquad (30)$$

If one compares Eq. (30) with the similar expression (23) for the electron—optical-phonon collision time, one finds again an effective electron velocity part ($T^{1/2}, T_n^{1/2}$). $T_{op}^{1/2}$ appeared in Eq. (23) as the collision with an optical phonon could significantly change the electron velocity, while this is never the case for acoustical phonons because the velocity of sound is small compared to the average electron velocity. The term in T/T_S is the memory of the phonon distribution; and the last term is a quantum phase-space restriction which takes place when the phonon energy $k_B(T_n T_S)^{1/2}$ is larger than $k_B T$: there is no exponentially small term due to the phonon distribution function as the electron energy is still larger than the phonon one and phonon emission is always possible, except for phase-space restriction in the electron gas.

Equation (30) shows that the electron—acoustical-phonon scattering probability first rises when the density is increased above the degenerate limit, up to a maximum value of the order of $k_B T_{dp}^2 T^2/T_A^{3/2} T_S^{3/2}$, reached for $T_n \sim T^2/T_S$; then the collision time starts to increase due to the phase-space restriction. The variation of $\hbar/\bar{\tau}$ with density is shown in Fig. 1.

V. EFFECT OF SCREENING

We have seen in Sec. II that if one takes into account the screening of the electron-phonon interaction, a screening factor

$$S^2(Q^2) = [(Q^2 + sQ_{sc}^2)/(Q^2 + Q_{sc}^2)]^2$$

appears in the integral equation (13) for φ. This will lead roughly to the division of φ by a factor $\hat{S}^2(Q_{eff}^2)$ where Q_{eff} is an effective transfer momentum which is of the order of the electron momentum K, i.e., $\sup\{1,(T_n/T)^{1/2}, (T_{op}/T)^{1/2}\}$, the term in $(T_{op}/T)^{1/2}$ appears of course only for a collision with optical phonons.

In the case of an electron-hole plasma, the parameter s is of the order of one so that $\hat{S}(Q^2)$ stays close to one no matter what Q_{eff}^2 is and the screening will not significantly change the results of Secs. III and IV.

For doped semiconductors, s is small and the screening factor will strongly modify the previous results if Q_{eff}^2 is much smaller than Q_{sc}^2 as $\hat{S}$ is then Q_{eff}^2/Q_{sc}^2 instead of one. From Eqs. (13) and (14), one finds that the effect of screening leads one to multiply the electron-phonon collision probability $\hbar/\bar{\tau}$ by

$$\frac{Q_{eff}^4}{Q_{sc}^4} \sim \frac{\sup\{T^2,T_n^2,T_{op}^2\}\sup\{T^2,T_n^2\}}{T_0 T_n^3} . \qquad (31)$$

if this quantity is less than one (T_0 is the rydberg defined in Sec. II). As $\sup\{T^2,T_n^2\} > T_n^2$ and $\sup\{T^2,T_n^2,T_{op}^2\} > T_n T$ or $T_n T_{op}$, one immediately finds that Q_{eff}^4/Q_{sc}^4 is larger than T/T_0 and T_{op}/T_0 no matter what T_n is; so that if $T_0 < T, T_{op}$, Q_{eff}^4/Q_{sc}^4 is always larger than one and the screening of the electron-phonon interaction will give a negligible change to the effective collision time, no matter what the electron energy is. This is, in particular, the case for a hot doped semiconductor when the thermal energy is larger than the rydberg.

For $T_0 > T$ and T_{op}, the screening factor decreases the collision probability only at intermediate electron density, as, from Eq. (31), one sees that Q_{eff}^4/Q_{sc}^4 is larger than one for T_n very small or T_n very larger. Tables I(a), I(b), and

TABLE I. Relative magnitudes of various quantities in different Q_{eff}/Q_{sc} regimes.

(a)	1		$\dfrac{T^4}{T_0 T_n^3}$		$\dfrac{T_n}{T_0}$	1
	$T\left[\dfrac{T}{T_0}\right]^{1/3}$		T		T_0	T_n

(b)	1		$\dfrac{T_{0p}^2 T^2}{T_0 T_n^3}$	$\dfrac{T_{0p}^2}{T_0 T_n}$	$\dfrac{T_n}{T_0}$	1
$\dfrac{T_{0p}^2}{T_0}$	$T\left[\dfrac{T_{0p}^2}{TT_0}\right]^{1/3}$		T	T_{0p}	T_0	T_n

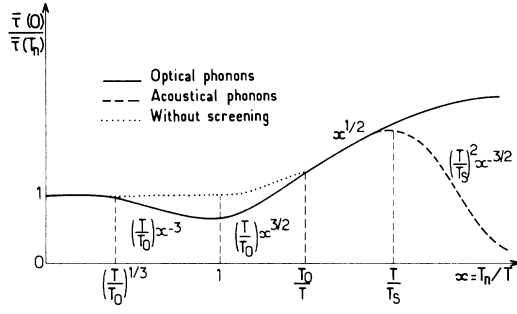| (c)	1		$\dfrac{T_{0p}^2}{T_n T_0}$	$\dfrac{T_n}{T_0}$	1
T	$\dfrac{T_{0p}^2}{T_0}$		T_{0p}	T_0	T_n

FIG. 2. Variation of $\bar{\tau}$ with x when the temperature T is smaller than the optical-phonon temperature T_{op} and than T_0.

I(c) give the screening contribution to the electron-phonon collision probability for $T_{op} < T$, $(T_{op}^2/T_0) < T < T_{op}$ and $T < (T_{op}^2/T_0)$, the case of acoustical phonons being the one where T_{op} does not appear, i.e., Table I(a).

If one adds these screening factors to the unscreened expressions of the collision times given in Eqs. (23) and (30) for optical and acoustical phonons respectively, one does not expect any significant change for an electron-hole plasma in any case and for doped semiconductors if the rydberg is less than the thermal energy or the optical phonon one. When the screening significantly modifies the previous results, i.e., for $T_0 > T$ *and* T_{op}, the resulting variations of the variations of the effective collision times are shown in Fig. 2 for acoustical phonons, and optical phonons if $T_{op} < T$, and in Fig. 3 for optical phonons if T is less than T_{op} but larger or smaller than T_{op}^2/T_0 [Figs. 3(a) and 3(b), respectively].
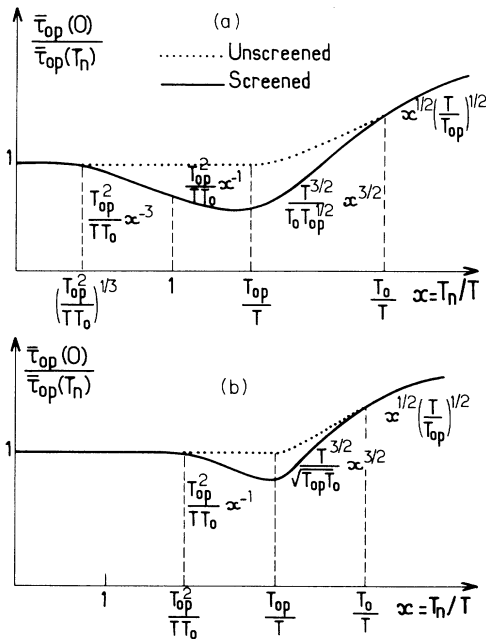

FIG. 3. Variation of $\bar{\tau}$ due to optical-phonon scattering in the two following regimes: (a) $T_{op}^2/T_0 < T < T_{op}$, (b) $T < T_{op}^2/T_0 < T_{op}$.

In the case of acoustical phonon, for $T_0 > T$, the effective collision time $\bar{\tau}(T_n)$ first increases up to a maximum value of the order of $\bar{\tau}(0)(T/T_0)$ reached when the electron gas starts to be degenerate and then decreases to a minimum value of the order of $\bar{\tau}(0)(T/T_S)^{1/2}$ reached when the acoustical-phonon energy is of the order of $k_B T$, and phase-space restrictions appear.

For optical phonons, when $T < T_{op}$, $\bar{\tau}(T)$ first increases with density to reach a maximum value of the order of $\bar{\tau}(0)(T_{op}/T_0)$ reached in the degenerate limit when the electron energy equals the optical-phonon ones and the emission of phonons becomes possible; then the effective collision time keeps decreasing when the density increases.

In all these calculations we did not include intervalley scattering. This can be done along the same lines.

VI. NUMERICAL VALUES FOR GERMANIUM

We discuss the density dependence of the electron-phonon scattering time in the specific case of germanium. The numerical values of the parameters introduced in the preceding sections are for this material:[11]

$$m = 0.2m_0 ,$$

$$T_{op} = 430 \text{ K} ,$$

$$D = 4.8 \text{ eV/Å} ,$$

$$E_1 = 9.5 \text{ eV} ,$$

$$V_S = 5.3 \times 10^3 \text{ m/s} ,$$

$$n_A = 4.4 \times 10^{22} \text{ cm}^{-3} ,$$

$$M_A = 1.2 \times 10^{-25} \text{ kg} .$$

This gives the following for the various temperaturelike parameters used previously:

$$T_S = 0.19 \text{ K} ,$$

$$T_A = 2.4 \times 10^5 \text{ K} ,$$

$$T_0 = 130 \text{ K} ,$$

$$T_{dp,op} = 2.1 \times 10^3 \text{ K} ,$$

$$T_{dp,ac} = 1.3 \times 10^2 \text{ K} .$$

Using Eqs. (23) and (30), the above parameters lead to the following values for the relaxation times $\tau(0)$, at room temperature ($T = 300$ K), when the density is zero

$$\tau_{op}(0) = 10^{-11} \text{ s}, \quad \tau_{ac}(0) = 0.5 \times 10^{-12} \text{ s}$$

in agreement with the usual values, of the classical limit.[5]

In the case of scattering with *acoustical phonons* the minimum value of τ_{ac}, that we have predicted to occur at $T_n \sim T^2/T_S$, should be reached for

$$n_c \sim 4 \times 10^{15} T^3 \text{ cm}^{-3} , \tag{32}$$

this gives $n_c \sim 3 \times 10^{19}$ cm^{-3} at $T = 20$ K but $n_c \sim 10^{23}$ cm^{-3} at $T = 300$ K, so that $\tau_{ac,min}$ can never be reached at room temperature, while the critical density for low temperature should now be accessible to experiments.

The corresponding minimum for the scattering time, given in Eq. (31), is

$$\tau_{ac,min} \sim 4 \times 10^{-9} T^{-2} \text{ s} . \tag{33}$$

This gives 10^{-11} s at 20 K. One can note that the decrease from the very low-density value $\tau_{ac}(0)$ of the acoustical scattering time is not expected to give a dramatic effect as

$$\tau_{ac,min}/\tau_{ac}(0) \sim (T_S/T)^{1/2} . \tag{34}$$

This gives 0.1 at 20 K (and 2×10^{-2} at 300 K; although impossible to reach experimentally, this value gives an order of magnitude of the possible decrease of τ_{ac} when the density increases at room temperature).

For $T < T_0$, i.e., at low temperature, τ_{ac} has a maximum in the case of doped semiconductors, due to the screening of the electron-phonon interaction [see Table I(a)]; this maximum is reached for $T_n \sim T$ and is such that

$$\tau_{ac,max}/\tau_{ac}(0) \sim T/T_0 . \tag{35}$$

This gives one order of magnitude change at 20 K. This effect should be larger for a semiconductor with a smaller effective rydberg (T_0).

In summary, experiments on doped germanium at low temperature (typically 20 K) should show, when the concentration of carriers is varied, first an increase of τ_{ac} by one order of magnitude, due to the screening of the electron-phonon interaction, then a decrease by two orders due to an increase of the average electron velocity, and finally again an increase of τ_{ac} due to phase-space restrictions. For optically excited germanium, for which the electron-phonon screening stays very small, the first increase of τ_{ac} is washed out.

In the case of scattering of *optical phonons*, the electron-phonon interaction screening is not expected to give a very large effect as $T_{op} > T_0$, so that the collision time τ_{op} should show a very shallow maximum. For carriers in the quantum limit, τ_{op} will simply decrease as $T_n^{-1/2} \sim n^{-1/3}$ when the density increases above $T_n \sim T_{op}$.

Note that the preceding results are based on Eqs. (23) and (30) and Table I where only the asymptotic behaviors in the leading parameters are stressed, and the numerical factors of the order of 1 are dropped out. This allows one

to show simply the relation between the qualitative behavior of $\tau(n)$ and the various physical effects as electron velocity, screening, and phase-space restrictions; but if one wants to make a precise comparison with experiment, one should trace these numerical factors back from Eq. (13) and eventually solve this integral equation numerically.

In this paper we have considered only *e-p* scattering. In doped semiconductors, the scattering of impurities has to also be considered at low temperature. In the range of doping, where τ_{ac} is maximum, the scattering time of impurities τ_{imp} is small and may hide part of the increase of τ_{ac}; but τ_{imp} is independent of temperature and a careful study of the mobility versus T should show the described effect.

VII. CONCLUSION

We have shown that the density dependences of the electron scattering time with acoustical phonons and optical phonons are much more complex than a simple increase which would have resulted from the screening of the electron-phonon interaction.

For optical phonons, $\tau_{op}(n)$ shows a maximum due to electron-phonon screening and then decreases, in the quantum limit, as the average electron velocity increases.

More interesting is the behavior of acoustical-phonon scattering time. After a maximum as similar as the one of τ_{op}, τ_{ac} shows a minimum followed by a final increase due to phase-space restriction in the extreme quantum limit. This minimum can be reached experimentally for dense carriers at very low temperatures (typically 20 K or lower in germanium).

The maxima of τ_{op} or τ_{ac} are washed out when the screening of the electron-phonon interaction is very small, i.e., for optically excited semiconductors or for hot-doped materials.

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