Superconductivity at High Temperature in the Cuprates

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A short review of the main properties of the superconducting cuprates is given in order to introduce the article of Deutscher and de Gennes which gives a spatial interpretation of the formation of localized hole pairs in these materials. The predictions and consequences of this model are given and discussed.

1. Introduction

Pierre-Gilles de Gennes (PGG) was very active in the field of superconductivity in the sixties. His famous textbook[1] remains a fundamental reference for students and researchers in that field. After that, he decided to switch to other problems and was very successful in many other fields, as described in this book. The discovery of high temperature superconductivity (HTSC) in cuprates compounds in 1986 by Bednorz and Müller[2] has been a great sensation in the physics community and has raised great expectations for applications which, we hope, will one day be fulfilled. More than twenty years after this discovery, the exact mechanism of HTSC is still not yet understood and remains a great challenge for solid state physicists. PGG, in spite of the fact he no longer worked in that field, was very interested in this remarkable property of this strange class of materials, the cuprates with lamellar structure. Several groups of chemists, physicists and crystallographers, started to work on these compounds in his school, the Ecole Supérieure de Physique et Chimie Industrielles (ESPCI), where he was the director during many years. At that time, I was at Ecole Normale Supérieure (ENS), and with Jacques Labbé, we published a paper[3] showing the importance of the so-called Van Hove singularities[3] in these almost 2D compounds. We stressed the role of peaks of the electronic density of states (DOS) in increasing the critical temperature T_c. PGG asked me to come to ESPCI, and to lead a new Solid State Laboratory

working on the superconductivity of the cuprates. I accepted his proposal, and we started this lab. A few months before his death, PGG had a new idea, concerning the formation of electron-pairs linked to the deformation of the lattice, i.e. a contraction of the Cu-O-Cu bond, which reduces the pair energy and favors its formation. This paper[4] was written with Guy Deutscher, submitted before his death, but only published afterwards.

Many reviews have been recently published on the HTSC[5,6] and I shall not give a complete description of the cuprates properties. After a rapid review of the well established properties of these materials, I shall discuss the proposed models for the formation of Cooper pairs and stress the contribution of Deutscher and PGG.[4]

2. Well Established Properties of the Cuprates

2.1. *Structure*

All the HTSC cuprates are layered materials containing CuO_2 planes. A typical structure is given in Fig. 1 for $YBa_2Cu_3O_{7-\delta}$, δ varying from 0.5 to 0. When $\delta = 0$, all the bands are stoechiometric and the CuO_2 planes contain no holes. The compound is insulating and antiferromagnetic (AF). These materials can be doped chemically. For example in $YBa_2Cu_3O_{7-\delta}$, when δ decreases from 0.5 to 0, the compound goes from a Mott insulator

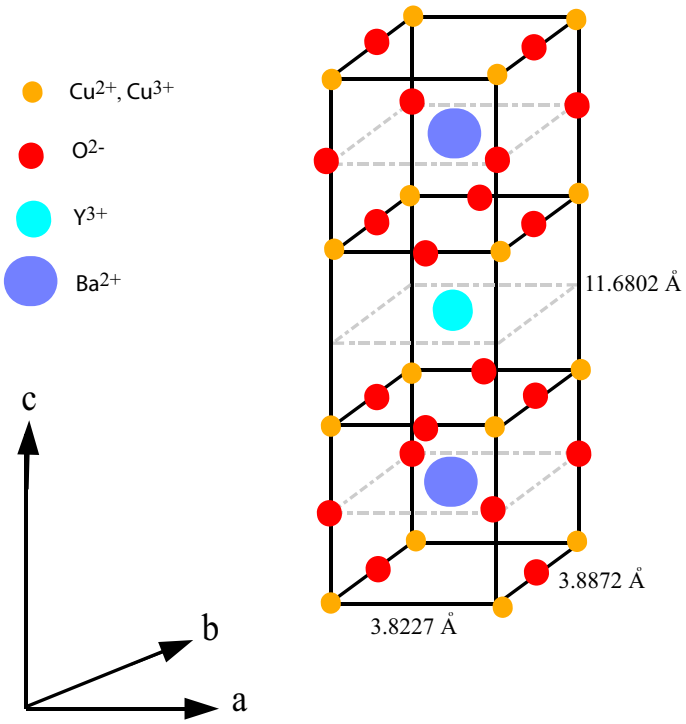

Fig. 1. Structure of $YBa_2Cu_3O_{7-\delta}$.

to a metal, and p the number of holes in the CuO_2 plane increases from 0 to 0.5 per Cu atom.

The universal phase diagram of these lamellar cuprates is illustrated in Fig. 2. For $p = 0$, the compound is an AF Mott insulator with a Néel temperature of about 300 K.

When more holes are added, the AF order is rapidly suppressed and disappears for $p \approx 0.03$. For $p \approx 0.05$ superconductivity appears, T_c versus p is shown in Fig. 2. The maximum T_c occurs for a value $p_0 \approx 0.17$ called "optimum doping." This maximum T_c is 40 K for LaSrCuO compounds, 93 K for YBCO compounds, and goes up to about 160 K for $HgBaCaCuO_y$ compounds. 168 K is the highest value obtained today.[6]

The properties of both the normal and the superconductivity phases have been intensively studied.

The main features are the following:

2.2. *Superconducting phase*

Experiments[5–7] have proved that there are electron pairs of charge $2e$ and 0 spin (singlet state). The two remarkable features are:

(1) The very short coherence length, of the order of 25 Å, (in comparison it is of the order of 1000 Å in the classical superconductors)
and
(2) The symmetry of the gap. In low temperature superconductors like Al, Ag, Sn, etc ..., the gap is isotropic, it has the same value in all directions. The electron pairs are described by an s-wave function. In HTSC cuprates, many experiments[6] show that d-wave functions are characteristic of electron pairs. A d-wave function changes sign with direction and this has several consequences. The angular shift is π instead of 2π and gives rise in certain conditions to vortices of quantum flux $h/4e$ instead of $h/2e$. This has been observed in certain structures[8] (tricrystals). Most of the experiments showing the d-wave character of the superconducting gap are surfaces or grain boundaries measurements. But some bulk experiments like measurements of the torque in a massive crystal in a magnetic field show a partial s character.[9] So there are still speculations about the pure d character of the pair wave function.[9]

2.3. *Normal phase*

Above T_c in the normal state of the phase diagram (Fig. 2) we can distinguish three regions:

(1) The underdoped region where $p < p_0$, where a feature called the "pseudogap" is observed.
(2) The region around the optimum doping p_0. The system is a bad metal with strange properties, different from a normal metal.
(3) The overdoped region, where the compound behaves like a usual classical metal, the conduction electrons forming a Fermi liquid.

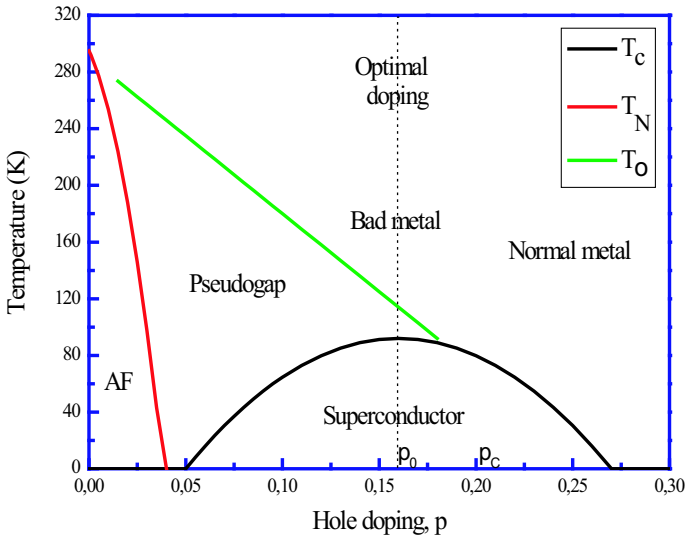

Fig. 2. Phase diagram of cuprates.

2.3.1. *The pseudogap region*

Many experiments, among them the measurements of various thermodynamical parameters (magnetic susceptibility, specific heat) and also transport properties, in particular electrical conductivity, and photoemission experiments, have shown a loss of quantum electronic states near the Fermi level (FL).[10] This appears as a dip in the density of states (DOS) near the FL and was coined the name "pseudogap" by Friedel. The pseudogap value kT_0 diminishes with the doping. In YBaCuO, it goes from 20 meV for $p = 0.1$ to 0 meV for $p = 0.25$ hole per Cu atom.[11] The origin of this pseudogap is still in discussion.[10,11] There are essentially two approaches: correlations[11] or preformed electron-pairs of binding energy kT_0 which condense at T_c; disorder may also play a role.[12]

2.3.2. *Around and above optimal doping*

The properties of high T_c cuprates in the normal state around the optimum doping p_0 are different from a good metal. The resistivity is linear in T (absolute temperature), the Hall coefficient is temperature dependant.[13] These were cited as examples of non Fermi-liquid behavior. But it can also be related to the electronic structure in these almost 2D crystals[14] (see the following chapter).

In the overdoped region, beyond the optimal doping, the normal state behaves like a usual Fermi-liquid.

3. Theoretical Models

Contrary to the experimental situation, there are still many discussions concerning the various proposals for high T_c theory.[5,6,11] I shall give only a few indications related to

the contribution of P.-G. de Gennes. Full developments are available in many review papers.[4–6,11]

3.1. *Electronic structure of the cuprates*

A one electron calculation is easy to perform.[3] A general feature of a 2D model is the presence of Van Hove singularity[3] (VHs) with logarithmic divergence of the DOS at an energy $E = E_S$. A simple calculation[14,15] gives the result shown in Fig. 3 for the constant energy surfaces (CES) in k-space. This is very well confirmed by the results of Ino[16] using angular resolved photoemission spectroscopy (ARPES) (see Fig. 4 of Ref. 16).

A topological transition is well seen for a doping value $p_c = 0.21$ hole per Cu atom. The CES are hole-like for $p < p_c$ and electron-like for $p > p_c$. The resulting VHs gives a peak in the DOS and thus increases the transition temperature whatever the pairing mechanism. The main consequences of this Van Hove scenario are given in Ref. 15.

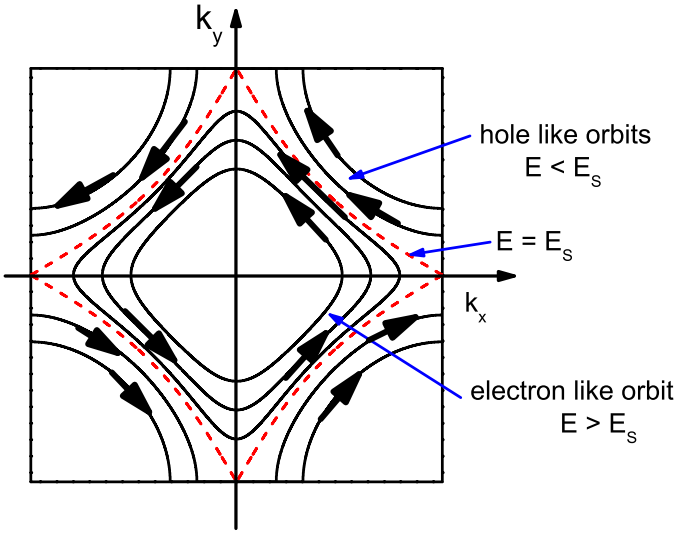

Fig. 3. Constant energy surfaces.

This approach is not valid for the underdoped region. The strong Coulomb repulsion U between two electrons on a same site is responsible for the fact that with $p = 0$ the cuprates are Mott-insulators with antiferromagnetic (AF) order. The AF order disappears rather rapidly with doping, but AF fluctuations remain, and decrease, until the optimum doping. This region of strong correlations is present and the valid approach is that of a doped Mott-insulator.[11] This is also seen in ARPES; some points of the Fermi surface disappear for underdoped samples.

3.2. *BCS condensation versus BE condensation*

There are two types of condensation of particles into a superfluid. The Bose Einstein (BE) condensation occurs for preexisting bosons below a critical temperature T_c (liquid Helium). The Bardeen-Cooper-Schrieffer (BCS) condensation occurs for electrons (fermions) forming pairs (carrying 0 or 1 spin) and condensing into a superfluid at the same temperature T_c. In the underdoped regime, the possibility for holes to form pairs at a temperature T_0 and then condensing at $T_c < T_0$ was stressed by several authors.[17] In other words it is a BE condensation of preformed pairs. Near optimum doping, however, the density of holes is high, the Fermi energy is larger than the binding energy of the pair and thus BCS condensation occurs.

3.3. *Mechanism of pairing*

Many theoretical models have been proposed for explaining the extraordinary properties of HTSC cuprates and it is not my purpose to give a complete review of them. Nevertheless I can classify them into two great categories:

(1) Models based on electron-electron interaction, which can be illustrated by the original model of Anderson, called the "Resonating Valence Bond" (RVB).[18] Lee *et al.*[11] give a good review of this approach. The overall philosophy is that the RVB idea of a spin liquid and its relation to superconductivity gives a good quantitative description of the underdoped phase diagram. It is in agreement with the observed AF fluctuations and with the d-wave symmetry of the superconducting gap.

(2) The other approach is electron-lattice interaction. It was the original idea of Müller and Bednorz,[2] who chose copper oxides, because the copper ion has a strong Jahn-Teller effect. The theory of strong coupling through bipolarons was developed by Alexandrov and Mott.[19]

An important experimental feature is the "anomalous isotope effect." T_c varies as $M^{-1/2}$ in low T_c superconductors, where M is the isotopic mass of the atom. This is considered as the main proof for electron-phonon interaction. In HTSC, the isotope effect disappears at optimum doping and this was considered as a proof for non-phonon interaction. But the isotope effect reappears for underdoped or overdoped samples. A simple interpretation for this effect has been given in the framework of the Van Hove scenario.[3]

An important property of underdoped samples is the formation of domains of alternating hole rich and hole poor regions. This is seen by Scanning Tunneling Spectroscopy (STM) of lightly doped cuprates.[20] Phase separation has also been seen in the formation of stripes.[21] The paper of Deutscher and de Gennes[4] proposes a new explanation for the formation of ladders of respectively small and large concentration of holes. The original idea of de Gennes is that the presence of two holes on adjacent copper atoms can be stabilized if the Cu-O-Cu bond contracts. This increases the transfer integral between the coppers. If the energy of the pair is sufficiently reduced, bound pairs or virtually bound pairs of holes can be formed. The model is developed in the following article "A

spatial interpretation of emerging superconductivity in lightly doped cuprates," by Guy Deutscher and Pierre-Gilles de Gennes, Ref. 4.

The main result is the formation of domains of width $4a$, where a is a cellular axis parameter, with two holes rich and two holes poor copper sites resulting from electron-lattice interaction and not from a purely electronic model as for stripes.

This model may explain several experimental results, in particular incoherent lattice fluctuations,[22] the vanishing of Cooper pairs approaching the Mott insulator,[23] the Raman effect,[24] and the isotope effect.[25]

One difficulty is that a pure attractive interaction favors the formation of s-wave pair functions instead of d-wave. Abrikosov has shown, however, that if a short-range repulsive interaction (which can represent either some part of the Hubbard repulsion at the copper sites or the interaction mediated by spin fluctuations) is added, then the order parameter can vary in sign and become negative at certain points of the Fermi surface.[26]

4. Conclusion

In conclusion, the bond contraction idea of de Gennes and its elaboration by Deutscher as presented in their paper Ref. 4 introduce a new and interesting concept for the formation of a pairing field in lightly hole doped crystals. This leads to a geometrical pattern of alternating hole-rich and hole-poor ladders with a periodicity of 4 times the lattice spacing. This pattern has been observed experimentally. It is also consistent with the isotope effect observed by scanning electron microscopy.[25] More experiments are needed to confirm this model.

Acknowledgments

I would like to thank Guy Deutscher and Jacqueline Bouvier for fruitful discussions.

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Physics/Solids, fluids: magnetic and electrical properties

A spatial interpretation of emerging superconductivity in lightly doped cuprates

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Abstract

The formation of domains comprising alternating 'hole rich' and 'hole poor' ladders recently observed by Scanning Tunneling Microscopy by Kohsaka et al., on lightly hole doped cuprates, is interpreted in terms of an attractive mechanism which favors the presence of doped holes on Cu sites located each on one side of an oxygen atom. This mechanism leads to a geometrical pattern of alternating hole-rich and hole-poor ladders with a periodicity equal to 4 times the lattice spacing in the CuO plane, as observed experimentally. ***To cite this article: G. Deutscher, P.-G. de Gennes, C. R. Physique 8 (2007).***
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Résumé

Cuprates supraconducteurs peu dopés : une interprétation des structures spatiales. Des arrangements électroniques réguliers ont été détectés récemment par Kohsaka et al. dans des cuprates sous dopés (via une sonde tunnel locale). Certaines paires Cu–O–Cu sont « actives », et forment une échelle. Les autres sites sont peu actifs. Pour expliquer ces structures, nous postulons que, lorsqu'une liaison Cu–O–Cu est occupée par deux trous, la distance (Cu–Cu) rétrécit et l'intégrale de transfert (t) est fortement augmentée. Ceci peut engendrer des paires localisées (réelles ou virtuelles). Aux taux de dopage étudiés, la période de répétition vaudrait 4 mailles élémentaires. ***Pour citer cet article : G. Deutscher, P.-G. de Gennes, C. R. Physique 8 (2007).***
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Keywords: Cuprates; Superconductors; Pairing; Domains; Electron–phonon interactions

Mots-clés : Cuprates ; Supraconducteurs ; Paires ; Domaines ; Interactions électron–phonon

Cette Note est le dernier article de Pierre Gilles de Gennes. Il s'était passionné pour ce retour à la supraconductivité, abandonnée après 1970. Sa disparition soudaine l'a empêché de répondre aux commentaires des rapporteurs. L'avenir dira si ses suggestions sont ou non pertinentes. Par respect pour sa mémoire ce texte est publié en l'état, mises à part quelques clarifications de détail apportées par son coauteur Guy Deutscher pour en préciser la lecture.

Comité de rédaction, Comptes rendus Physique

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Version française abrégée

On voit par effet tunnel, sur des cuprates peu dopés, des domaines rectangulaires de largeur $4a$ (où a est la maille du réseau plan CuO). Le centre du rectangle est formé par une colonne d'atomes O et deux cuivres voisins, qui apparaît comme « active » en effet tunnel, et est entourée de colonnes inactives.

Nous proposons une interprétation de ces arrangements à partir du postulat suivant : si deux trous occupent deux sites Cu voisins, il se produit une contraction de la liaison Cu–O–Cu. Celle-ci augmente significativement l'intégrale de transfert t entre les Cu, car t fait intervenir deux fois le recouvrement des orbitales d du Cu et p de l'oxygène, et est donc très sensible à la distance. Nous montrons, par une méthode variationnelle, que l'énergie d'une paire est notablement abaissée (Eq. (2)). Si cet abaissement dépasse la demi-largeur de bande pour des trous isolés, on arrive à une paire liée. Si il est un peu inférieur, on aura un état lié virtuel, avec une durée de vie finie. On arrive à une conclusion similaire si l'on considère des segments O–Cu–O–Cu–O avec deux trous localisés de préférence sur les oxygènes, comme indiqué Fig. 2.

Les conséquences de ce modèle apparaissent sur la Fig. 3 où la paire active est formée par les atomes $2'$–1–2. La distance suivante (entre 3 et $3'$) est augmentée : donc cette colonne doit avoir une faible intégrale de transfert, et sera inactive.

La propagation de paires ne peut alors se faire que dans la direction perpendiculaire. Pour un taux de dopage voisin de la moitié du dopage optimum, comme c'est le cas dans les expériences, on engendre ainsi des rectangles, de largeur $4a$, dont la longueur est probablement conditionnée par des défauts. Avec le taux de dopage utilisé ($\sim$10%), le nombre minimum de Cu requis pour avoir une paire active est de l'ordre de 20. La concentration locale sur les colonnes actives est alors comparable à l'optimum de supraconductivité.

Y-a-t-il des excitations, de type quasi particule, mobiles ? La conduction à l'état normal est faible. Si une certaine cohérence entre domaines subsiste, elle porterait probablement sur des excitations orientées à 45° des domaines. On sait que les cuprates peu dopés ont une surface de Fermi incomplète formée de petits arcs autour de la direction (11).

La supraconductivité macroscopique n'est pas asservie à la structure interne des domaines : on attendrait plutôt un paramètre d'ordre Δ fort dans les directions (01).

1. Introduction

Recent imaging of Scanning Tunneling Spectroscopy (STM) of lightly doped superconducting cuprates has revealed the existence of rectangular domains of width $4a$, where a is the lattice spacing in the CuO plane [1]. Inside each domain the carrier concentration is non-uniform, with a sharp contrast between a central ladder consisting of a column of oxygen atoms and the two neighboring Cu–O–Cu columns where the carrier concentration is high, while it is low on similar ladders at the edges of the domain. The general aspect of the images shows domains having two possible orientations at right angles to each other, with less well organized domains spread randomly across the surface. Tunneling characteristics measured at sites belonging to the central ladder show at low bias a structure reminiscent of a superconducting density of states, with pronounced peaks particularly on the Cu–O–Cu columns, while at the edge ladders only a small conductance dip is visible.

In this communication we wish to point out that this geometrical pattern can be easily understood if one assumes the existence of a mechanism that favors the presence of a pair of doped holes on Cu–O–Cu, or perhaps O–Cu–O–Cu–O segments, accompanied by a small contraction of the Cu–O distances. This mechanism can be due to an increase of the transfer integral t_{OCu}. t_{OCu} depends critically on the overlap between Cu(d) orbitals and O(p) orbitals. If the energy gained by contraction is sufficient, a bound state of the hole pair can be formed.

2. The model

Our interpretation of the pattern observed in [1] proceeds in three steps: (i) Pair formation; (ii) Formation of hole-rich and hole-poor regions; (iii) Pair propagation.

2.1. Pair formation

We discuss on Fig. 1 the binding energy of a hole pair on a Cu–O–Cu segment. We estimate it by a variational function containing four states α, β, γ', and γ'' as defined in Fig. 1(b). We are interested here in the spin singlet,

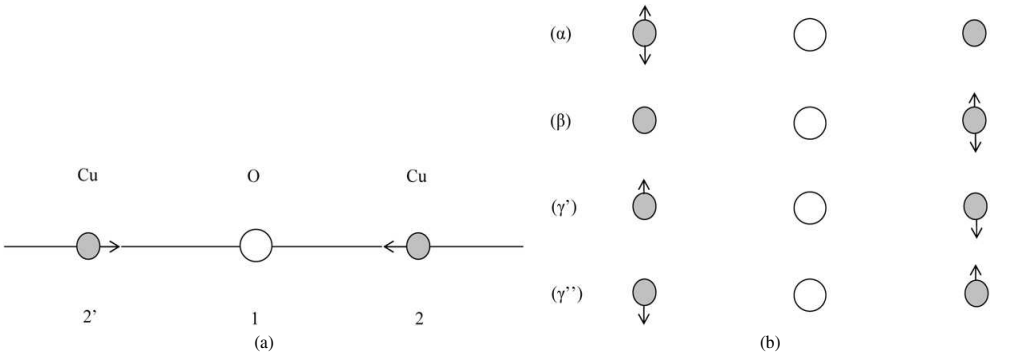

Fig. 1. (a) A two hole pair on two copper atoms (2) and (2′), around one oxygen atom (1). We assume here that the two coppers move in when each of them carries a hole: the transfer integral between 2 and 2′ is increased. (b) The four states participating in the construction of a spin singlet pair.

Fig. 1. (a) Une paire de trous sur deux atomes de cuivre (2) et (2′), autour d'un même atome d'oxygène (1). On suppose ici que les deux suivres se rapprochent de l'oxygène (quand chacun porte un trou) : l'élément de matrice de transfert entre (2) et (2′) est alors augmenté. (b) Les quatre états quantiques participant à la construction d'un singulet de spin.

Fig. 2. (a) An oxygen centered O–Cu–O–Cu–O segment with holes located on the edge oxygens. Energy is lowered by admixture with states as shown in Fig. 2b.

Fig. 2. (a) Un segment O–Cu–O–Cu–O centré sur un atome d'oxygène, avec un trou sur les oxygènes aux extrémités du segment. L'énergie est réduite par mélange avec des états comme montres Fig. 2b.

for which the amplitudes are even ($\alpha = \beta$, $\gamma' = \gamma''$). The unperturbed energy of (α) and (β) is U and is large. The eigenvalue equation for the energy ϵ is then:

$$\epsilon(\epsilon - U) = 4(t_{\mathrm{OCu}})^2 \tag{1}$$

This leads (for large U) to an energy:

$$\epsilon_{\mathrm{pair}} = -4(t_{\mathrm{OCu}})^2/U \tag{2}$$

(We are indebted to W. Harrison for pointing out the factor 4 showing up in the singlet energy.) We must compare this to the energy of two independent holes, both at the bottom of the band, $2\epsilon_0$, where $\epsilon_0 = 4t_0$ is the half band width. The pairs win if:

$$t_{\mathrm{OCu}} > (Ut_0/2)^{1/2} \tag{3}$$

On the whole, this idea is very tentative but it has one merit: the ratio t_{Ocu}/t_0 is very sensitive to the (Cu d)(O p) overlap; this could explain why ions which are isoelectronic to Cu cannot compete with the cuprates.

There is, however, a difficulty with holes residing on copper atoms, in view of the high ionization potential of the Cu^{++} ion [2]. Holes may reside preferentially on oxygen atoms. This can be achieved if we consider a O–Cu–O–Cu–O segment, with holes on the edge oxygen atoms (Fig. 2). Here, energy can be gained by admixture with states where a hole is transferred from one of the coppers to the central oxygen. The energy gained is still proportional to $(t_{\mathrm{OCu}})^2$.

2.2. *Formation of hole-rich and hole-poor regions*

So let us assume that two holes have formed a bound state, and let us examine the consequences for the neighboring sites, as shown in Fig. 3. Following Kohsaka et al. we label 1 the site of the central oxygen atom; 2 and 2′ the neighboring copper sites; 3 and 3′ the following copper sites, followed by the oxygen sites 4 and 4′. Since the contraction, or dimerisation, of the segment (2–2′) goes hand in hand with a local increase of the hole density, it follows that the

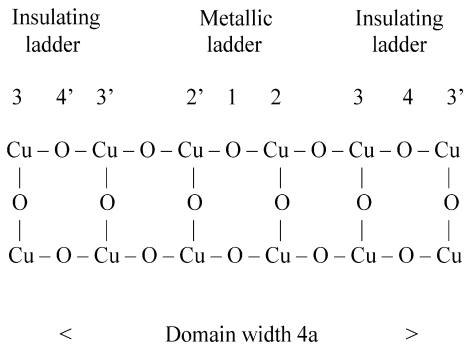

Fig. 3. The pattern of hole concentration in a domain consisting of one high concentration central ladder centered on a column of oxygen atoms with two low concentrations ladders on each side. The pattern leads to a periodicity of 4 times the lattice parameter of the CuO plane.

Fig. 3. Répartition des trous dans un domaine : une échelle centrale riche en trous et deux échelles pauvres de part et d'autre. Ceci engendre une maille multipliée par 4.

hole density on neighboring sites such as copper sites 3 and 3′ is decreased. A charge density wave can be triggered by local pair formation, dimerisation and the charge density wave being coupled phenomena, the two reinforcing each other. We then arrive at the following sequence: the central Cu–O–Cu bond (2′–1–2) is hole-rich, the neighboring bonds (3–4) and (4′–3′) are hole-poor. Over a distance of 4*a*, there are two hole-rich and two hole-poor Cu sites. Defects, due primarily to independent nucleation of other dimers at right angles with the one under consideration, will pin the charge density wave. The pair on 2′–2 cannot propagate along its axis. These conclusions still hold if the hole pair is localized on a five site segment.

2.3. Pair propagation

There is nothing to prevent pair propagation in the perpendicular direction, which will reduce the kinetic energy. This will lead to the formation of domains consisting of one central hole-rich and two lateral hole-poor ladders, the central ladder being centered on an oxygen column. Pair propagation on the central ladder can lead to incipient superconductivity. A periodicity of 4*a* (1), as has been observed, with alternating conducting (and may be superconducting), and insulating (and may be antiferromagnetic) domains can be favorable in some range of doping, see below a discussion of this point.

Domain formation is the result of a nucleation process of the first bound state. This nucleation process occurs randomly along the (10) and (01) directions, resulting in domains at right angles randomly distributed, again as observed. There is no long range order in this pattern.

3. Discussion

The essential difference between the proposed model and that of stripes [3–5] is that ours is not a purely electronic one reflecting the competition between AF and superconducting orders, where the lattice plays no role (or only a very secondary one). Rather, the contraction of Cu–O bonds plays here a central role. Both models predict oxygen centered structures, in agreement with experiments [1,6], but differ in some important ways. While stripes are basically one dimensional (namely their length should be much longer than their width), our model being based on a local bond contraction leads to a pattern of domains, at right angles to each other, that are not necessarily very long, in better agreement with what is observed [1], and with several pieces of experimental evidence for an Electronic Cluster Glass in underdoped samples ([7] and references therein), rather than one dimensional stripes. Lattice disorder below the temperature of formation of the domains is a natural consequence of our local pair formation model, since bond contraction is local and occurs randomly in two different directions. Lattice disorder has indeed been found to increase below a temperature of about 150 K, which we surmise is that of formation of the domains [8,9].

The hole doping level of the samples studied by Kohsaka et al. is of the order of 0.1 per Cu site. The minimum number of Cu sites necessary to have at least one hole pair per domain is then 20, which means that slightly elongated

domains as observed do contain a few hole pairs. Conduction can take place along the central ladder. If we assume that the hole concentration is peaked on the central ladder and is negligible on the edge ladders, the hole concentration on the central ladder is then of the same order as the average concentration at optimum doping. The edge ladders are then at a concentration near that of the pure antiferromagnetic phase. Barring complications due to the one-dimensional character of the central ladders, one may also expect the formation of a superconducting gap of the same order of magnitude as that of the gap at optimum doping, i.e. of a few 10 meV, again as observed.

The emerging picture of a lightly doped cuprate is then that it is composed of conducting and superconducting ladders weakly coupled together, either laterally through insulating ladders or at right angles. Conduction and superconduction on the macroscopic scale will necessarily require coupling between ladders oriented at right angles. Since translational and rotational symmetry are broken in the ladder pattern, which is highly disordered, it is not obvious that quasi-particles can exist with a well defined wave vector formed in lightly doped cuprates. Indeed, the normal state is known to be weakly insulating. If some degree of coherence can be achieved, one may expect that it will be for quasi-particles having their momentum at 45 degrees from the ladders directions, which is the (11) orientation and equivalent. Clearly, there is no possibility of propagation along the (10) and equivalent directions because of the interruption of the domains. This is in agreement with the known fact that lightly doped cuprates have an incomplete Fermi surface consisting of small arcs around the (11) directions [10]. We note that strong renormalization effects near the Fermi level around the (11) direction over an energy range of the order of 100 meV have recently been attributed to electron–phonon interaction [11].

Superconductivity on the macroscopic scale can be very different from that in the individual domains, again because it requires coupling between domains that are at right angles from each other. A gap anisotropy is to be expected, with the strongest gap values being along the (10) and equivalent directions. It may be that this coupling is also at the origin of the d-wave symmetry of the macroscopic superconducting order parameter.

Note added by Guy Deutscher at the time of submission of the final version

Pierre-Gilles de Gennes passed away a few days before we were to have a final discussion of the manuscript. He had left clear instructions that I was to take care of the final version. It is under these very sad circumstances that this revised version is being submitted. Had we met as planned, the final version might of course have been somewhat different but in view of the intense correspondence that we had in the period of several weeks preceding his death, I feel reasonably sure that this version is close to what he wanted it to be. In any case, I have done the best I could. Warm thanks are due to Philippe Nozières for a critical reading of the manuscript.

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