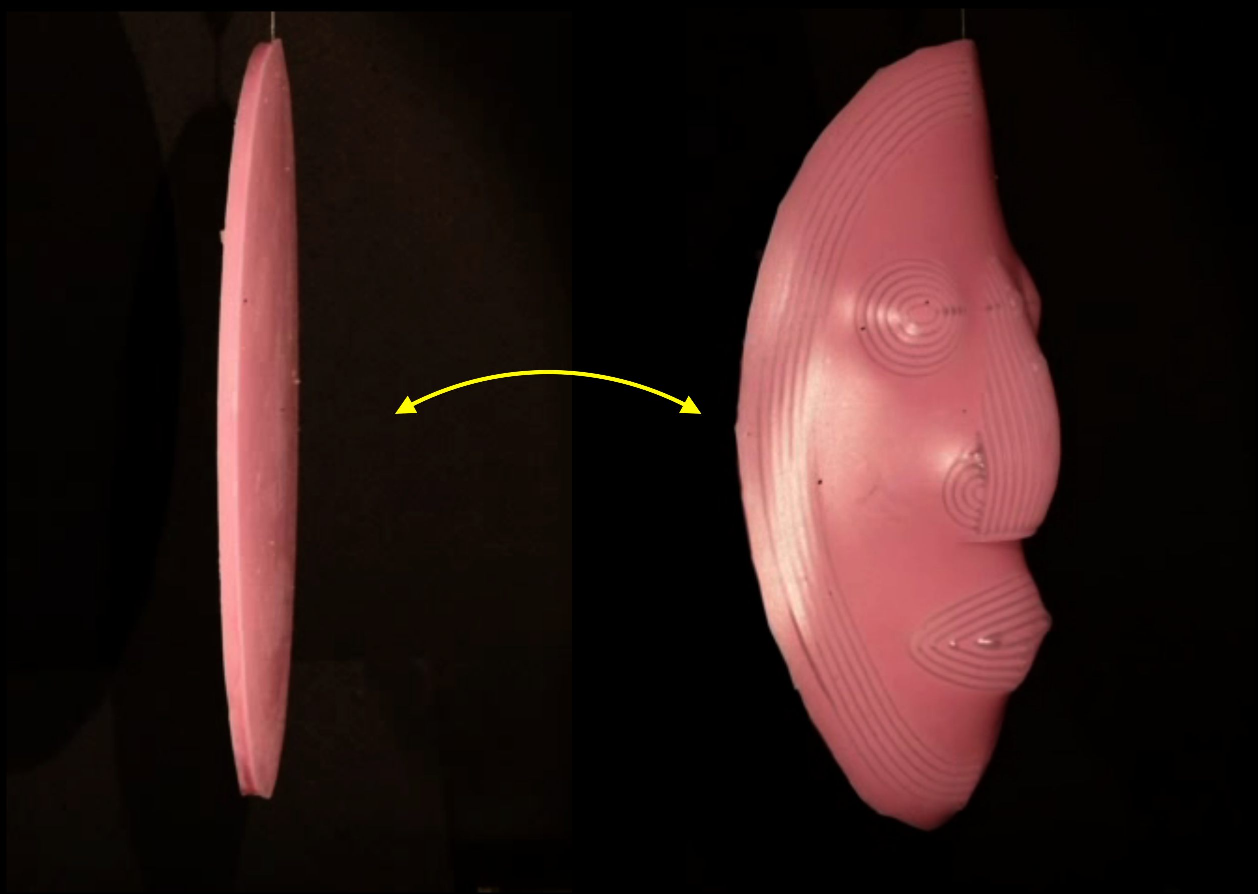
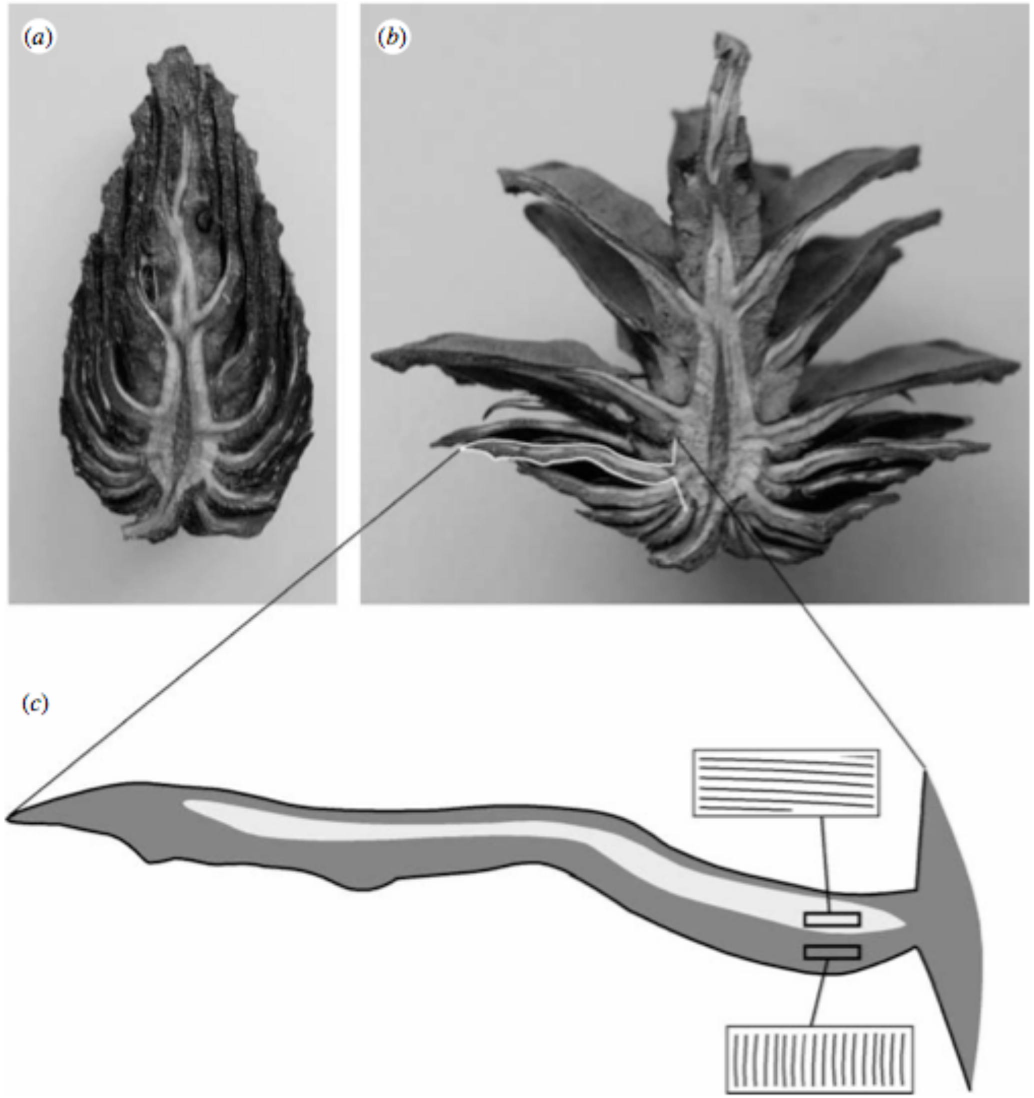
Mises en forme

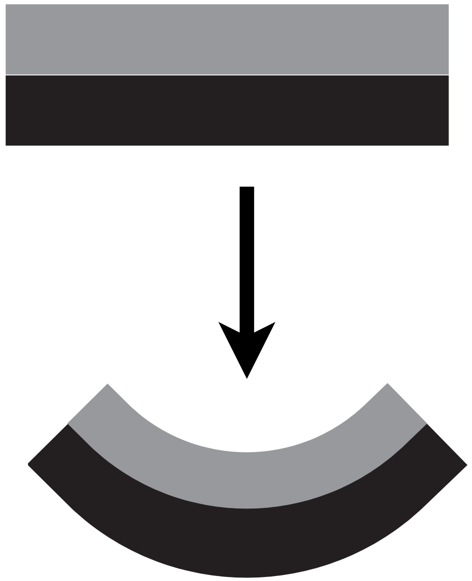


José Bico, MecaWet Group PMMH ESPCI

Drying pine cones

Bilayer scales

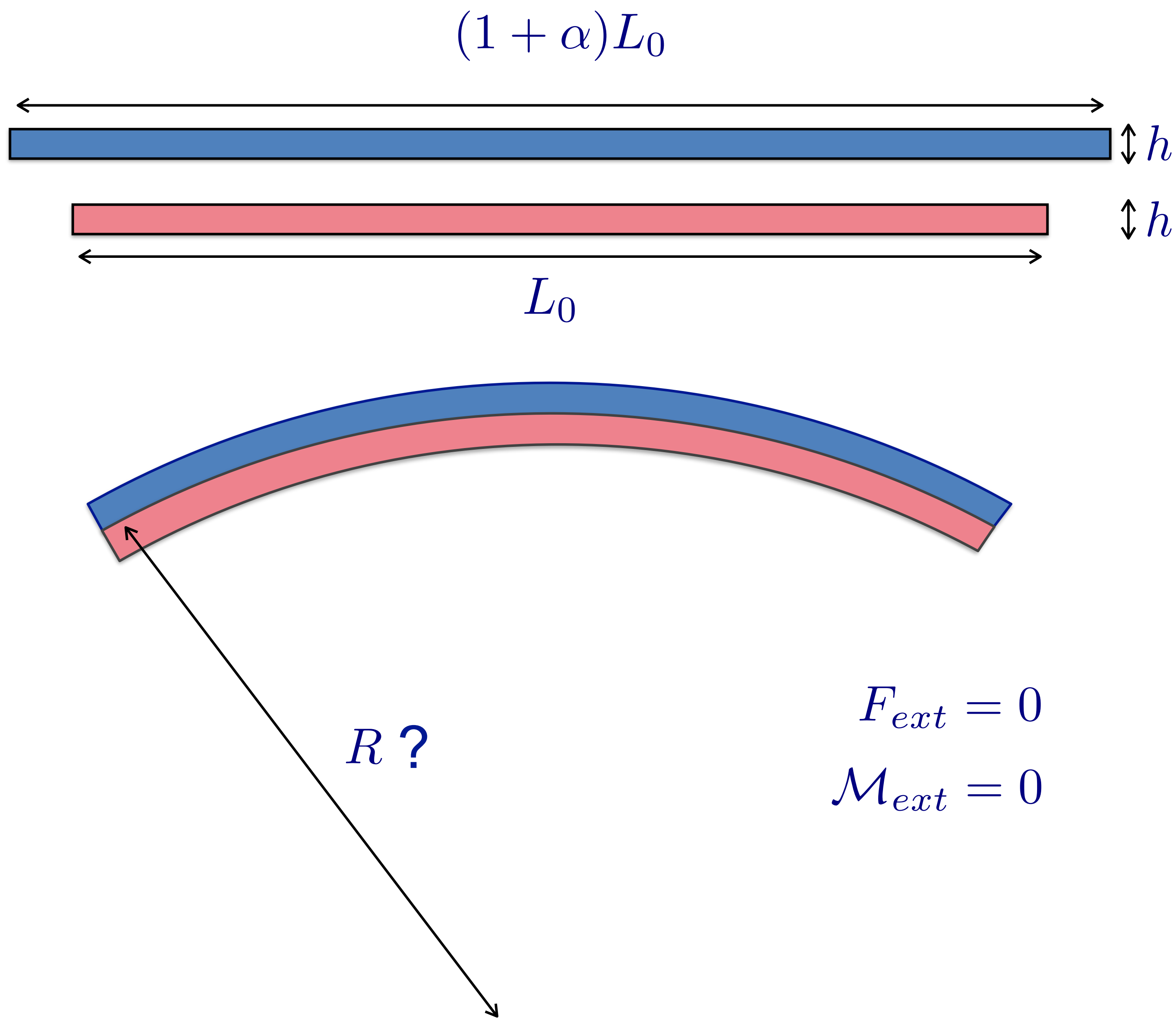




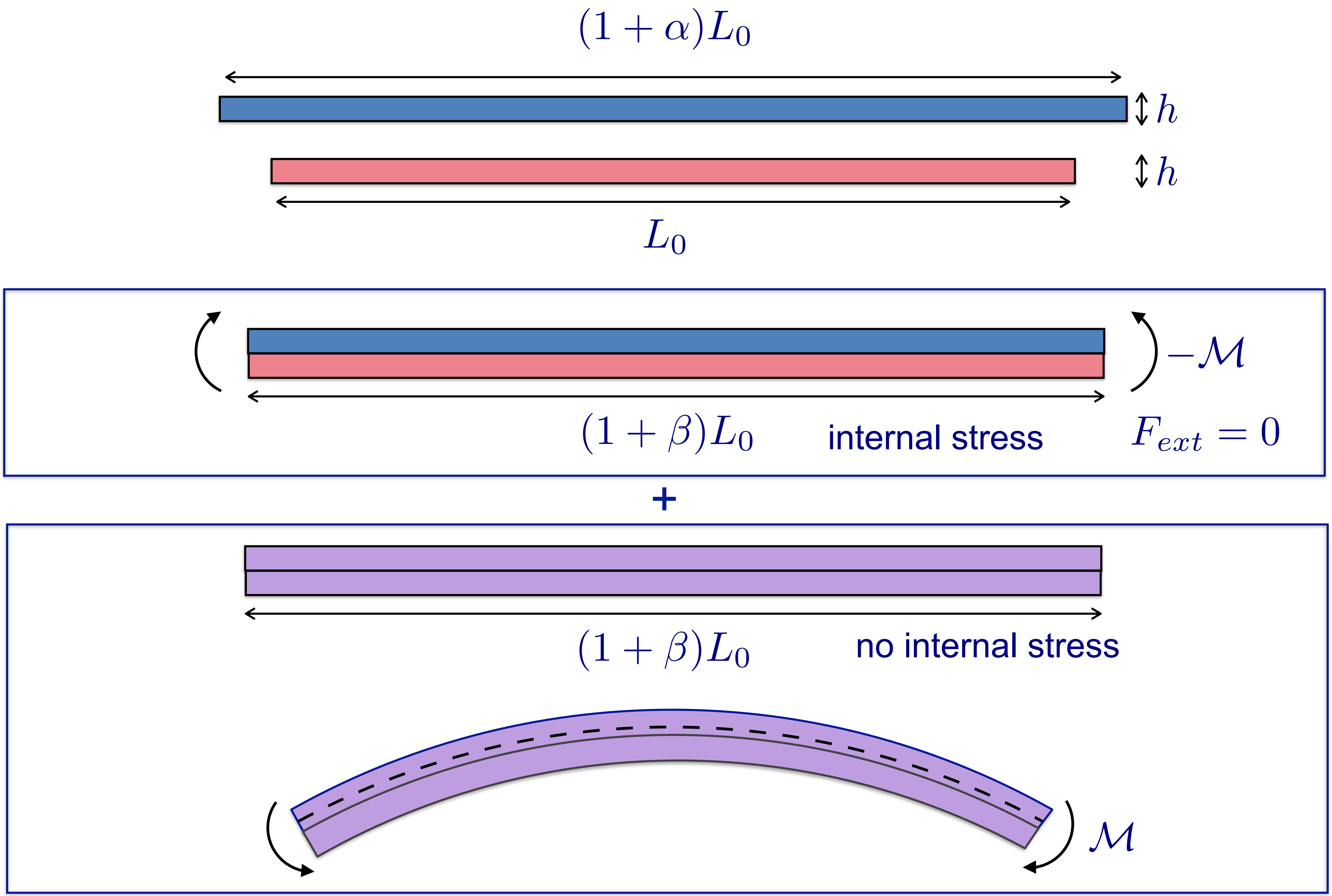
Dowson & Vincent, Nature (1997)

Reyssat & Mahadevan, J.R.Soc Interface (2008)

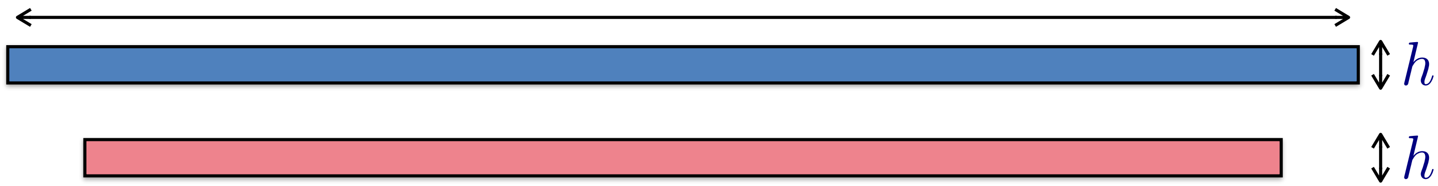
Bilayers

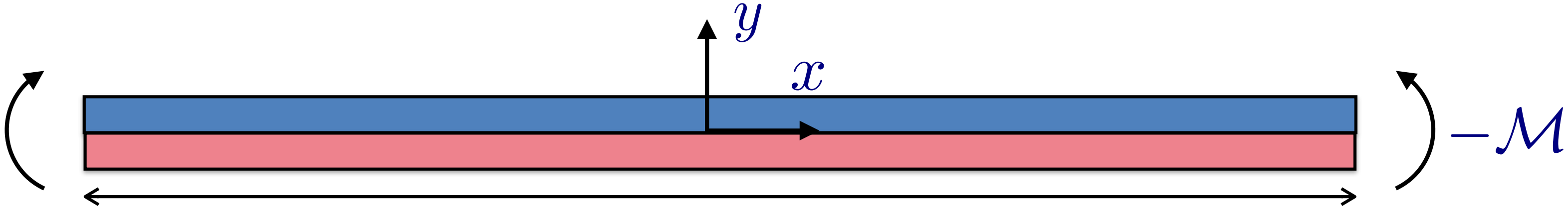


Superposition



Superposition



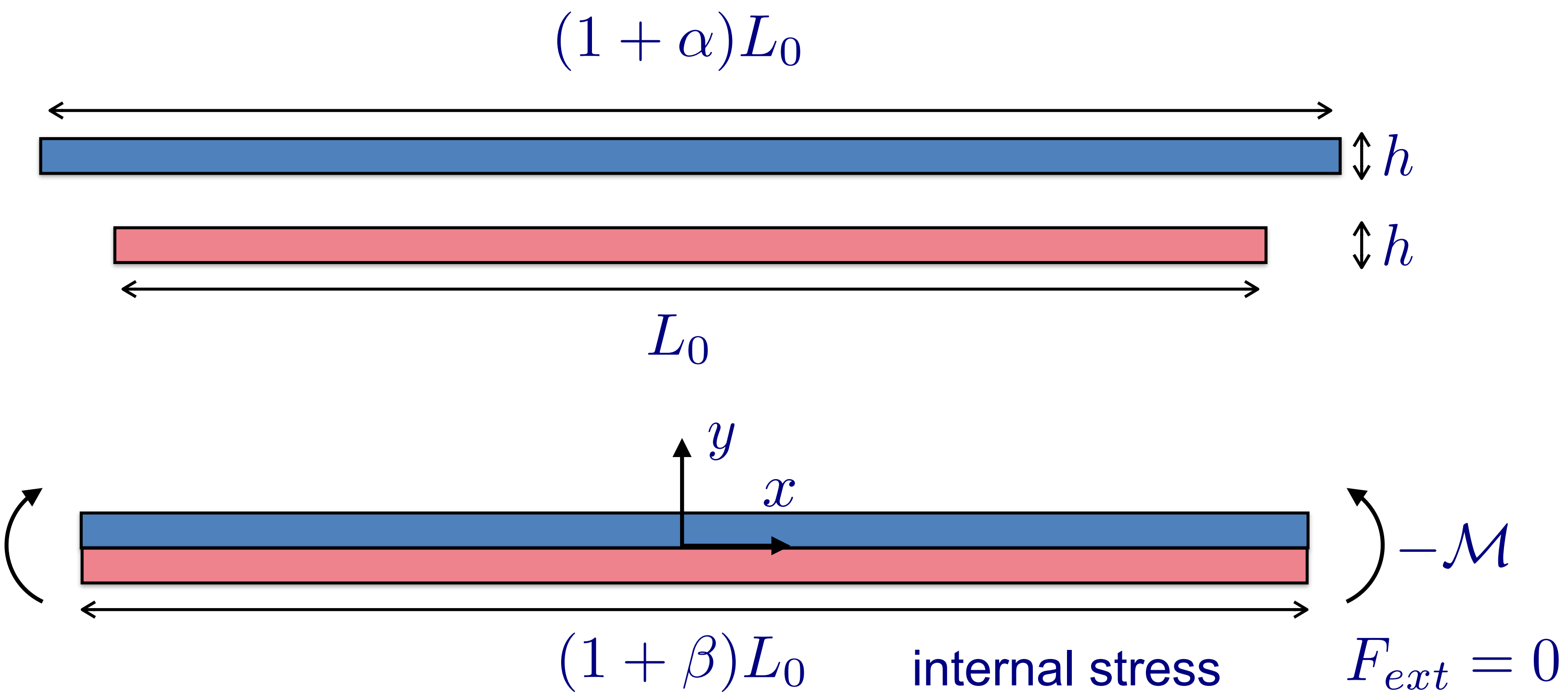


$$-h \le y \le 0 \qquad \varepsilon_{xx} = \beta \qquad \sigma_{xx} = \beta E$$

$$0 \le y \le h \quad \varepsilon_{xx} = \frac{(1+\beta)L_0 - (1+\alpha)L_0}{(1+\beta)L_0} \simeq (\beta - \alpha)$$

$$\sigma_{xx} = (\beta - \alpha)E$$

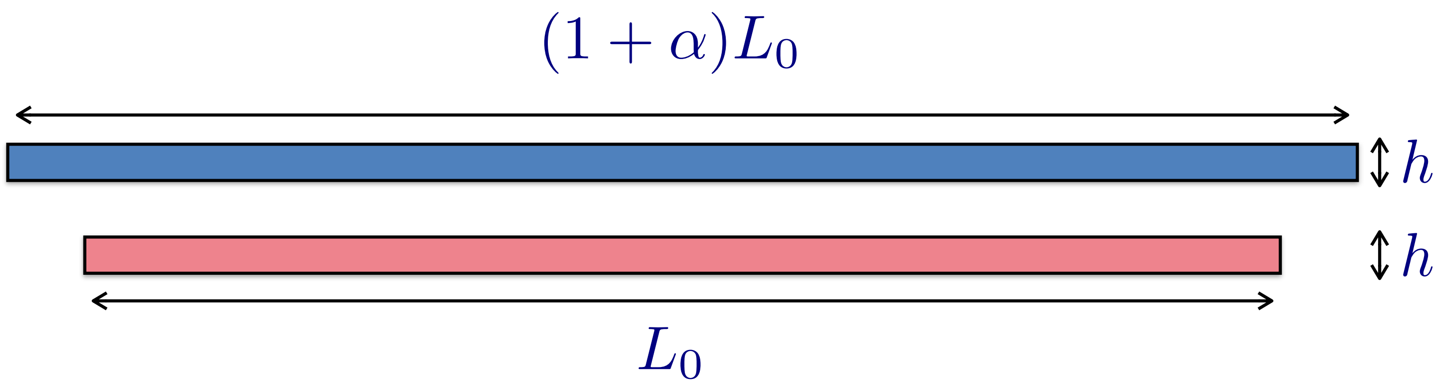
Superposition

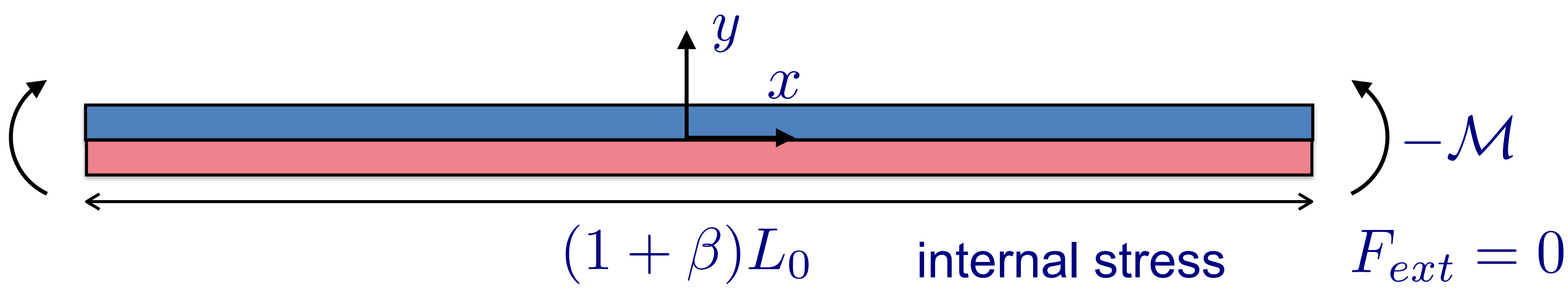


$$F_{ext} = 0 \quad \Rightarrow \quad \int_{-h}^{h} \sigma_{xx}(y)\, dy = 0$$

$$E(\beta - \alpha)h + E\beta h = 0 \qquad \boxed{\beta = \alpha/2}$$

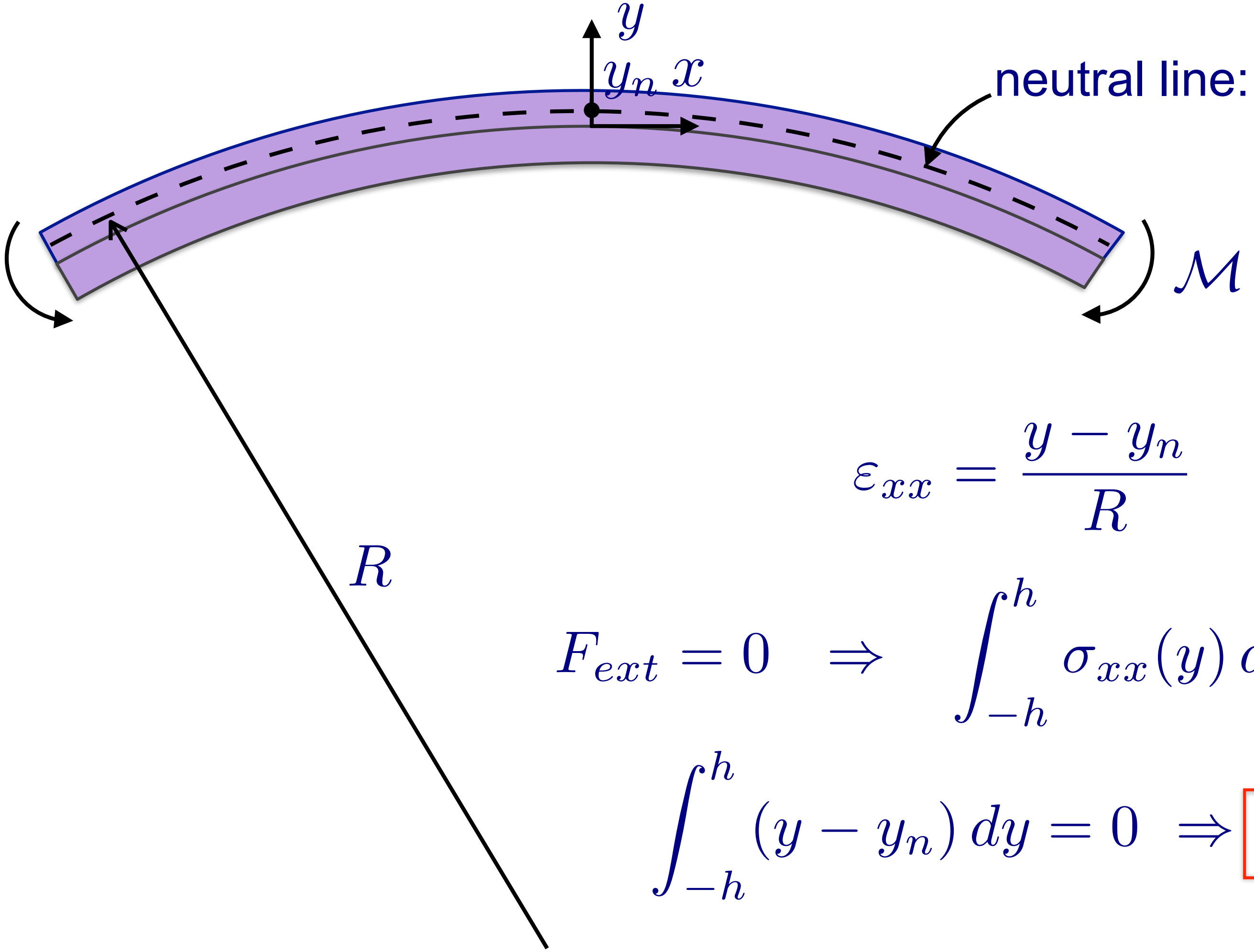
Superposition





$$\mathcal{M}/\ell = \int_{-h}^{h} \sigma_{xx}(y) y \, dy = \int_{-h}^{0} -\frac{E\alpha}{2} y \, dy + \int_{0}^{h} \frac{E\alpha}{2} y \, dy = \boxed{\frac{E\alpha h^2}{2}}$$

Superposition

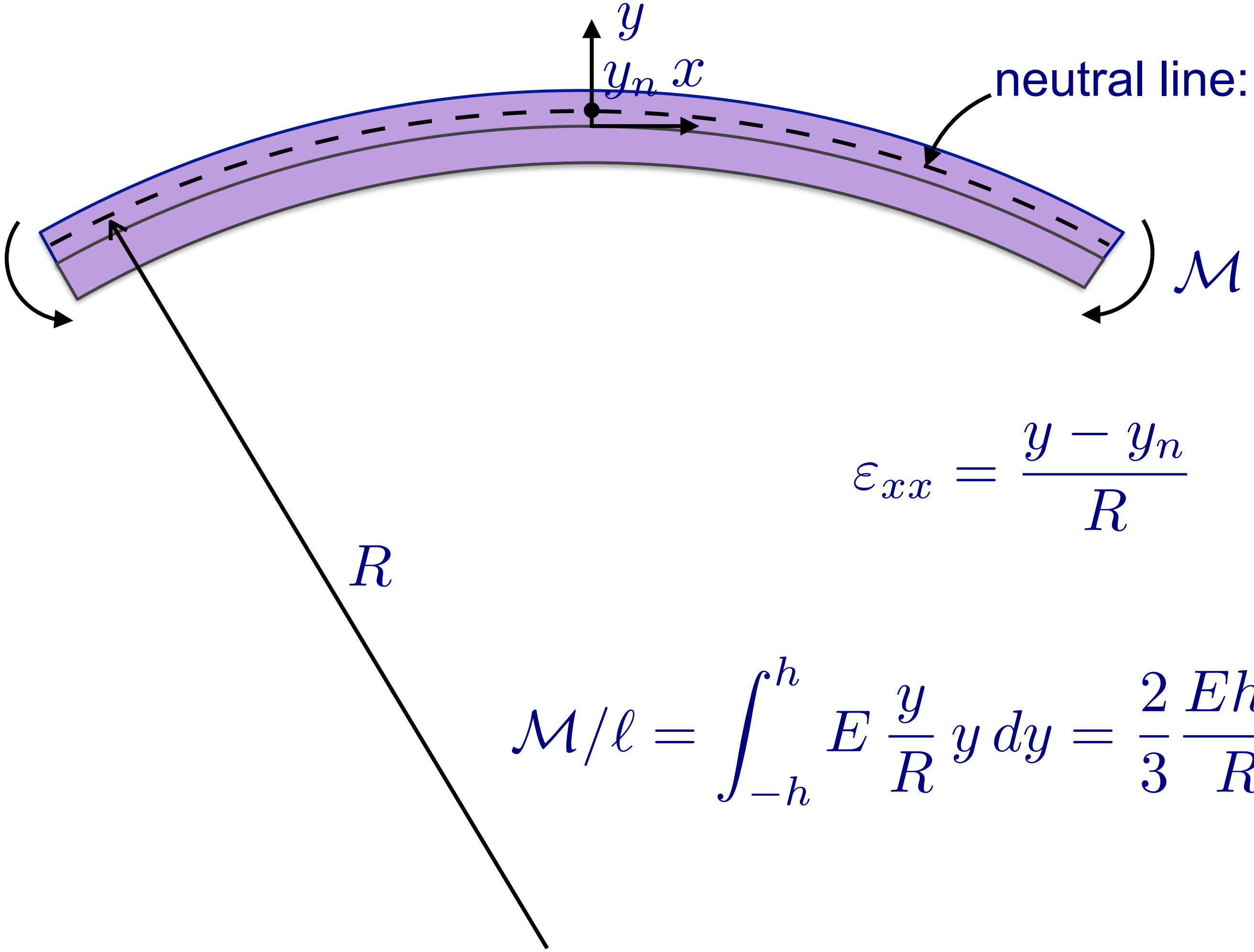


$$\varepsilon_{xx} = \frac{y - y_n}{R}$$

$$F_{ext} = 0 \quad \Rightarrow \quad \int_{-h}^{h} \sigma_{xx}(y)\, dy = 0$$

$$\int_{-h}^{h} (y - y_n)\, dy = 0 \quad \Rightarrow \quad \boxed{y_n = 0}$$

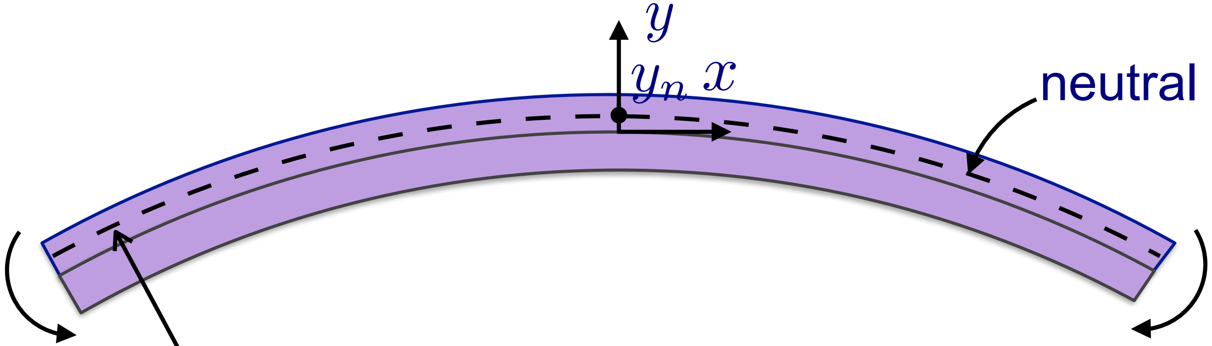
Superposition



$$\varepsilon_{xx} = \frac{y - y_n}{R}$$

$$\mathcal{M}/\ell = \int_{-h}^{h} E \, \frac{y}{R} \, y \, dy = \frac{2}{3} \frac{Eh^3}{R} = \frac{E\alpha h^2}{2}$$

Superposition



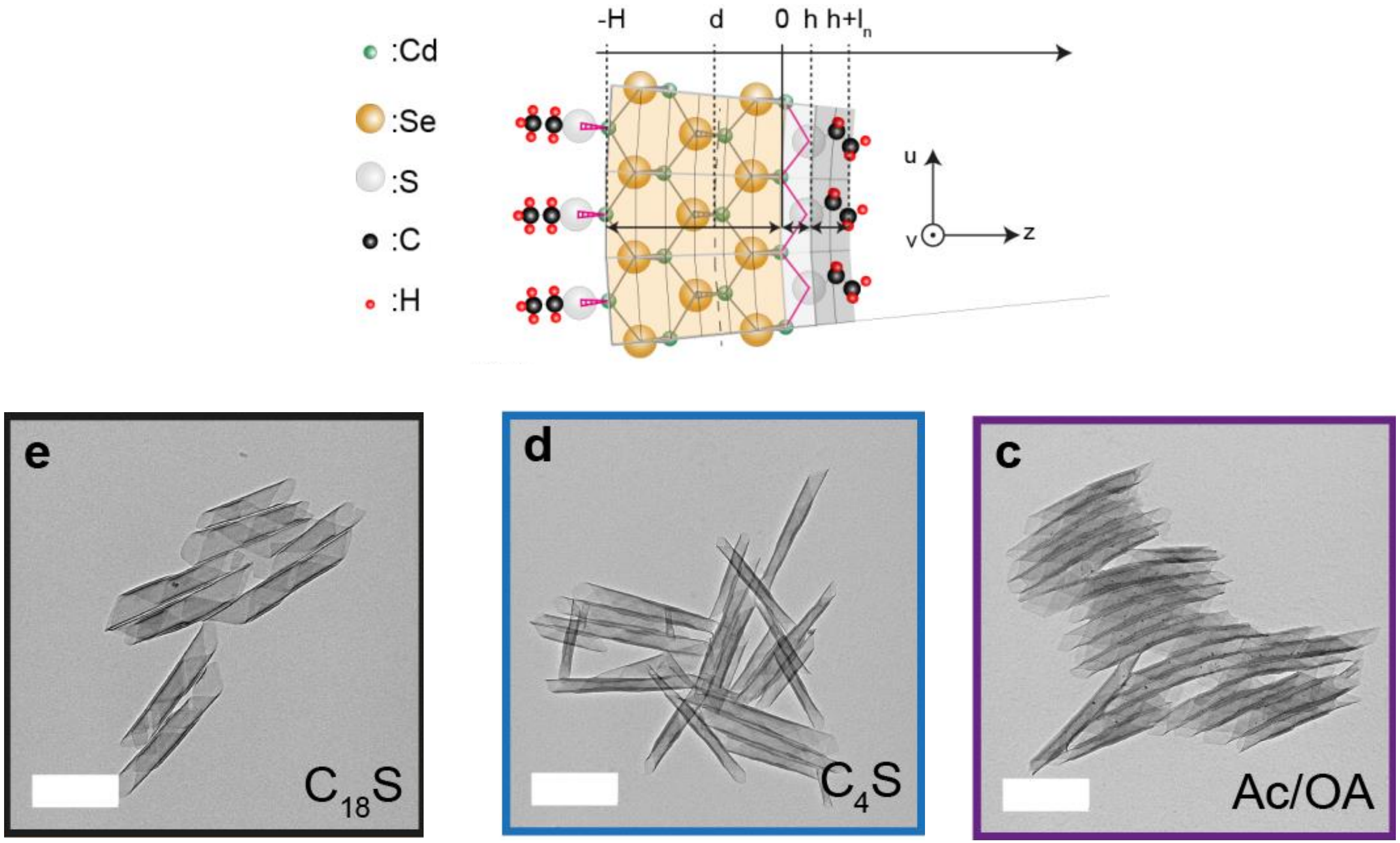
$$R = \frac{4}{3}\frac{h}{\alpha}$$

general case

$$R = \frac{E_1 h_1^4 + E_2 h_2^4 + 6E_1E_2h_1^2h_2^2 + 4E_1E_2h_1h_2(h_1^2+h_2^2)}{6E_1E_2h_1h_2(h_1+h_2)\alpha}$$

and more complex if $\eta_1 \neq \eta_2$

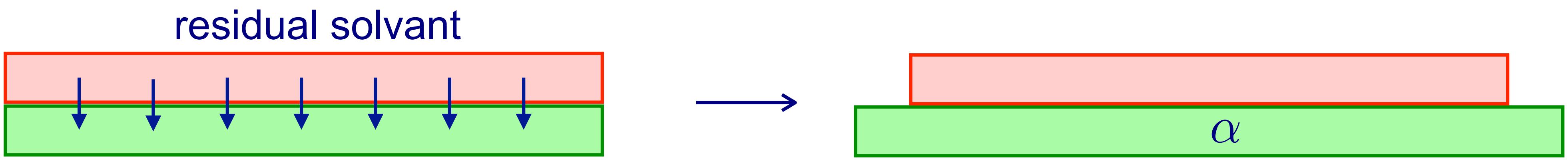
Also at small scale



CdSe nanoplatelets + ligands ⇒ surface stress

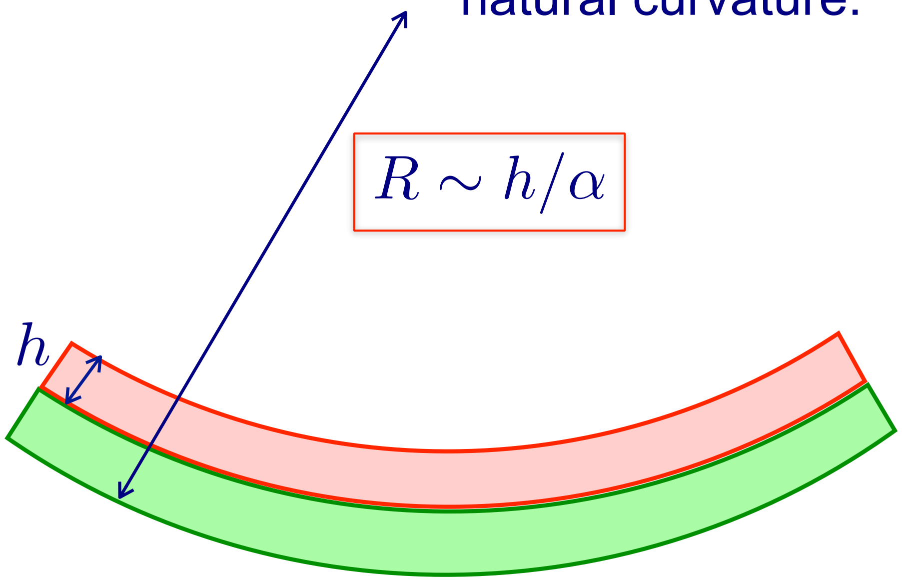
Hong Po & Sandrine Ithurria

Bilayer sheets



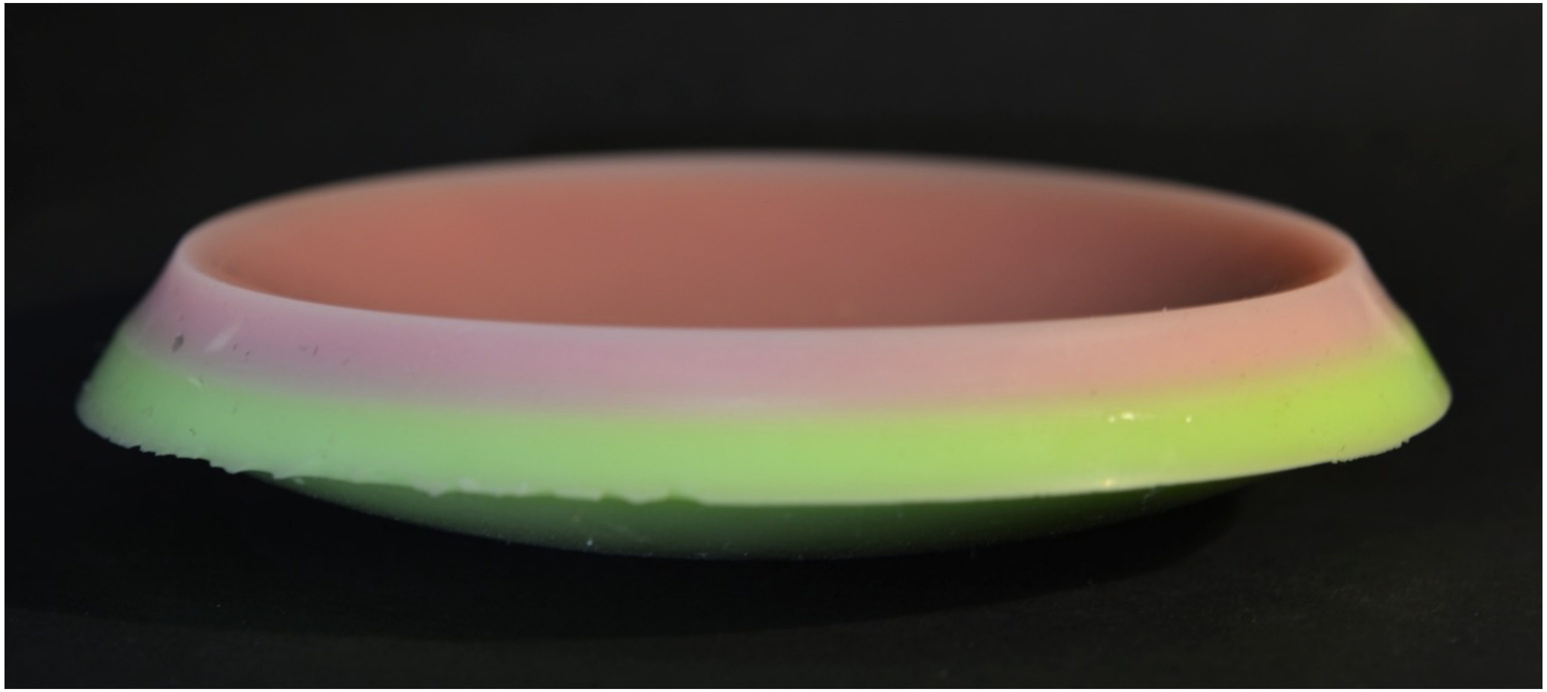
natural curvature:

$$R \sim h/\alpha$$



in 2D ?

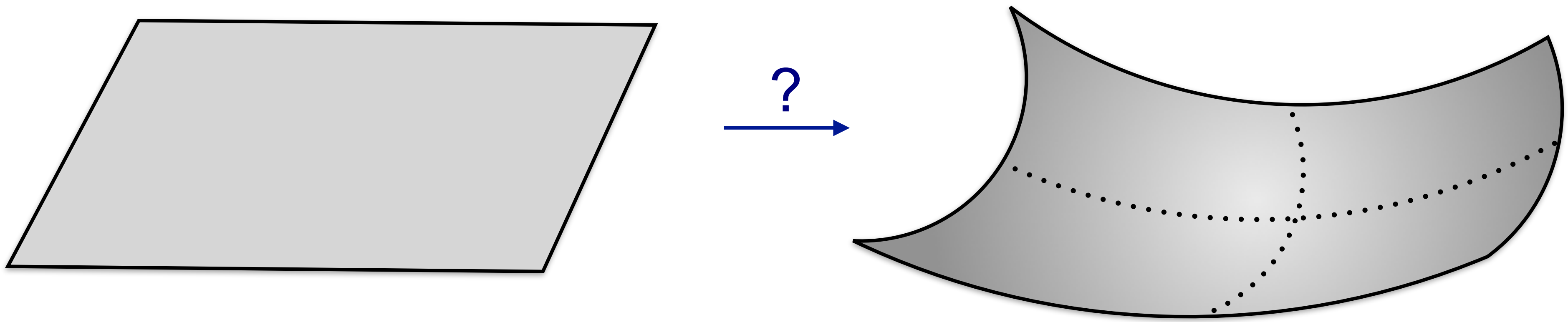
Thick bilayer



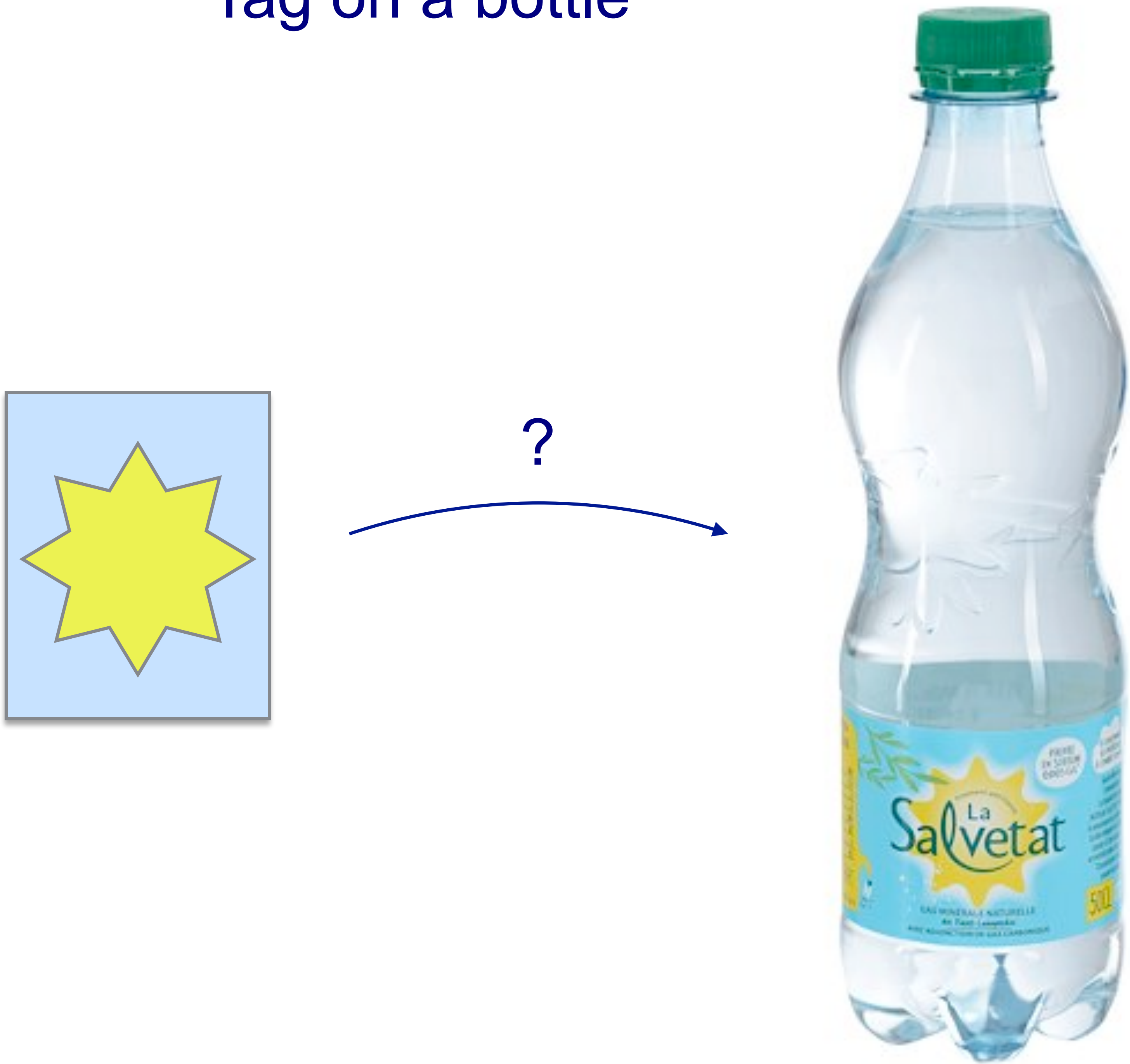
Thin bilayer: frustrated rolling

M.Pezzulla, P. Nardinocchi & D.Holmes (2016)

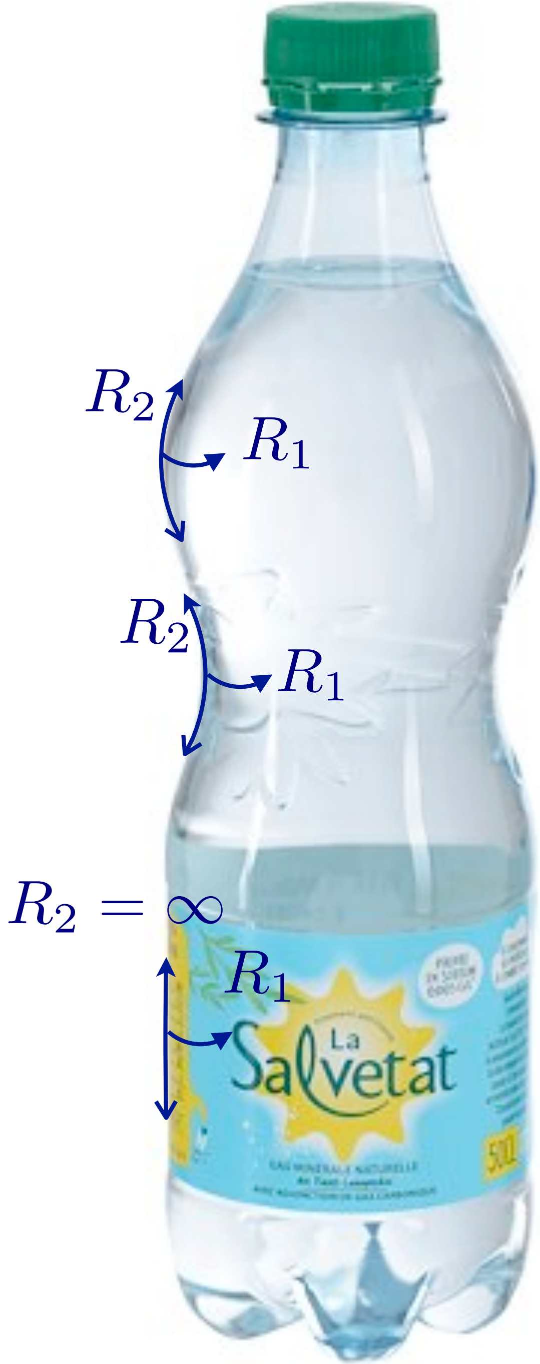
Bending a sheet in 2 directions?



Tag on a bottle



Curvatures



Gauß theorem

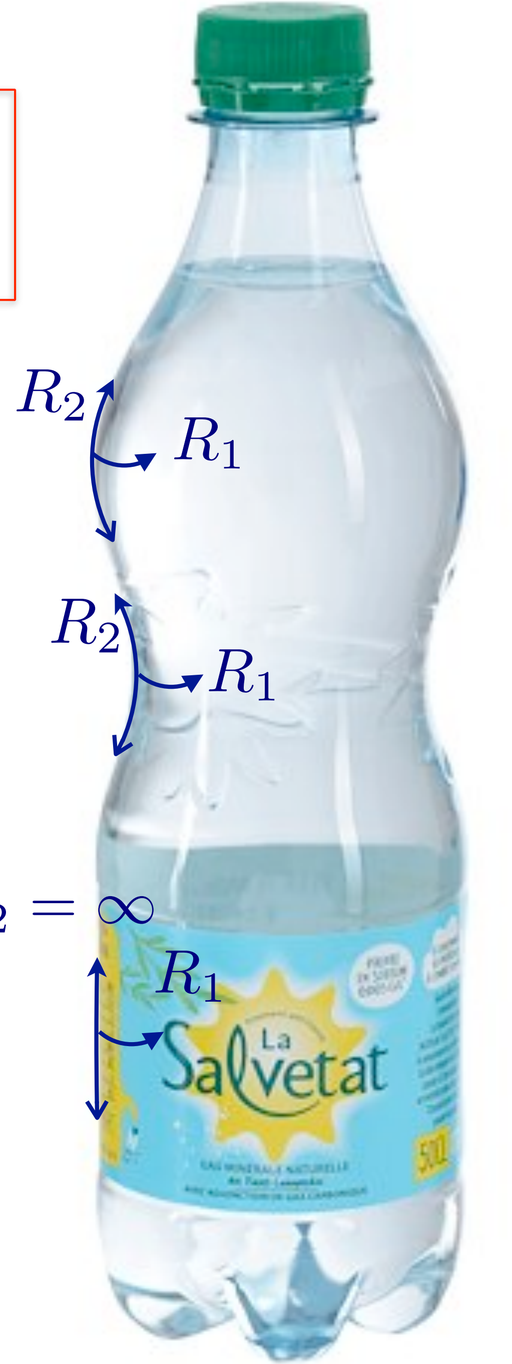
Gaussian curvature $$K = \frac{1}{R_1}\frac{1}{R_2}$$

$K > 0$

$K < 0$

$R_2 = \infty$

$K = 0$



Gauß theorem

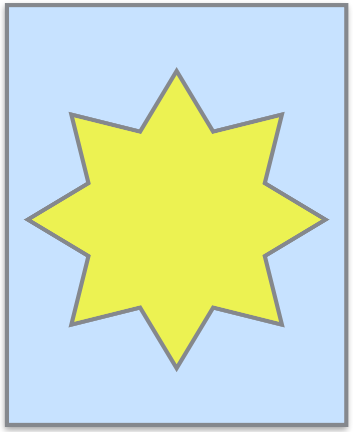
Gaussian curvature $$K = \frac{1}{R_1}\frac{1}{R_2}$$

$K > 0$

distances not preserved

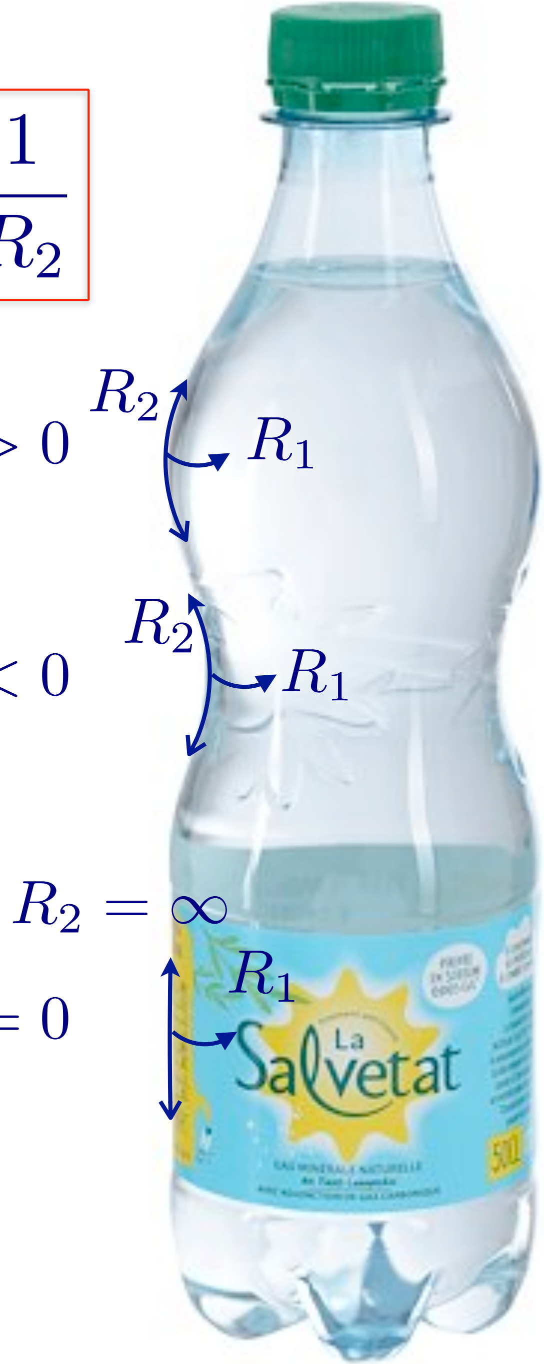
$K < 0$

$K = 0$



$K = 0$

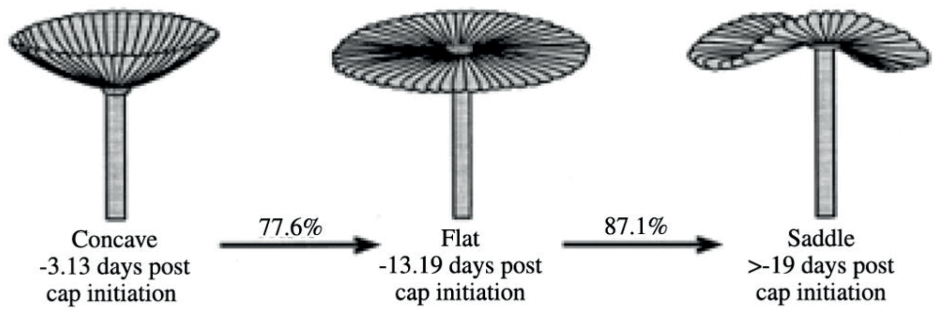
distances preserved



Challenging Gauß theorem ?

the cap of *acetabularia*

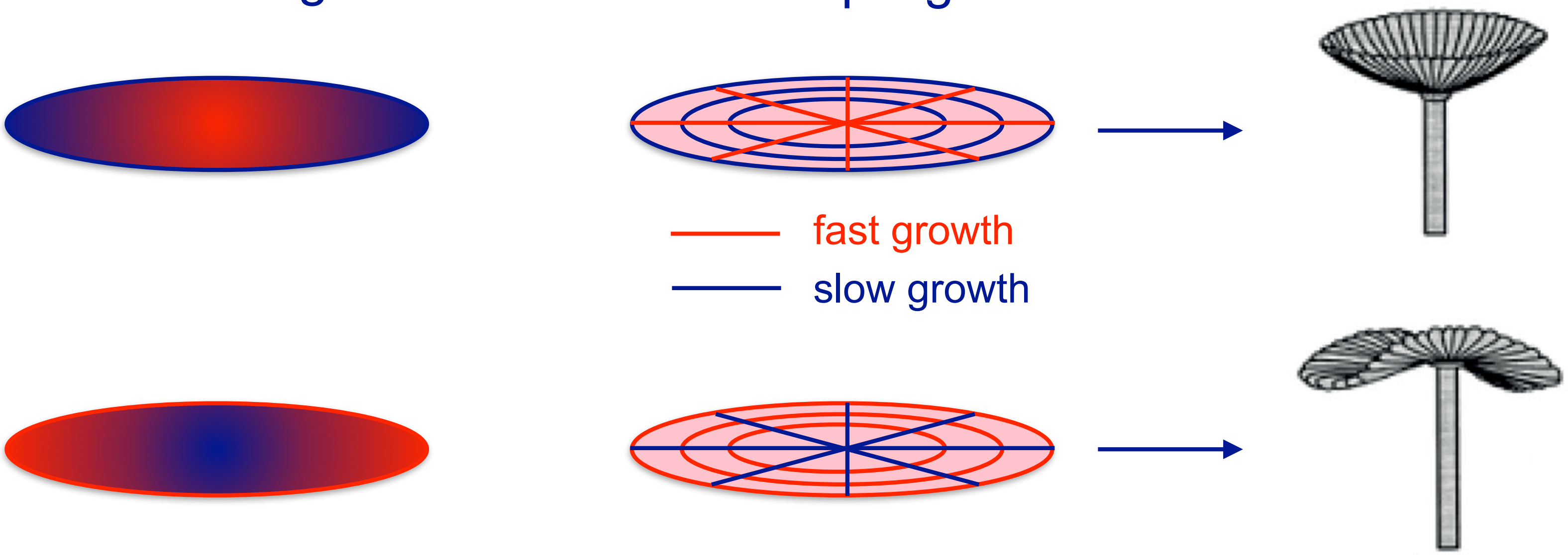
Non uniform/isotropic growth



Serikawa & Monadoli Planta (1998)

non uniform growth

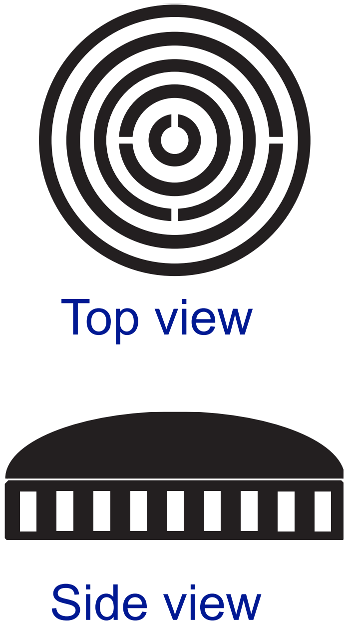
non isotropic growth

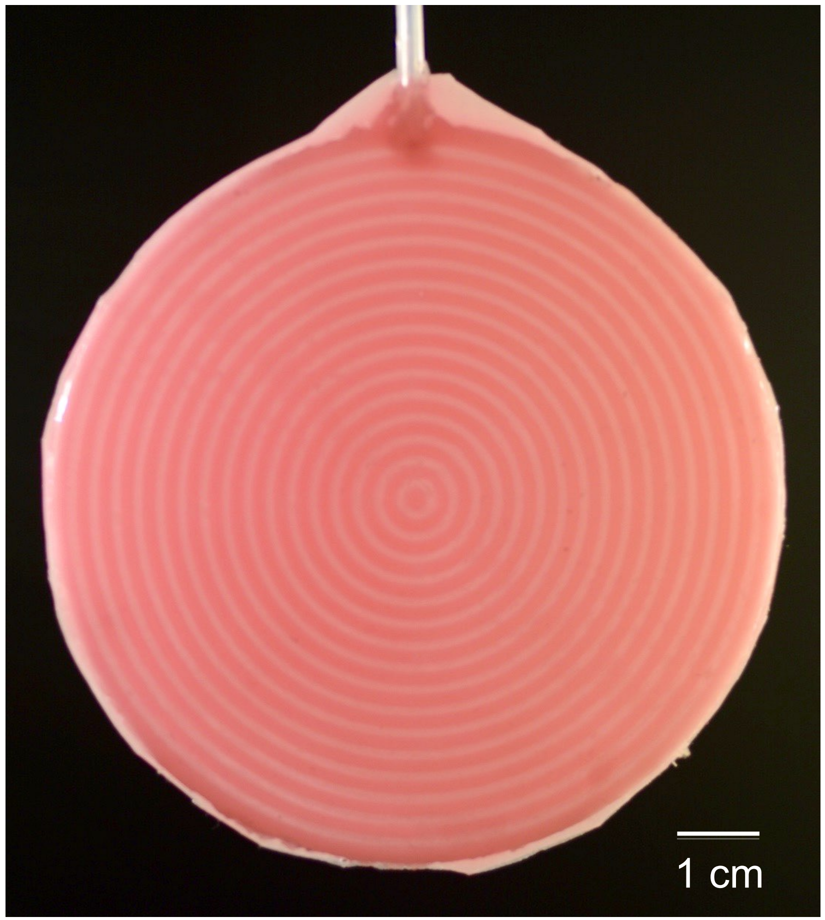


Dervaux & Ben Amar PRL (2008)

leaf growth: Arezki Boudaoud

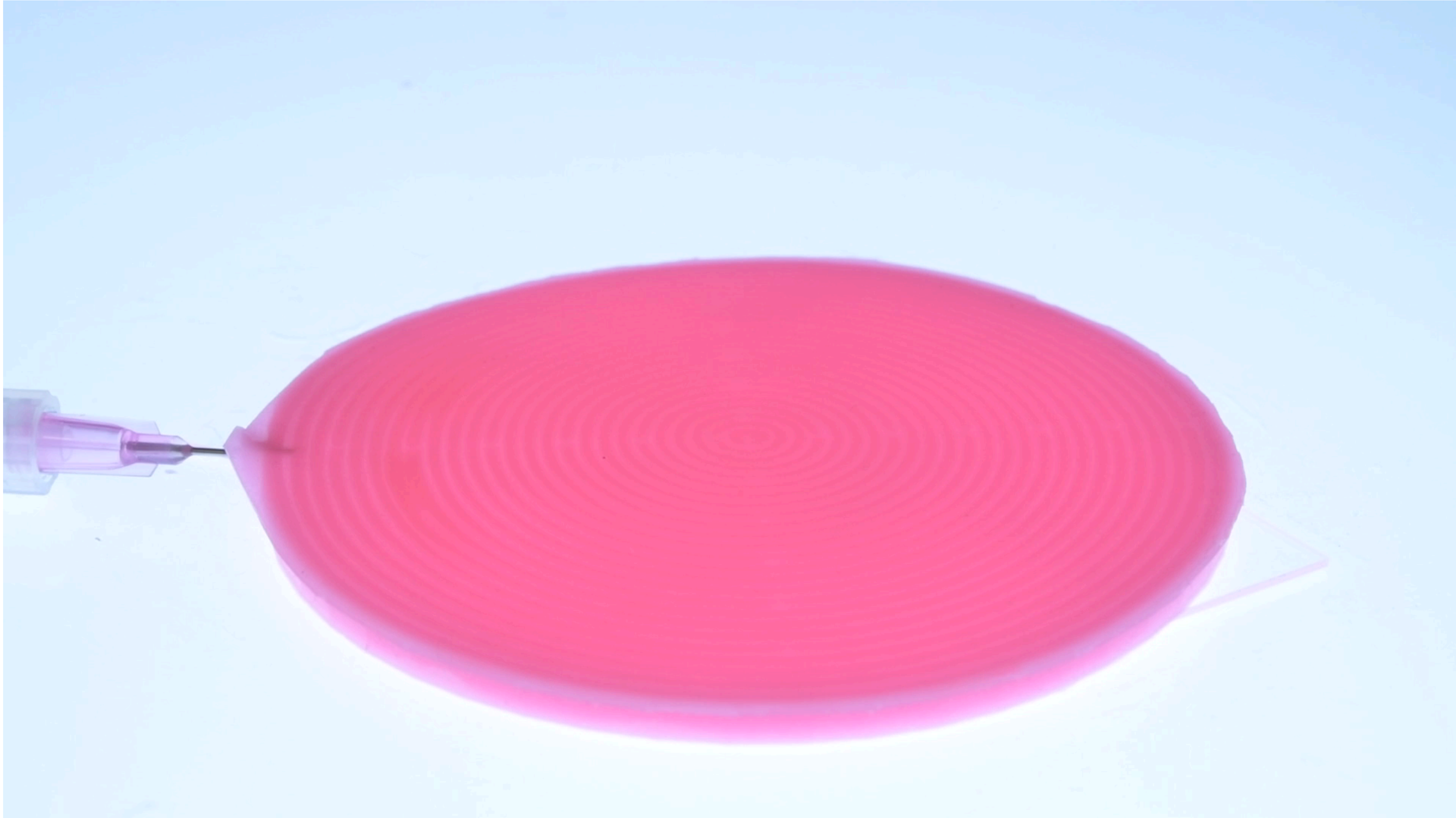
Inflatable rubber patches



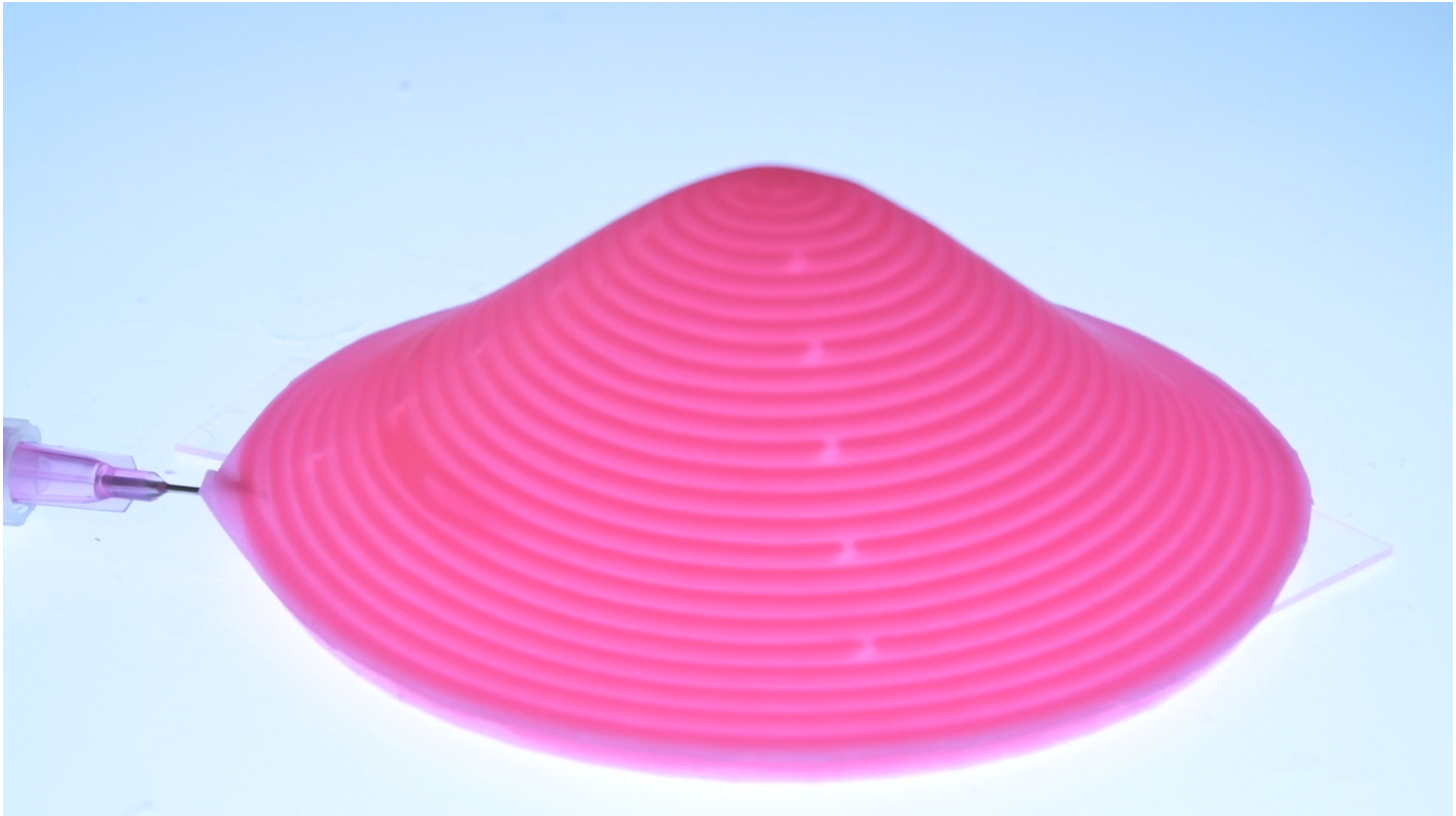


Siéfert et al., Nature Materials (2018)

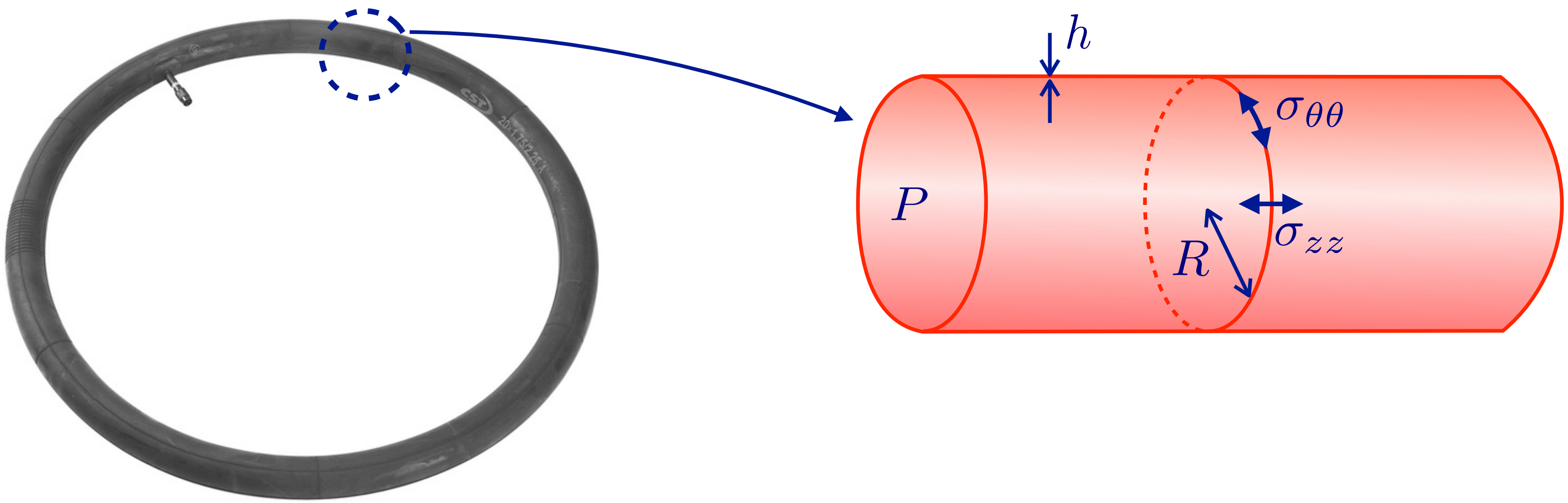
Inflatable rubber patches



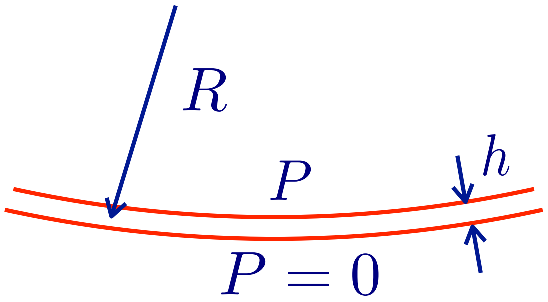
Inflatable rubber patches



Inflating a tube

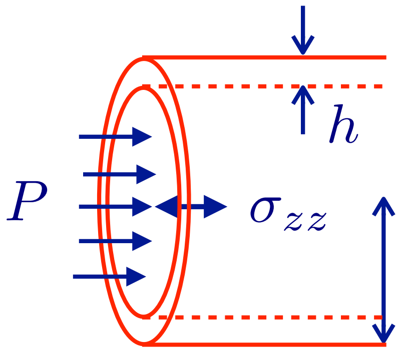


Inflating a tube



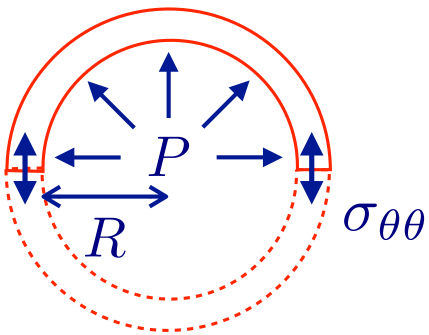
$$h \ll R$$

$$\boxed{0 \leq |\sigma_{rr}| \leq P}$$



$$\pi R^2 P = 2\pi R h \, \sigma_{zz}$$

$$\boxed{\sigma_{zz} = \frac{R}{2h} P}$$



$$2R\,P = 2h\sigma_{\theta\theta}$$

$$\boxed{\sigma_{\theta\theta} = 2\sigma_{zz} = \frac{R}{h} P} \gg \sigma_{rr}$$

Inflating a tube

Hook's law for linear elasticity:

$$\varepsilon_{11} = \frac{1}{E}\left[\sigma_{11} - \eta(\sigma_{22} + \sigma_{33})\right]$$

$$\varepsilon_{12} = \frac{1+\eta}{E}\sigma_{12}$$ + permutations $1,\ 2,\ 3$

with $\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$

$$\varepsilon_{\theta\theta} = \frac{1}{E}(\sigma_{\theta\theta} - \nu\sigma_{zz}) = \frac{3}{4}\frac{R}{Eh}P$$

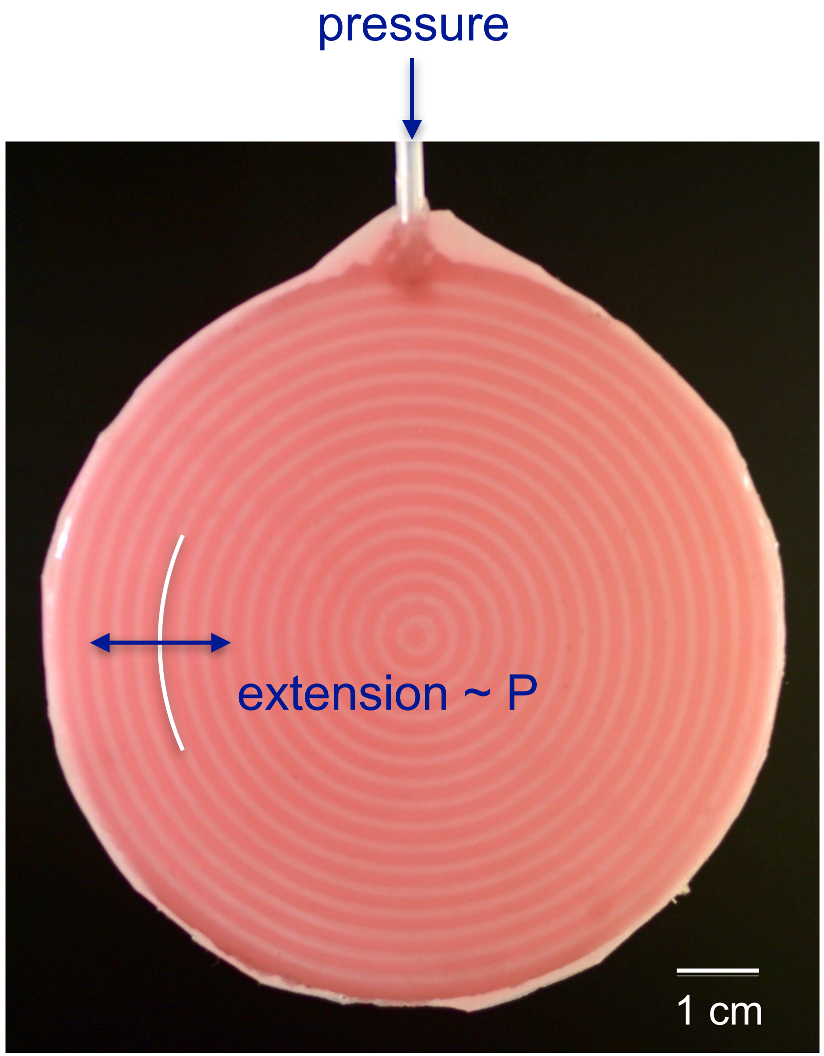
$$\varepsilon_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu\sigma_{\theta\theta}\right) = 0$$

$\nu \simeq 0.5$ for rubber

Pipe failure

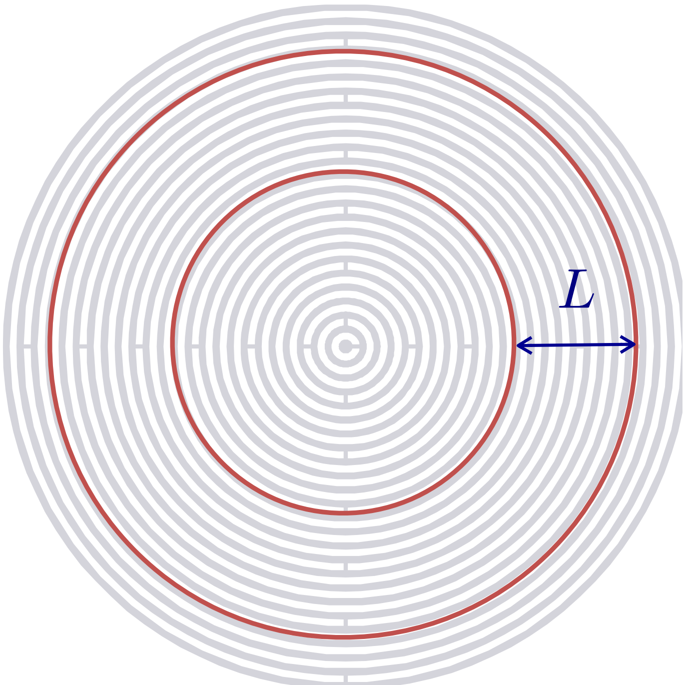
$$\sigma_{\theta\theta} = 2\sigma_{zz}$$

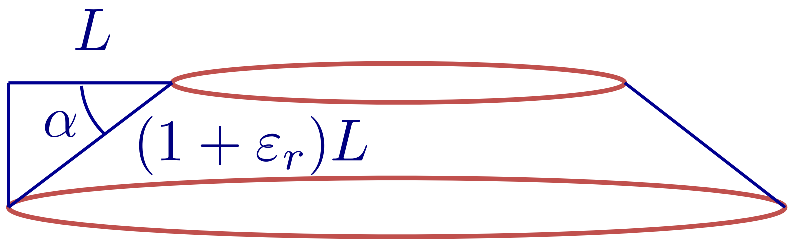
Inflatable rubber patches



Siéfert et al., Nature Materials (2018)

Geometrical limit
(bending negligible)

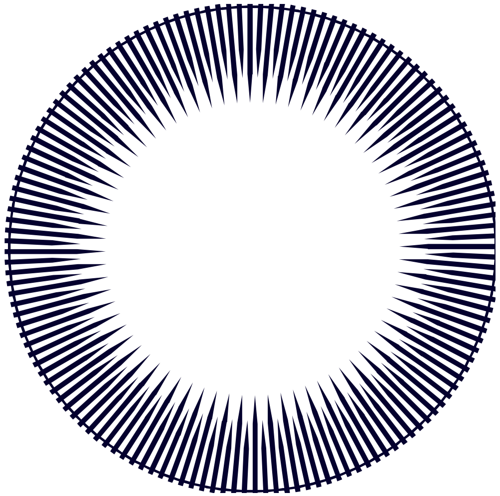


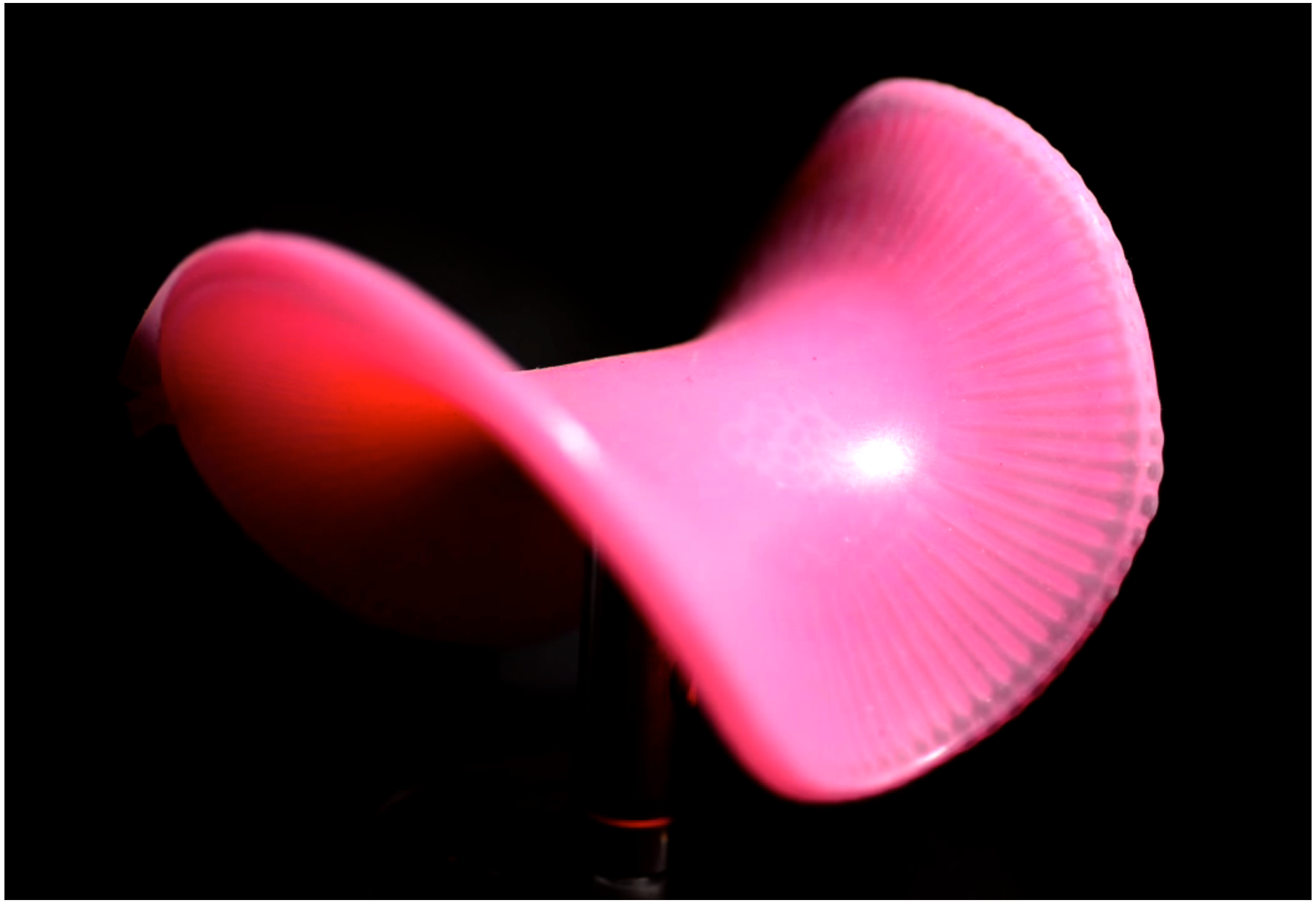


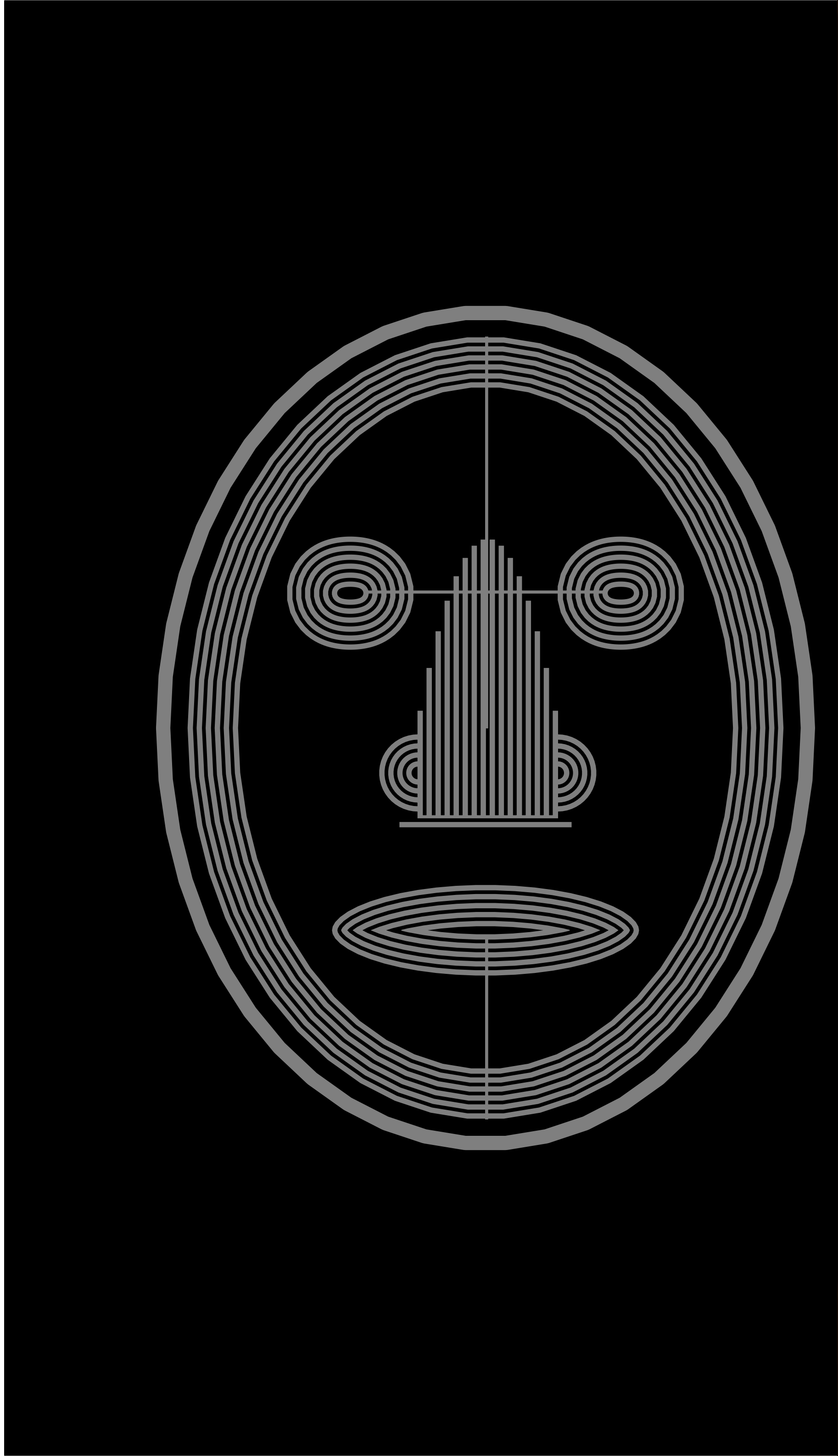
$$\varepsilon_\theta \simeq 0$$

$$\cos \alpha = \frac{1}{(1 + \varepsilon_r)}$$

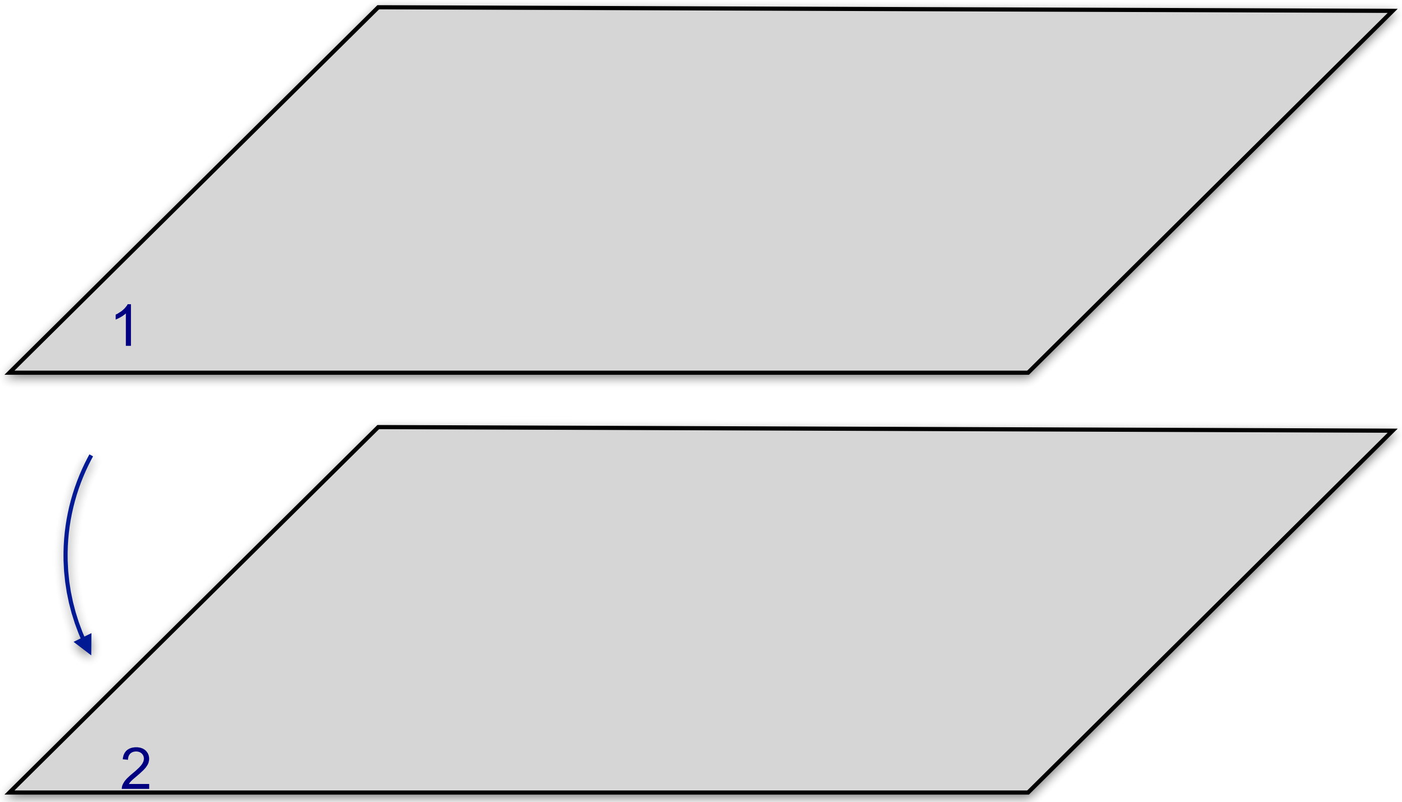
Radial channels








Inflatable fabrics



fabric coated with thermo-sealable polymer

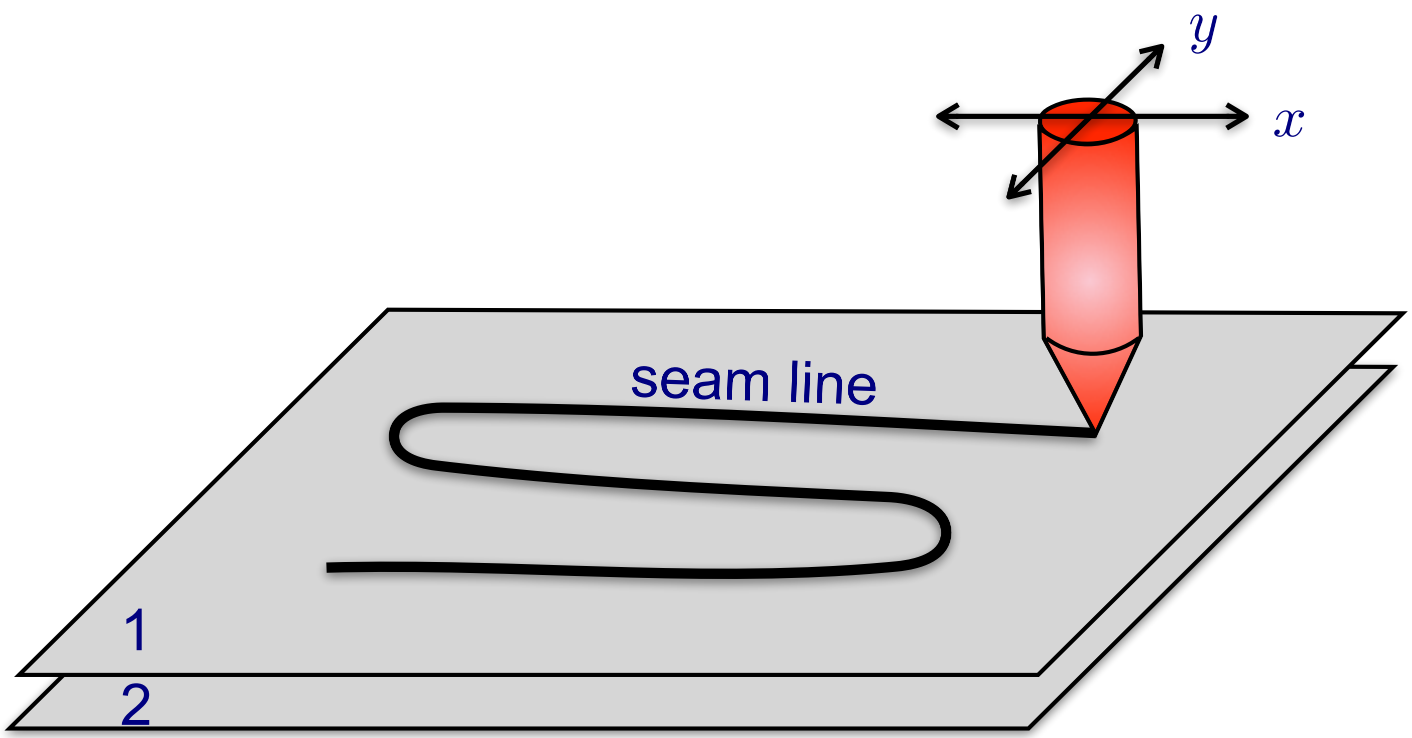
Siéfert et al., Soft Matter (2020)

Inflatable fabrics

fabric coated with thermo-sealable polymer

Siéfert et al., Soft Matter (2020)

Inflatable fabrics



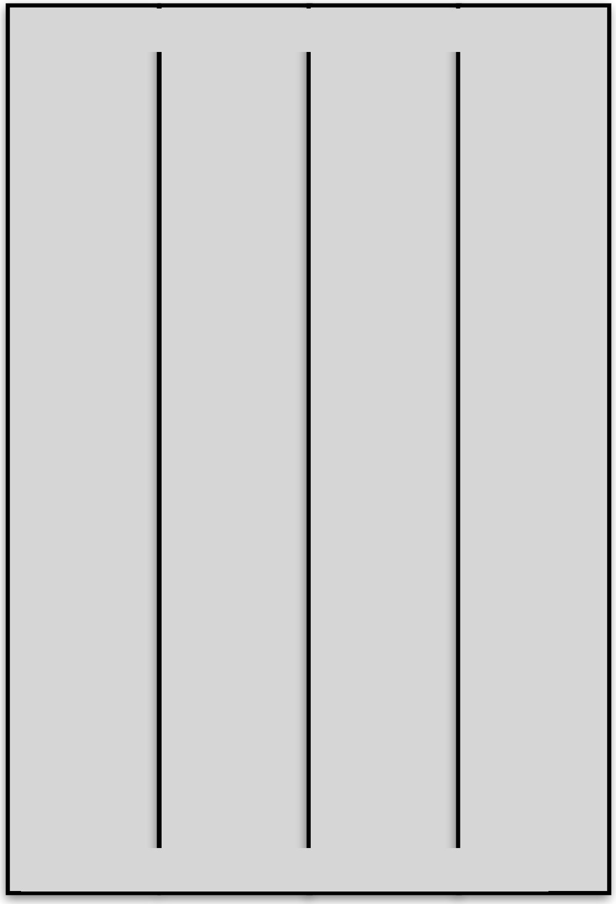
fabric coated with thermo-sealable polymer

Siéfert et al., Soft Matter (2020)

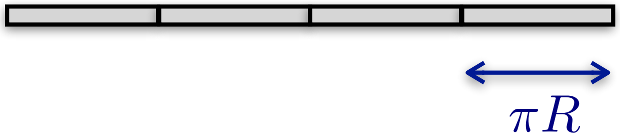
Inflatable fabrics

$P = 0$

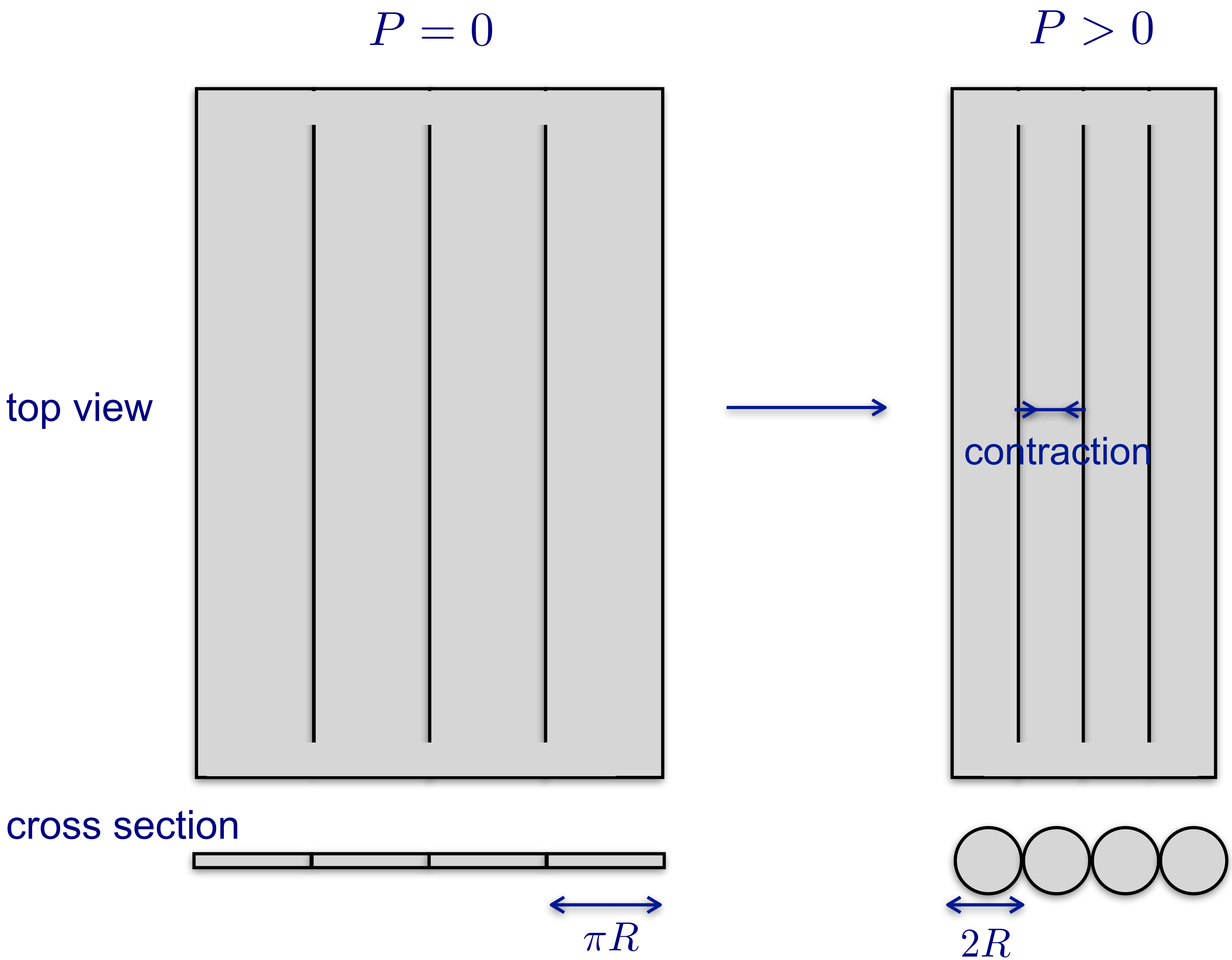
top view



cross section



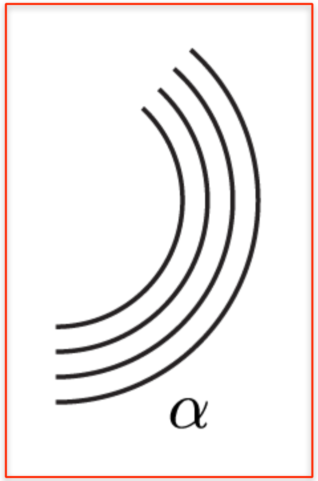
Inflatable fabrics



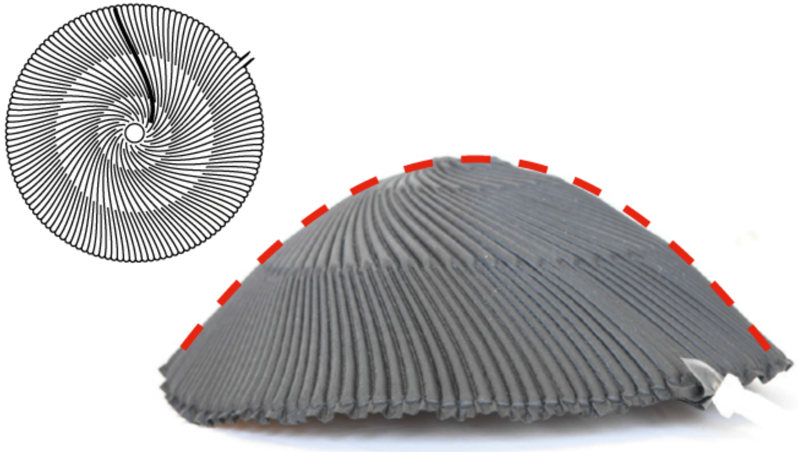
Inflatable fabrics

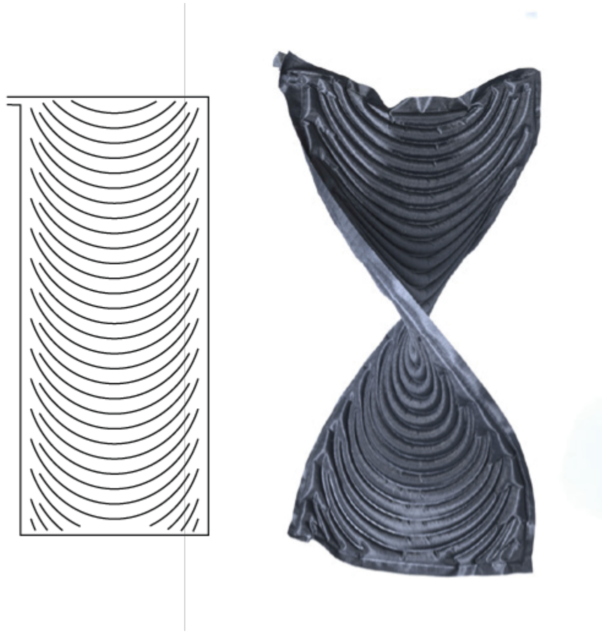
Inflatable fabrics

Different morphing strategies



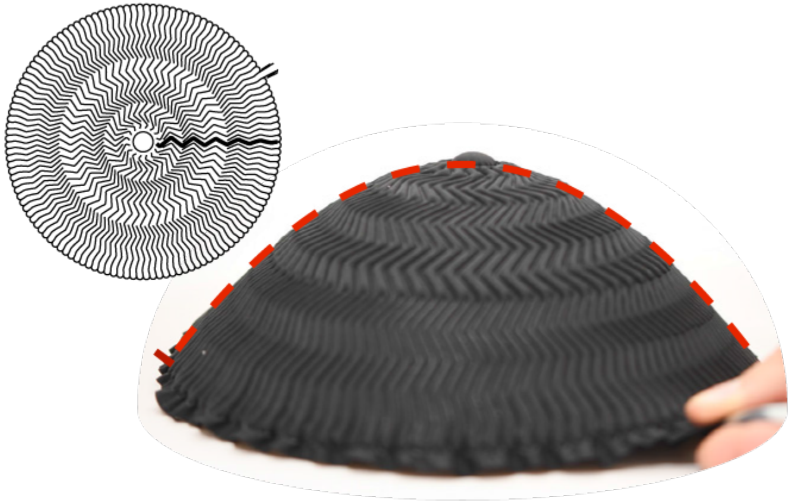
channels direction

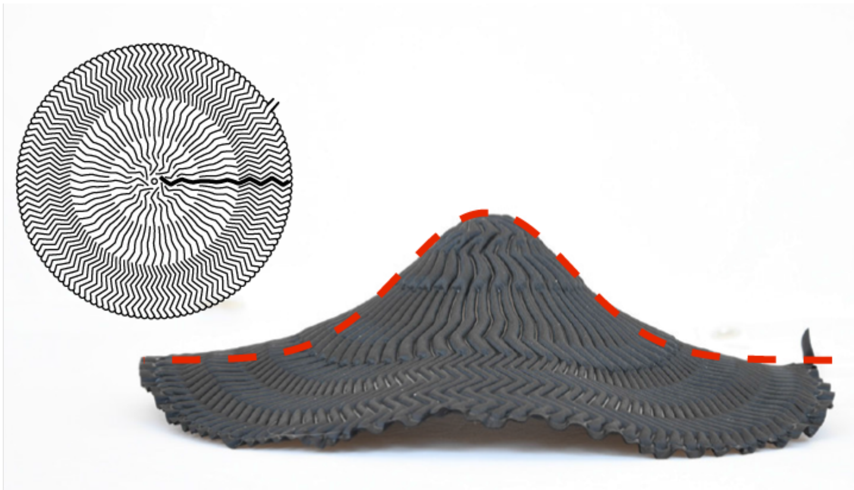


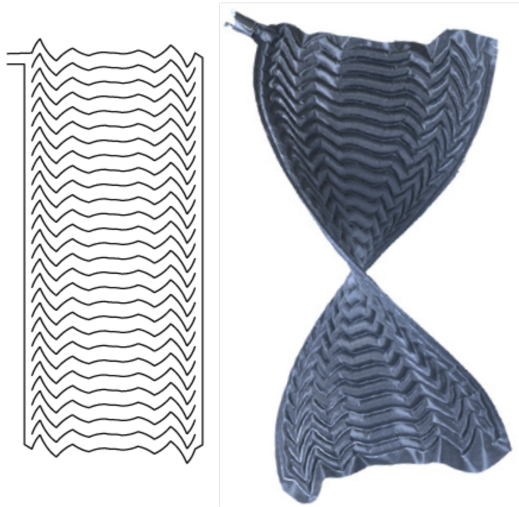


Zigzags

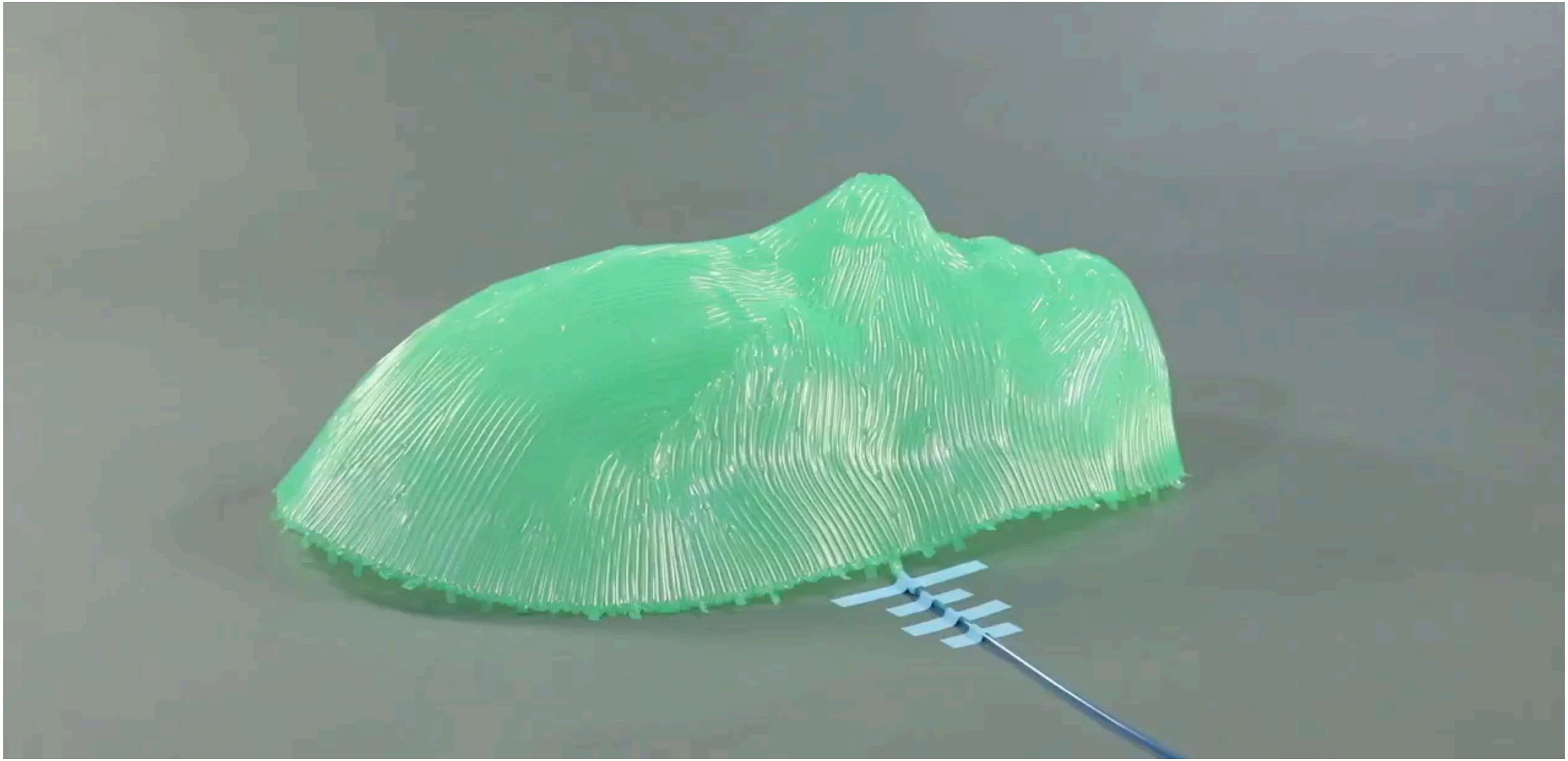
zig-zags
↕
orientation
+
contraction ratio







Reverse engineering?



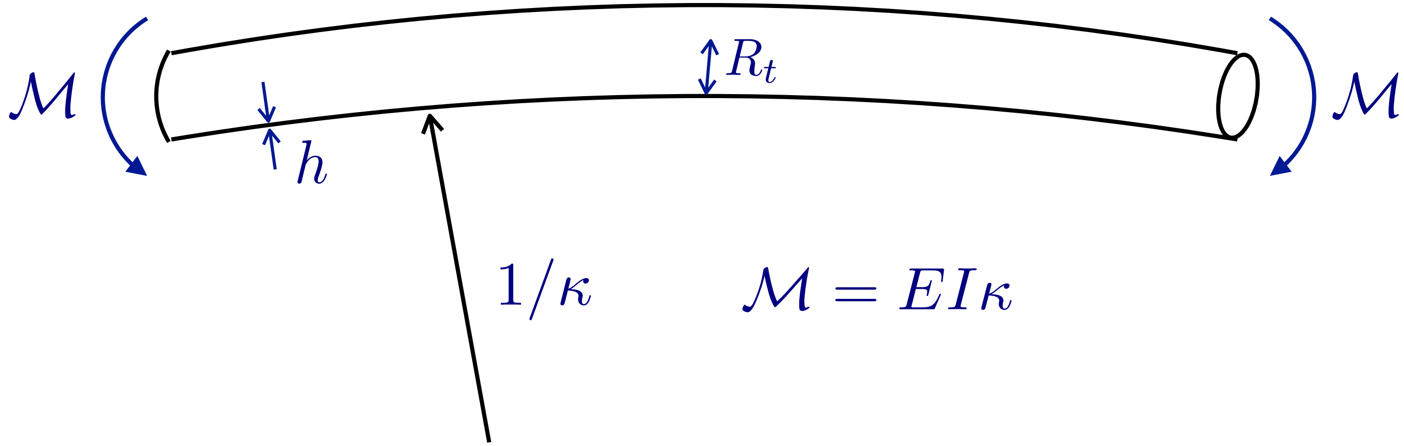
collaboration Julian Panetta & Mark Pauly (EPFL)

Panetta et al., ACM Transactions on Graphics (2021)

Large scales

Large scales

Stiffness



$$\mathcal{M} = EI\kappa$$

$$I = \frac{\pi}{4}(R_t + h)^4 - \frac{\pi}{4}R_t^4 \simeq \pi R_t^3 h$$

But failure for $\sigma_{zz} \lesssim 0$ $\quad \frac{R_t}{2h}P - ER_t\kappa_c \simeq 0$

inflation bending

$$\kappa_c \simeq \frac{1}{2h}\frac{P}{E}$$

$$\boxed{\mathcal{M}_c \simeq \frac{\pi}{2}R_p^3}$$