Collaborative oscillatory fracture
Supplemental Material

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I. GENERAL PROPAGATION OF FRACTURE AND DEBONDING FRONT

We give here a quick review of several fracture situations (previously described in other works) and put them in the same framework developed in this article. These configurations serve as the basis for comparison, and were used to prepare initial conditions for 3-crack interaction in the present study.

A. Isolated channel cracks

We start by the well described isolated “channel crack” in a material under bi-axial stress. In our notations, the energy released during propagation is $\gamma eh$ on each side of the fracture cut (per unit increment of crack area). We recall that $e$ is the elastic energy per unit area in the film and that $\gamma$ is a non-dimensional number that depends on the relative mechanical properties of the layer and substrate. Griffith’s criterion therefore reads [1]

$$2\gamma eh = G_c$$

where $G_c$ is the fracture energy.

It was shown that when adhesion is decreased, fracture propagation is favored through the collaborative effect of delamination which results into two remarkable modes of regular fracture propagation, the duo and the follower modes [2, 3], that we review now.

B. Duo of cracks.

In the duo mode, two cracks are observed to propagate parallel to each other, and are joined at their tips by a straight delamination front with a very specific length $l_d$ (Fig. S1a).

We therefore consider the general situation of two cracks delimiting an extended debonding zone. Here we take the energy difference with the pristine state as

$$\delta E_r = eA + ef(l) + \gamma eh(S_+ + S_-),$$

where $l$ is the length of the debonding front, and $S_+$ (resp $S_-$) the curvilinear abscissa for the position of the upper (resp inferior) fracture tip along its trajectory. Griffith’s criterion for crack $S_+$ reads

$$d\delta E_r = \Gamma dA + G_c h dS_+,$$

where $\Gamma$ is the debonding energy. Experimentally the adhesion energy is found to increase with the velocity of the delamination front (see section III for the experimental characterization of the adhesion energy) consistently with a simple linear relation:

$$\Gamma(v) = \Gamma_0 \left(1 + \frac{v}{\eta}\right)$$

when $\Gamma \gtrsim \Gamma_0$ where $\eta$ is a kinetic parameter that has to be determined experimentally. This variation of the adhesion energy with velocity is based on experimental observations, but other generic kinetic laws are considered in section IV of Supplemental Material.

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Denoting \( \mathbf{n} \) the unit vector normal to the front, \( \mathbf{u} \) its tangent vector pointing towards the crack \( S_+ \) chosen as reference, and \( \mathbf{T}_+ \) its direction of propagation (see the sketch in Fig. S1a), simple geometry gives \( 2dA = \int (\mathbf{u} \cdot \mathbf{T}_+) \, dS_+ \) and \( dl = (\mathbf{u} \cdot \mathbf{T}_+) \, dS_+ \) so that Griffith’s criterion can be rewritten as

\[
\frac{1}{2} \left[ e - \Gamma(V+) \right] \mathbf{n} \cdot \mathbf{T}_+ \, dS_+ + e f'(l) \mathbf{u} \cdot \mathbf{T}_+ \, dS_+ + \gamma eh dS_+ = G_c h dS_+ ,
\]

which we rewrite in a condensed way as

\[
\mathbf{F}_+ \cdot \mathbf{T}_+ + \gamma eh = G_c h ,
\]

with the notation \( \mathbf{F}_+ = \left[ e - \Gamma(V+) \right] (l/2) \mathbf{n} + e f'(l) \mathbf{u} \). We recognize on the left-hand side of equation (3) the Energy Release Rate for the fracture (integrated over the thickness). We assume that fracture propagates in the direction of maximum energy release rate, so in direction \( \mathbf{T}_+ \) aligned along \( \mathbf{F}_+ \). In that case, equation (3) becomes \( |\mathbf{F}_+| + \gamma eh = G_c h \) and sets the value of adhesion energy \( \Gamma(V+) \), and therefore the velocity of the crack \( V_+ \) through (2).

A similar treatment for crack \( S_− \) gives

\[
\mathbf{F}_- \cdot \mathbf{T}_- + \gamma eh = G_c h ,
\]

with the notation \( \mathbf{F}_- = \left[ e - \Gamma(V-) \right] (l/2) \mathbf{n} - e f'(l) \mathbf{u} \). These equations provide similarly the direction of propagation \( \mathbf{T}_± \) (aligned along \( \mathbf{F}_± \)) and the speed \( v_± \).

We have therefore found the propagation rules for the two cracks, and it is found in general that they converge toward a “duo” state with a constant distance \( l_d \), in which both vectors \( \mathbf{T}_± \) are perpendicular to the debonding front (therefore along \( \mathbf{n} \)), which imposes that \( l_d \) obeys \( f'(l_d) = 0 \). Since \( f(l) = \beta h - \alpha l^2 \), with \( \alpha \) and \( \beta \) dimensionless constants (see main text, and [2, 3]), we finally obtain \( l_d = \beta h/2\alpha \).

**C. Follower crack**

We observe that sometimes a fracture propagates nearly parallel to the path of a preexisting cut (as observed for example in spiral crack propagation in Fig. S1b). We call it a case of “follower crack”. We note that the debonding front joins the crack, that we note \( S \) to a point with abscissa \( s \) along the pre-existing crack (see the sketch in Fig. S1b). For simplicity we consider the case of a straight pre-existing fracture, along a direction vector \( \mathbf{t} \).

We now must compute the energy difference with respect to state including the pre-existing crack. The energy difference therefore is \( \mathcal{E}_r = eA + e f(l) + \gamma eh S - \gamma eh S \), where \(- \gamma eh S\) corresponds to the stress already released by the pre-existing crack. We use the same geometrical convention with vector \( \mathbf{u} \) pointing towards the crack tip. The dynamics of the debonding front may be determined by applying Griffith’s criterion, for a fixed crack position.

**D. Three cracks interactions**

Here the energy difference with respect to the pristine state is

\[
d\mathcal{E}_r = eA + e f'(l) dl \pm \gamma eh ds
\]
where, (+) holds for a part in yellow in Fig. S2, while (-) holds for the part in grey. As in the main text, we choose here to consider the case where the front edge s− lags behind the center crack s+. The surface variations in terms of $ds_\pm$ and $dS_\pm$ (see Fig. S2) writes,

$$dA_\pm = \frac{l_\pm \mathbf{n}_\pm \cdot (\mathbf{T}_\pm ds_\pm + t_\pm ds_\pm)}{2},$$

and expressing $dl_\pm$ in terms of $ds_\pm$ and $dS_\pm$, we find

$$dl_+ = u_+ \cdot (-t_+ ds_+ + T_+ ds_+)$$
$$dl_- = u_- \cdot (-t_- ds_- + T_- ds_-)$$

The Griffith criterion, accounting for energy conservation, indicates that the elastic energy released is compensated by the energy expense of cracks propagation and delamination. For the case of a middle crack linked to the upper front, this is, $dE_r = \Gamma (dA_+ + dA_-) - G_c(h ds_+ + dS_+ + dS_-)$ leads to,

$$[(C_+ (\Gamma_+ \mathbf{n}_+ + D_+ \mathbf{u}_+) \cdot \mathbf{T}_+ - (G_c h - \gamma e h)] dS_+$$
$$+ [(C_- (\Gamma_- \mathbf{n}_- + D_- \mathbf{u}_-) \cdot \mathbf{T}_- - (G_c h - \gamma e h)] dS_-$$
$$+ \left[ (C_+ (\Gamma_+ \mathbf{n}_+ + D_+ \mathbf{u}_+) \cdot \mathbf{T}_+ - (G_c h - \gamma e h) dS_+ + \gamma e h ds_-ight] = 0$$

where, $C_\pm = (e - \Gamma_\pm)l_\pm/2$ and $D_\pm = e f'(l_\pm) = e (\beta h - 2\alpha l_\pm)$. For simplicity used the notation that shows before where $F_\pm = (C_\pm \mathbf{n}_\pm + D_\pm \mathbf{u}_\pm)$ and $f_\pm = (C_\pm \mathbf{n}_\pm - D_\pm \mathbf{u}_\pm)$. In order to derive the propagation equations for cracks and follower front, we apply independent progression for each variable. This is,

$$F_+ \cdot T_+ + \gamma e h = G_c h$$
$$F_- \cdot T_- + \gamma e h = G_c h$$
$$f_- \cdot t_- - \gamma e h = 0$$
$$f_+ \cdot t_+ + \gamma e h = G_c h,$$

which were presented in the main text.

II. SYMMETRIC 3 CRACKS

In the case of symmetric propagation of three cracks (as in the inset of main figure 2), we may directly take equation (12) with $ds_\pm = ds$ and $t_\pm = t$, and find the equivalent of (13-15) in this symmetric case:

$$(f_+ + f_-) \cdot t = G_c h$$
$$F_\pm \cdot t + \gamma e h = G_c h$$

where $F_\pm$ must also be aligned with the direction of propagation $t = \cos \theta n - \sin \theta n$, where $\theta$ is the angle between the delamination front $n$ and the direction of propagation $t$. Together with this information, equation (18) can be replaced by $|F_\pm| = G_c h - \gamma e h$ together with

$$\sin \theta = -e(2\alpha l_T - \beta h) \sin \theta = \frac{(e - \Gamma)l_T}{2(G_c h - \gamma e h)}$$

where $l_T$ is the length of the debonding fronts. Equation (17) for the central crack writes $(e - \Gamma)l_T \cos \theta + 2e(\beta h - 2\alpha l_T) \sin \theta = G_c$, which, using (19) leads to

$$\cos 2\theta = \frac{G_c}{2(G_c h - \gamma e h)}$$

Finally, using this value for $\theta$, equations (19) lead to

$$l_T = \frac{\beta h + G_c h}{2\alpha} + \frac{G_c h}{2\sqrt{2}\alpha e} \sqrt{\frac{\gamma e}{G_c} - 1} \left(\frac{\gamma e}{G_c} - \frac{1}{2}\right)$$

$$\Gamma = e - \frac{\sqrt{2}G_c h}{l_T} \sqrt{\frac{\gamma e}{G_c} - 1} \left(\frac{\gamma e}{G_c} - \frac{3}{2}\right)$$

III. EXPERIMENTAL CHARACTERIZATION
OF THE ADHESION ENERGY

In Fig. S3, we plot the variation of the velocity of the delamination front $v$ as a function of the adhesion energy $\Gamma$ normalized by the adhesion energy at vanishing speed $\Gamma_0$. The dashed line is a linear fit $v = \eta(\Gamma/\Gamma_0 - 1)$ of the experimental data.
FIG. S4. Dimensionless wavelength $\lambda/h$ (a) and dimensionless amplitude $A/h$ (b) of oscillation vs. $e/\Gamma_0$. Solid lines represent power velocity laws $v = \eta(\Gamma/\Gamma_0 - 1)^\epsilon$, dashed red lines are obtained for an exponential law $v = v_0\left[1 - e^{-\varphi(\Gamma/\Gamma_0 - 1)}\right]$ where $v_0 = 4.5\, \mu$m/s and $\varphi = 1$. All curves are traced with $\eta = 4.5\, \mu$m/s and $\gamma e/G_c = 0.49$.

This description is also compatible with independent experimental measurements in other similar systems [5, 6]. To show the generality of the instability process, we next test the robustness of the oscillations to other kinetic laws (i.e. the dependence of the adhesion energy with velocity) and the effect of the kinetic law on the selection of the geometry of the oscillations.

IV. EFFECT OF KINETIC LAW ON WAVELENGTH AND AMPLITUDE OF OSCILLATIONS

In order to investigate the influence of the delamination speed on the oscillation features, we compute numerically the wavelength and amplitude and oscillations for different kinetic laws, namely, power laws: $v = \eta(\Gamma/\Gamma_0 - 1)^\epsilon$, with $\epsilon = 0.5, 1$ and $2$, and an exponential law based on experimental observations on similar systems [5]: $v = v_0\left[1 - e^{-\varphi(\Gamma/\Gamma_0 - 1)}\right]$, where $v_0$ is a limiting speed and $\varphi$ a decay coefficient of order 1. Figure S4 compares wavelength and amplitude of oscillations for power laws, with $\eta = 4.5\, \mu$m/s and for an exponential law with $v_0 = 4.5\, \mu$m/s and $\varphi = 1$, such that in the limit of $\varphi(\Gamma/\Gamma_0 - 1) \ll 1$, the exponential tends to the linear law with the same value of $\eta = 4.5\, \mu$m/s. In all cases, oscillatory cracks are observed and the instability process appears for all the kinetic laws considered, albeit with different pattern geometry.