Time-reversal in an ultrasonic waveguide

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The aim of this letter is to study, by using a time-reversal mirror (TRM), ways to overcome the distortions induced by multipaths in an acoustic transmission. A set of experiments is performed with a TRM made of 96 reversible transducers and results will be presented. One result is related to the high focusing property obtained with a TRM working in a bounded channel. It will be shown that the time-reversed beam is much thinner than the one measured in an unbounded medium. The second result concerns the time compression observed for the time-reversed acoustic pulse. The influence of the number of TRM elements on the time compression is discussed. [S0003-6951(97)02914-8]

Time-reversal invariance of the acoustic wave equation means that for every burst of sound diverging from a source—and possibly reflected, refracted or scattered by any propagation medium—there exists in theory a set of waves that precisely retraces all of these complex paths and converges in synchrony, at the original source, as if time were going backwards. Based on this concept, time reversal cavities and time reversal mirrors have been developed in our laboratory to focus pulsed waves through heterogeneous media.[1,2] Time-reversal mirrors (TRM) are made of reversible transducers arrays, allowing the incident acoustic field to be sampled, time reversed and reemitted.

The aim of this letter is to study a way to overcome the problem of multipath distortion in acoustic waveguides. For example, TRM may compensate for the type of multipath propagation found in oceans that limits the capacity of underwater communication systems. The problem arises because, in shallow water, acoustic transmission bounces off the ocean surface and floor, so that a transmitted pulse gives rise to many copies of itself arriving at the receiver. The same problem occurs in aerial acoustics in reverberating rooms. Theoretical approaches of these problems have been conducted by various authors.[3,4] In this letter, we study experimentally both the temporal compression and the spatial focusing observed with a TRM working in a waveguide.

We present a set of experiments conducted through an ultrasonic water channel bounded by two plane interfaces. A TRM made of 96 reversible transducers is used in these experiments and two interesting results will be emphasized.

The first result is the time compression observed for the time-reversed acoustic multipath pulse which shows that the multipath effects are compensated.

The second result is related to the excellent focusing property obtained with a time reversal mirror operating through a bounded channel. It will be shown that the time-reversed beam is much thinner than the one observed in a similar experiment made in unbounded water.

The influence of both the duration of the time-reversed window and the number of elements of the TRM on time compression and focusing will be discussed. We conclude with the fact that, compared to TRM used in an unbounded medium, TRM used in a waveguide amplifies the peak value of the pressure field at the source point.

The principle of the experiments is presented in Fig. 1. The water channel is bounded by two plane and parallel interfaces (two steel/water interfaces). The total length of the channel along the y axis is $L=740$ mm, which is of the order of 20 times the water depth $H=40$ mm along the x axis. The medium is invariant under any translation along the z axis. This permits to restrict the study of the propagation in the Oxy plane. A pointlike ultrasonic source (a single transducer) is located on one side of the waveguide and can be used either as a source or as a hydrophone. A time reversal mirror, made of a linear array of 96 transducers identical to the transducer source, is located at a distance L from the source and is located along the x axis. The element size along x axis is 0.39 mm, and the spacing between two elements is 0.42 mm. Thus the total aperture of the array is 40 mm. The mirror works with a central frequency of 3 MHz with a 50% relative bandwidth at -6 dB (central wavelength $=0.5$ mm). The wave front transmitted by the impulsive source is recorded by the array elements. Each transducer element has its own amplifier, an 8 bit analog-to-digital (A/D) converter, a storage memory and an 8 bit digital-analog (D/A) converter working at a 20 MHz sampling rate, which permit to retransmit a time-reversed version of the selected part of the recorded signals.

A complete time reversal experiment is divided in three steps: first, the transducer source located at S transmits a pulsed wave. In the second step, a temporal window T selected one part of the transmitted wave which has been recorded by the array and these data are time reversed and retransmitted along the same channel. In the last step, the time-reversed field is measured in the source plane by the transducer source which is used as a hydrophone. This hydrophone can be translated around the initial position S along the x axis to measure step by step the time-reversed pressure field $p_{\mathrm{tr}}(x,t)$. In this case we are interested both in the temporal compression and in the spatial focusing of the time-reversed field around the initial source. The quality of the time compression at the source can be deduced from the curve $p_{\mathrm{tr}}(0,t)$ and the directivity pattern of the time-reversed beam is obtained from the curve $d(x)=\max_t\{p_{\mathrm{tr}}(x,t)\}$.

Figure 2(a) shows the transmitted field recorded by the array after the propagation through the channel. After arrival

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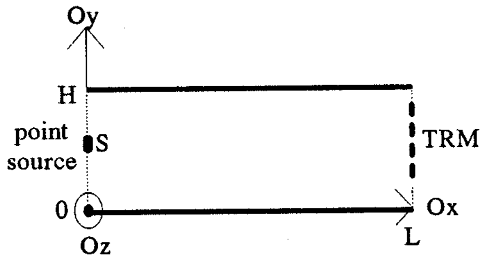

FIG. 1. Description of the acoustic waveguide.

of a first cylindrical wave front corresponding to the direct path, we notice the arrival of a set of multipath signals corresponding to the multiple reflections of the incident wave on the interfaces. Figure 2(b) represents the signal recorded on one transducer of the TRM.

After the time-reversal experiment, Fig. 3(a) shows a remarkable time compression that means that multipath effects are compensated. The signal observed at the source point is nearly identical to the one received in a time-reversed experiment made in unbounded water [Fig. 3(b)]. The signal duration is only related to the acousto-electric responses of the transducers which are taken into account in the complete time-reversal process. The transfer function of the propagation medium has been completely compensated by the time-reversal process. Indeed, the time reversal process enables the realization of an ''optimal'' spatio-temporal filter matched to the waveguide transfer function.[5] This is related to the reciprocity theorem which states that the positions of a source and receiver can be interchanged without altering the resulting field. Thus, if the transfer function is characterized by its impulse response or equivalently

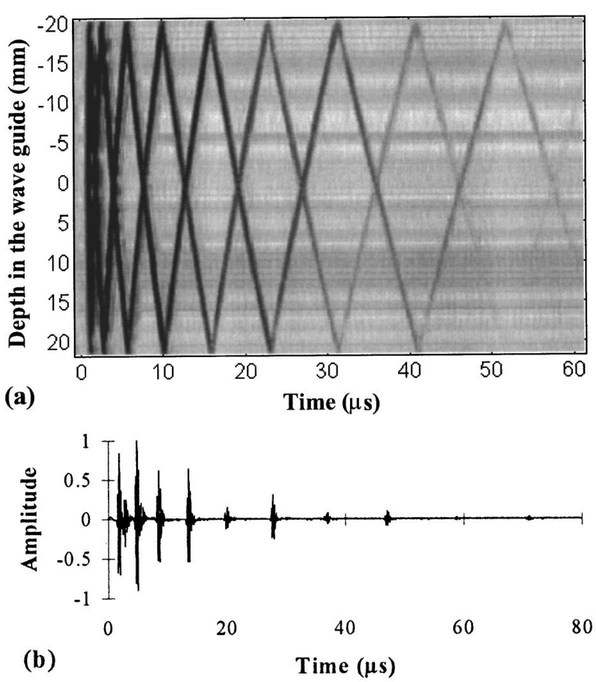

FIG. 2. (a) *B* scan of the incident acoustic field received by the TRM. The *x* axis is the time, the *y* axis is the position of the transducer in the waveguide; the amplitude of the field is in dB. (b) Temporal signal measured on one transducer of the array.

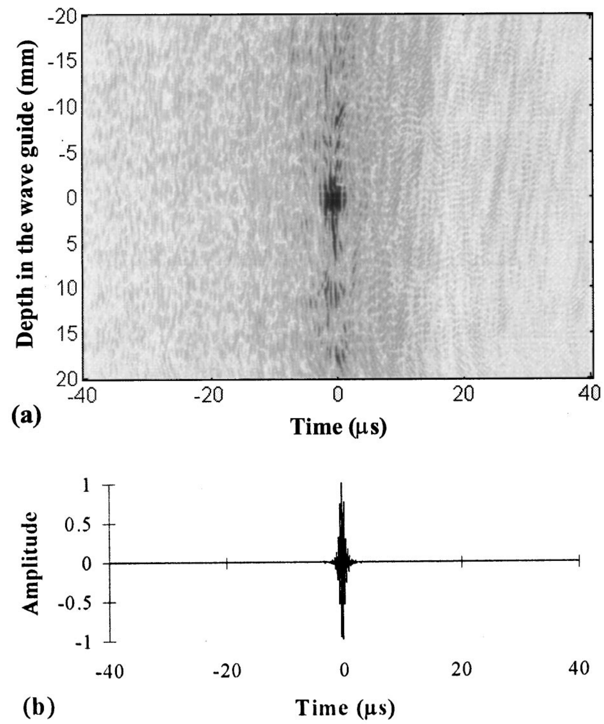

FIG. 3. (a) *B* scan of the time-reversed field measured in the plane of the point source, (b) time-reversed signal measured at the point source.

Green's function defined at point r_i and time t by $G(r_i,t|r_s,0)$ from a source at point r_s and time 0, reciprocity means that $G(r_i,t|r_s,0)=G(r_s,t|r_i,0)$. Under this assumption, and if we do not take into consideration the acoustic responses of the transducers, the time-reversed field observed at the source location becomes

$$P_{\mathrm{tr}}(r_s=0,t)=\sum_{i=1}^{N} G(r_i,t|r_s=0,0)\otimes G(r_i,T-t|r_s=0,0), \tag{1}$$

where N is the number of transducers in the TRM. It is a symmetrical function with a maximum at time T.

Each individual contribution i is the autocorrelation function of the impulse response between the point r_s and the transducer i at point r_i: it is a symmetrical signal with a maximum at time T for each transducer. However, for a given transducer i, the sides lobes of

$$G(r_i,t|r_s=0,0)\otimes G(r_i,T-t|r_s=0,0)$$

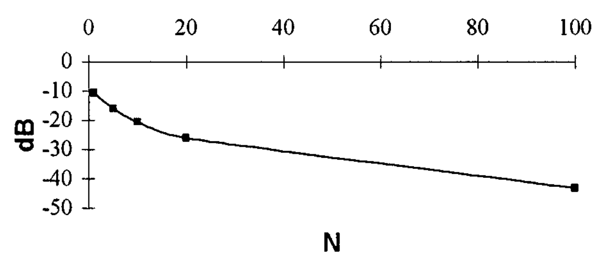

FIG. 4. Noise to signal ratio measured after time reversal at the point source vs the number of transducers in the TRM aperture.

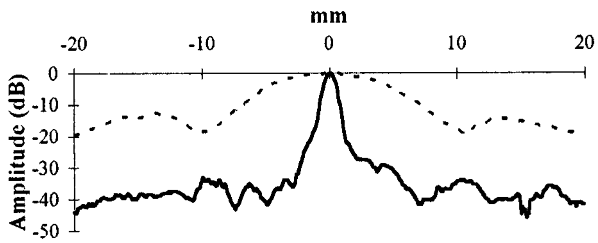

FIG. 5. Directivity pattern of the time-reversed field in the plane of the point source: dotted line corresponds to an unbounded water medium, full line to the waveguide.

are located at different times that depend on the various multipaths between the source and the transducer. The fundamental result is that, when we add a sufficient number of channels, the side lobes interfere destructively, while all the individual maxima add constructively. In the limit of a TRM sampling all the available aperture, the time-reversed field can be approximated by a Dirac δ function.

This result is achieved in our experiment when the TRM contains 96 transducers sampling the 40-mm-high water channel. The analysis of Fig. 3(b) shows that the ratio between the peak signal and the side lobe level is of the order of 45 dB. If we limit the number N of transducers in the time-reversed aperture, the signal to noise ratio decreases as shown on Fig. 4. In the limit of a TRM made of only one transducer, the signal to noise ratio is 10 dB. In the early sixties, time-reversal experiments made with only one transducer were conducted by Parvulescu[6] in shallow water at sea. The approach of Parvulescu was to consider the ocean as a correlator but his experiments did not embody the most important property of time-reversal mirrors: spatial focusing.

The second step of the experiment conducted with the 96 element TRM is the spatial focusing measurement. Figure 5 shows the directivity pattern of the time-reversed beam observed in the source plane. It shows that the time-reversed beam is focused on a spot which is much smaller than the one observed in unbounded water. In our experiment, the 6 dB lateral resolution is improved by a factor of 9.

This can be easily interpreted by the principles of mirror images in a medium bounded by two mirrors. For an observer, located at the source point, the TRM can be considered as a periodic vertical array of TRM images as shown in Fig. 6. Each reflected each received by the real TRM in the waveguide corresponds to one image of the TRM. When taking into account the first ten echoes received, the theoretical aperture of the TRM is thus ten times larger than the real aperture, since the first echo corresponds to the real TRM, and the following nine can be interpreted as nine TRM images. However, in practice as the echo arrives later, it's strength will be smaller [Fig. 2(b)]. When considering TRM images, this phenomenon can be explained by the apodization of the emitted signal: the angular directivity of the transducers implies that the echo strength decreases as the ''angle of image'' increases. Figure 7 shows the effect of the duration of the time-reversed window on the width of the focal spot. We observed first that it increases linearly with the inverse of the number of echoes selected in the window. But, when more than eight echoes are selected, the width of the focal spot no longer changes due to the apodization of the TRM images. This clearly shows that the effective aperture of the TRM is directly related to the duration of the time-reversal window.

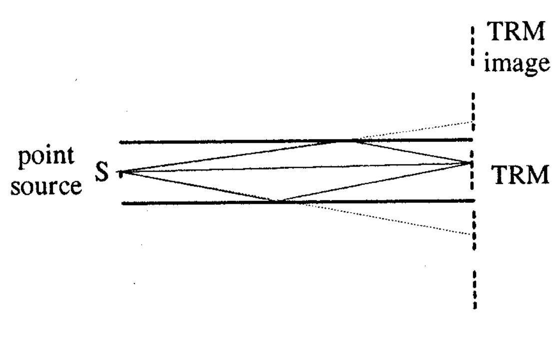

FIG. 6. The principle of mirror images applied to the waveguide.

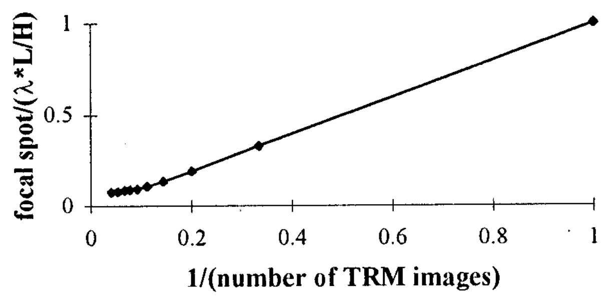

FIG. 7. Width of the focal spot vs the inverse of the number of TRM images.

The classical value of the 6 dB beamwidth $\Delta \approx \lambda L/H$ (where L is the focal length and H the aperture dimension) is replaced by $\Delta \approx \lambda L/H'$, where H' is the effective aperture of the TRM: in the studied waveguide, we measure $H' = 9H$.

To conclude on time reversal in waveguides, we notice that both time and spatial compression of the time reversed wavefield can be used for practical applications: the temporal compression may be of great interest for underwater acoustic transmission in shallow water. The spatial focusing may be used for medical imaging using ultrasonic waveguide. In addition, the time compression may be used to obtain, with low electrical power input applied to the transducers, a pressure peak of high value. Our group is now working on shock wave generators using this principle.

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