Tunneling of electromagnetic energy in multiple connected leads using ϵ-near-zero materials

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A realization of a reflectionless power splitter is proposed by use of a metamaterial junction. To design the junction, the electromagnetic wave transmission in multiple connected leads is investigated theoretically and numerically. A closed analytical form is derived for the scattering matrix of any geometry of the interconnected leads. We show that the use of a junction made of ϵ-near-zero (ENZ) material allows production of perfect transmission. This can be achieved by reducing the area of the ENZ junction (squeezing effect) and by tuning the widths of the output leads with respect to the input lead. It is also shown that the same effect is obtained without squeezed junction by using a match impedance zero index material (MIZIM junction).

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Recently, zero-index metamaterials (ZIMs) have been the subject of growing interest. ZIMs are structures consisting of permittivity ϵ-near-zero (ENZ) media, permeability μ-near-zero (MNZ) media, or matched impedance zero-index material (MIZIM), where ϵ and μ vanish simultaneously over a specific frequency band. Original devices and applications involving ZIMs have been investigated experimentally and theoretically [1,2]. The use of such media has been proposed to tailor the emission and the phase of the radiation patterns of arbitrary sources [3], and to realize ultrasensitive defect sensors [4]. Other interesting applications concern the tunneling and squeezing of electromagnetic waves in ZIM channels to realize total transmission or total reflection. This has been considered first in [5] for subwavelength channels and bends. Then, the tunneling and squeezing effects have been considered in the case of defects embedded in ZIM channels for potential applications to invisibility and perfect shielding [6,7]. These works exemplify the interest of ENZ metamaterials in problems of guided waves. Obviously, they have potential applications in the miniaturization of optical energy transports but they also open perspectives for applications in other contexts of wave propagation, in acoustics, or in hydrodynamics [8].

Until now, interest has been concentrated to geometries involving a channel with a unique inlet and a unique outlet. However, many configurations involve interconnected leads and, thus, possibly multiple inlets and outlets. This is the case of waveguide splitters that divide an input signal into N outputs. These elements are important in a variety of applications, including the power splitter networks used in transmission lines. In this Letter, from closed form expressions of the scattering matrix, we show that tunneling and squeezing effects can be used to control the scattering of a ZIM junction connected to multiple leads. It appears that by properly choosing the widths of the leads, it is possible to get perfect transmission, allowing the design of nearly perfect power splitters.

We first characterize the scattering matrix of the junction with a ZIM connecting multiple leads, as shown in Fig. 1(a). The geometry of the junction, the directions, and the sections of the leads are arbitrary.

In the following, the time dependence of the electromagnetic fields is $e^{-i\omega t}$, with ω as the angular frequency. The whole geometry is two dimensional, invariant to translation along the z direction, and we consider TM polarization of the magnetic field $\mathbf{H}(x,y) = H(x,y)\mathbf{e_z}$. The magnetic field $H(x,y)$ satisfies the Helmholtz equation in inhomogeneous media:

$$\nabla\left(\frac{1}{\epsilon}\nabla H\right) + k^2\mu H = 0. \qquad (1)$$

It is subject to the boundary conditions at the walls, and we consider that the walls are perfectly electrically conducting (Neumann boundary condition, $\partial_n H = 0$). It is also subject to the boundary conditions at each boundary L_m, $m = 0, \ldots.., N$ between the ENZ junction and the lead m, that imply the continuities of the fields H and $\partial_n H/\epsilon$. From Eq. (1), we get the variational formulation for any function Q. The integration has been done over the region of the ZIM junction, denoted $\mathcal{A}$, with boundary $\partial\mathcal{A}$. The boundary conditions on $\partial_n H/\epsilon$ over $\partial\mathcal{A}$ are used:

$$-\int_{\mathcal{A}} \frac{1}{\epsilon}\nabla H \nabla Q + \int_{\mathcal{A}} k^2\mu H Q + \sum_{m=0}^{N} \int_{L_m} \frac{1}{\epsilon_m}\partial_n H^{(m)} Q = 0. \qquad (2)$$

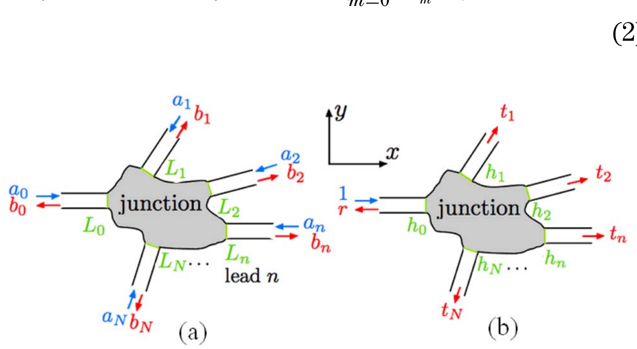

Fig. 1. (a) Geometry of the leads connected to a ZIM junction. (b) Geometry of the power splitter with N outlets.

In Eq. (2), ϵ and μ are the relative permittivity and permeability of the ZIM, ϵ_m is the relative permittivity in the leads m, and $H^{(m)}$ is the magnetic field in the lead m. For ENZ and MIZIM junctions, $\epsilon \to 0$, so that the first term in Eq. (2) is $O(1/\epsilon)$ and it has to vanish. Since Q is any function, this implies that ∇H vanishes in $\mathcal{A}$. Thus, the field $H = H_0 + O(\epsilon)$ is constant at dominant order and we consider this dominant order in the following. Denoting a_m and b_m the amplitudes of the incoming and of the outgoing waves in the lead m (Fig. 1), we have $\partial_n H^{(m)}/\epsilon_m = -iZ_m k(a_m - b_m)$, where the $Z_m \equiv \sqrt{\mu_m/\epsilon_m}$ is the impedance of the lead m. The scattering matrix S, defined by $b_n = S_{nm} a_m$ can be easily determined using the continuity of H at the lead boundaries, $a_n + b_n = H_0$. This allows us to deduce the scattering matrix:

$$S_{nm} = \frac{2Z_m h_m}{\sum_{j=0}^{N} Z_j h_j - ik\mu A} - \delta_{nm}, \qquad (3)$$

with h_m the width of L_m and A the area of the junction. Eq. (3) gives an exact solution for the scattering matrix in the configuration of Fig. 1. Next, we will use the above-derived result to solve the configuration of interest of a power splitter. The geometry is shown in Fig. 1(b). We discriminate the inlet of height h_0 connecting the ZIM junction at L_0 from the remaining N outlets. It is important to note that the geometry of the junction, of area A, is completely arbitrary and the relative dispositions of the inlet/outlet are arbitrary, as well.

The wave is incident from the inlet lead and we are interested in the reflection and transmission coefficients r and t_n, $n = 1, \ldots\ldots, N$. It is sufficient to use Eq. (3) with $a_0 = 1$ and $a_{n>0} = 0$ to get the reflection coefficient in amplitude $r \equiv S_{00}$ in the inlet and the transmission coefficient $t = S_{n0}$, $n = 1, \ldots\ldots N$:

$$\begin{cases} r = \frac{1 + ik\mu A/h_0 - \sum_{n=1}^{N} h_n/h_0}{1 - ik\mu A/h_0 + \sum_{n=1}^{N} h_n/h_0}, \\ t_n = 1 - r, \end{cases} \qquad (4)$$

where we considered that all the leads have the same impedance $Z_n = 1$ (for leads filled with air). Note that the transmission in amplitude t_n is the same at each outlet. According to Eq. (4), low reflection occurs for $\sum_{n=1}^{N} h_n = h_0$ and $k\mu A/h_0 \ll 1$. By scaling the area of the ENZ junction with h_0, $A = \alpha h_0^2$, we get the squeezing condition for the ENZ junction $\alpha k h_0 \ll 1$. Incidentally, note that the low phase variation of the magnetic field in the ENZ material imposes also a condition on the geometry. Denoting L the typical length of the junction, we should have $\sqrt{\epsilon} kL \ll 1$.

Alternatively, small reflection can be obtained by using a MIZIM ($\mu = \epsilon$) with $\sum_{n=1}^{N} h_n = h_0$, leading reflectionless junction $r = 0$ at dominant order. In this case, the low phase variation condition is $\epsilon kL \ll 1$.

For the above reflectionless junction, the transmitted power flux is $T_n = |t|^2 h_n/h_0 \simeq h_n/h_0$ in each outlet. It depends only on the section h_n and an equal repartition of the transmitted power can be obtained by choosing the same width for the N outlets, $h_n = h_0/N$.

Now, we confirm our analytical results in a particular geometry of power splitter, by comparison with numerical simulations using finite element software package COMSOL. We set the widths of the inlet and outlet leads $h_0 = 0.3$ m and $h_n = h_0/3$, $n = 1, 2, 3$. The frequency f varies between 0 and 2 GHz. This corresponds to a dimensionless frequency kh_0 between 0 and 4π. Figure 2(a) shows the magnetic field distribution when a TM plane wave is incident from the left in the lead 0 connecting a MIZIM junction with $\epsilon = \mu = 0.001$. The incoming wave transmits through the junction with very low reflectivity, and a transmission with an equal repartition of the energy in the three output leads is obtained. Here, the area of the junction $A \simeq h_0^2$. As expected, replacing the MIZIM by a nonmagnetic ENZ material in the junction produces an increase in the reflectivity, because of the mismatch of impedance at the interfaces of the ENZ junction, as shown in Fig. 2(b). Then, squeezing the geometry of the ENZ junction to $A \simeq 0.1 h_1^2$ produces the same low reflectivity as for the MIZIM junction [Fig. 2(c)]. Finally, when the ENZ material is removed, i.e., when the junction is filled with air (or vacuum), high reflectivity is observed; this is shown in Fig. 2(d).

Because we consider a nearly perfect ZIM with small but finite values of ϵ and μ, it is of interest to quantify the robustness of the reflectionless splitter when the frequency is increased. Indeed, as already stated, the squeezing effect and low phase variation need $\mu k A/h_0 \ll 1$ and $\sqrt{\epsilon\mu} kL \ll 1$. The reflected power $R = |r^2|h_0$ as a function of the normalized frequency $kh_0/(2\pi)$ is shown on Fig. 3(a). The three curves correspond to the cases of a ENZ, MIZIM, or air junction in the squeezed geometry [geometry shown in Figs. 2(b) and 2(c)]. Low reflectivity for both the ENZ junction and the MIZIM junction is observed at sufficiently low frequency. As expected, when the ZIM material is removed (air junction) high reflectivity is observed, except in the static limit. In our squeezed geometry, $\alpha \simeq 0.1$ and $L \sim h_0$. For the ENZ junction, the squeezing effect is efficient for $kh_0 \ll 1/\alpha \sim 10$ and the assumption of low phase variation of the magnetic field inside the ENZ junction is relevant for $kh_0 \ll 1/\sqrt{\epsilon} \sim 30$. This is consistent with the variation of the reflection coefficient observed on Fig. 3. In the case of a MIZIM

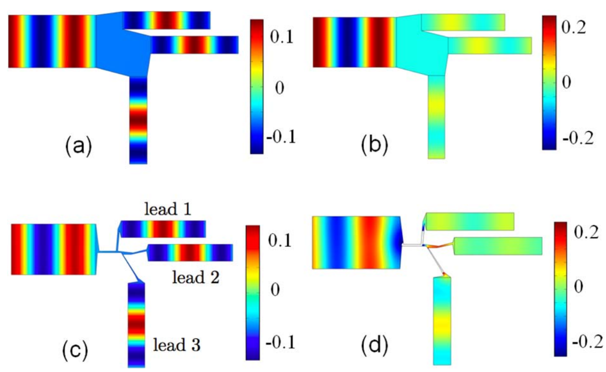

Fig. 2. Real part of the magnetic field $H(x, y)$ for a three output power splitter. (a) The junction is made of a MIZIM with $\epsilon = \mu = 0.001$ and a junction area $A \sim h_0^2$. (b) Same geometry as in (a) with a nonmagnetic ENZ material; $\mu = 1$ and $\epsilon = 0.001$. (c) ENZ junction with a squeezed junction $A \sim 0.1 h_0^2$. (d) Removing the ENZ material in the squeezing geometry $\mu = \epsilon = 1$. All fields have been calculated at frequency $f = 0.8$ GHz ($kh_0 = 1.6\pi$).

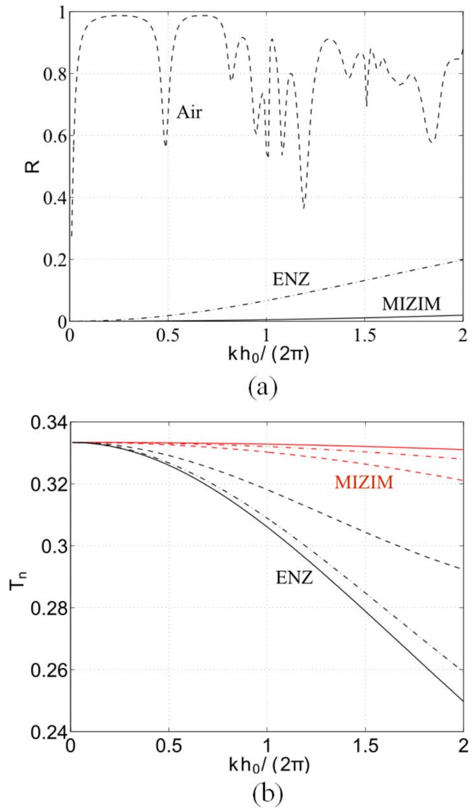

Fig. 3. (a) Reflection coefficient in energy R at the inlet lead as a function of the normalized frequency $kh_0/(2\pi)$ in the squeezed geometry for three different junctions. (b) Corresponding transmission coefficients $|T_n|$ at the three output leads [as indicated in Fig. 2(c)]: $n = 1$, solid line; $n = 2$, in dashed–dotted lines; and $n = 3$, dotted line.

junction, the same criteria give $kh_0 \ll 1/(\alpha\epsilon) = 10^4$ and $kh_0 \ll 1/\epsilon = 10^3$, well above our maximum working frequency. Nearly perfect reflection is indeed observed in Fig. 3 in the whole range of frequencies.

Figure 3(b) shows the corresponding power transmission coefficients $T_n = |t_n|^2 h_n$ for the three output leads $n = 1, 2, 3$. This representation illustrates the slight loss of the equipartition of the power among the three leads when the frequency is increased. We found a maximum deviation to the power equipartition of 4% for a MIZIM junction and a maximum of 6.3% in the case of a ENZ junction.

In conclusion, we have shown that efficient multiple output dividers can be designed using ZIMs. These dividers have the advantage of being compact for any geometry, without restriction on the number or on the directions of the outlets. We have considered here the case of an equipartition of the power, but one can design easily a configuration with another power repartition. Finally, note also that such a divider can work as a combiner as well in symmetric modes: in this case, by considering $a_0 = 0$ and $a_n = 1$, $n = 1, \ldots..N$, we get the transmission coefficient at the lead 0 $t' = \sum_{n=1}^{N} S_{0n}$ from Eq. (3):

$$t' = \frac{2\sum_{n=1}^{N} h_n/h_0}{1 - ik\mu A/h_0 + \sum_{n=1}^{N} h_n/h_0}, \tag{5}$$

and $t' \simeq 1$. This means that the N inlets are reflectionless when excited simultaneously and that the power is entirely transmitted in the unique outlet $n = 0$. Experimentally, such a structure could be realized in planar waveguides with a ZIM junction based on a metamaterial or a ZIM junction made of a narrow channel close to the cut-on frequency.

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