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Mean-Flow Correction as Non-Linear Saturation Mechanism.

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PACS. 47.20Ky – Nonlinearity (including bifurcation theory).

Abstract. – We are interested in identifying the mean-flow correction in the development of an instability. In the presented configuration (a 2D jet confined in a rectangular cavity that presents an oscillating instability), numerical simulations allow the identification of this mean-flow correction. We show that this distortion contributes to the non-linear saturation of the instability and that it is itself generated by the instability development: the action of the Reynolds tensor due to oscillating terms produces the mean-flow change.

Introduction. – The mean-flow distortion in the development of instabilities is a mechanism present in various configurations. Stuart [1, 2] predicts, in the plane Poiseuille flow, that the fundamental mode and its harmonics will grow until their amplitudes are sufficient to modify the mean flow, contributing to the saturation mechanism. Similarly, in the Taylor-Couette instability, Stuart [1] and Hall [3] show the existence of a stationary mode that modifies the profile of a circular Couette. In the first non-linear works in the Rayleigh-Bénard convection [4] this zero-wave-number non-linear mode was obtained for the temperature perturbation and analysed as a slave mode. After these works, Zippelius and Siggia [5, 6] show that an equation of Landau-Ginzburg type is not sufficient and that one more equation has to be considered to take into account the non-slave behaviour of the stationary mode. In the Görtler instability, theoretical studies [7] have shown the existence of a homogeneous mode (with zero spatial wave number), that is added to the Blasius profile of radial velocity component in the basic flow. In the case of open flows instabilities, numerical simulations of the vortex shedding in wakes performed by Hanneman and Oertel [8] allow to observe a structural change in the mean flow during the transition from the linear-growth stage to the saturation state. As we will see, this last situation is very close to the problem discussed here.

The configuration presented in this paper is a 2D jet confined in a rectangular cavity. This flow, characterized by an impingement free shear layer, behaves as an oscillator: it displays intrinsic dynamics contrary to the free jet, sensitive to external noise and unable to sustain a well-defined oscillation frequency (for a review see [9] and [10]). The flow is found to undergo a transition to an oscillatory state for a Reynolds number ($Re = Ud/v$, where U is the maximum velocity at the jet entrance and d the nozzle width) of 102.5. In fig. 1 an experimental visualization of a 2D jet of water penetrating into a cavity full of water (a)) and streamlines obtained in numerical simulation (b)) are shown. We have checked that the

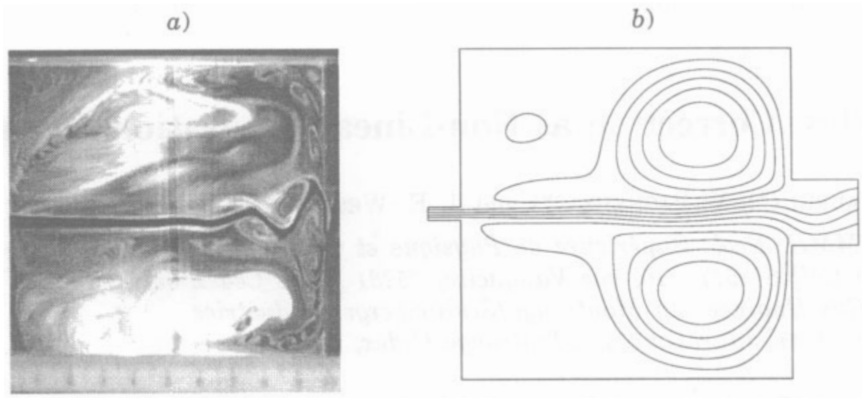

Fig. 1. – *a*) Visualization of the unstable flow, *b*) streamlines from the numerical simulation at $Re = 115$.

numerical simulations are in good agreement with the experimental results [11]. The study of the instability has been performed in previous works [12].

The originality of this paper is the use of numerical simulations to identify the mean-flow correction during the temporal development of the oscillations from the linear regime to the saturation state. We show that this correction acts on the oscillating part of the perturbation and we observed that the velocity profiles have been stabilized. We propose to interpret this correction using a method borrowed from the turbulence analysis with the Reynolds tensor, that underlines the actions of the oscillatory part of the perturbation on the mean flow.

Characterization of the instability. – The numerical simulations were performed with Nekton code [13], based on a spectral finite element method. The domain decomposition contains around 10 000 nodes. The boundary conditions are the following: 1) at the entrance, a parabolic profile is imposed. 2) The condition at rigid walls implies that the two velocity components are zero. 3) At the exit, a classical outflow condition is imposed. This condition implies that the stress tensor at the surface exit is zero [14].

The simulations have been performed for Reynolds numbers between 105 and 125. The results presented in this letter correspond to a simulation at $Re = 115$. Figure 2 shows the temporal evolution of both velocity components, u_x and u_y, at a point located on centreline.

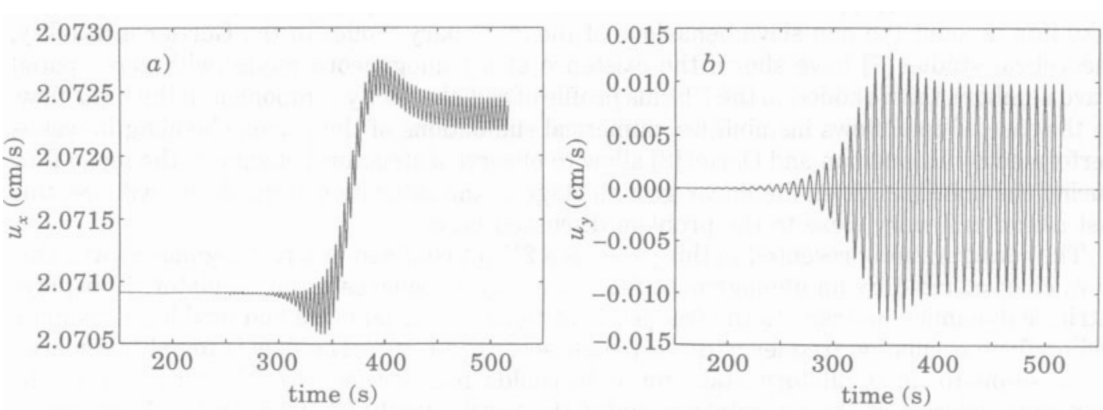

Fig. 2. – Temporal evolution of the u_x (*a*)) and u_y (*b*)) velocity components at $Re = 115$ for a point on the centreline at 3 cm from the cavity entrance.

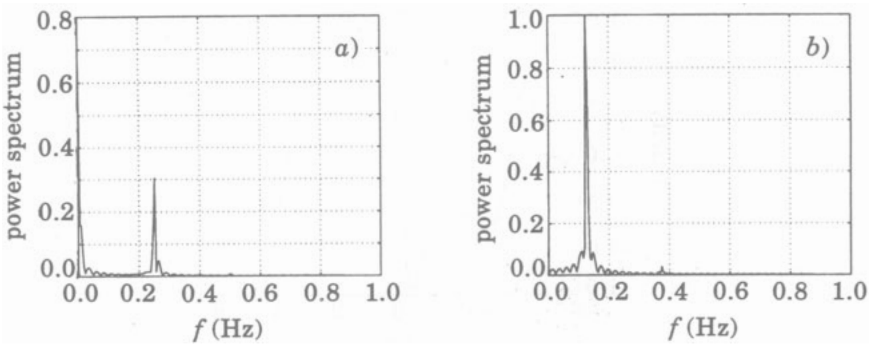

Fig. 3. – Power spectra for the u_x (*a*)) and u_y (*b*)) velocity components ($Re = 115$).

Between $1 = 150$ s and around 200 s, we observe a nearly stationary behaviour of both velocity components: this stationary behaviour is in common for the entire cavity. We have checked that the initial stationary flow corresponds to the basic flow field. The instability starts to grow exponentially with a characteristic growth rate λ (from t around 200 s to 300 s): this behaviour corresponds to the linear regime, where $s = \lambda + i\omega$ is the eigenvalue selected in the linear problem and ω is the pulsation. From t around 300 s to 500 s, non-linearities become significant and start to saturate the signal. From t around 400 s to 500 s, the signal conserves the same oscillation amplitude: non-linearities have completely saturated it and we observe the saturation state. Spectra of these signals (fig. 3) show the following points:

1) The instability contains mainly one frequency f for the u_y-velocity of points located on centreline.

2) The u_x-velocity testifies to the existence of the harmonic with $2f$ frequency, this frequency being a result of a quadratic term in non-linearities [1, 8, 15].

3) For the u_x-velocity, we observe a temporal evolution of the mean velocity value. This change testifies to a structural evolution of the mean flow during the non-linear development of the instability, as underlined by Hanneman and Oertel [8]. Note that the mean-flow correction from the linear to the saturation state may be also interpreted as a consequence of a non-linearity of same order as the $2f$ harmonic.

4) The contribution of other harmonics than $2f$ and 0 can be neglected.

Mean-flow distortion. – We are interested here in the influence of the oscillatory part of the signal on the mean flow. To obtain the mean flow, we subtracted the oscillating perturba-

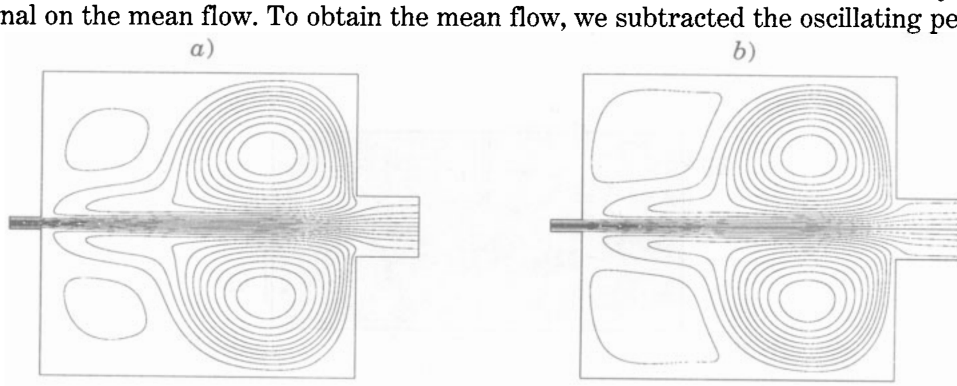

Fig. 4. – Streamlines for the basic flow, *a*) Ψ_B and *b*) in the saturated state $\Psi_B + \Psi_0$ ($Re = 115$).

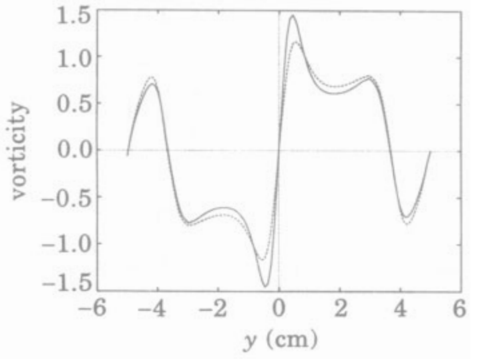

Fig. 5. – Vorticity profile at $Re = 115$ for a point on the centreline at 9 cm from the cavity entrance.

tion with f and $2f$ frequencies from the total flow field [12]. On the other hand, by forcing the symmetry on the centreline, the non-bifurcated solution of the Navier-Stokes equations is obtained corresponding to the basic flow. This procedure has been performed by simulating a half-cavity with a symmetry condition on the centreline and we have checked that the obtained field corresponds to the observed mean-flow field in the linear regime.

In the saturation regime, the mean flow obtained differs from the basic flow and we note the stream function $(\Psi_B + \Psi_0)$. These results are shown in fig. 4: *a)* shows the streamlines for the basic flow field Ψ_B and *b)* the streamlines for the mean-flow field at saturation $(\Psi_B + \Psi_0)$. The distortion of the mean flow between the linear and non-linear regime corresponds to the development, due to non-linearities, of a zero frequency mode Ψ_0.

Figure 5 shows the vorticity profile for the basic flow and the mean flow at saturation. It appears that the maximum of vorticity for the basic flow is higher than the maximum vorticity for the mean flow at saturation. This shows that the flow has been stabilized during the oscillation development. Actually, the velocity gradient has decreased with the oscillations development and the shear is thought to be responsible for the instability.

Consider the Navier-Stokes equations (1), where $\vec{\vec{\sigma}}$ is the stress tensor:

$$\varrho\left(\frac{\partial \vec{u}}{\partial t} + \mathrm{div}\,(\vec{u}\otimes\vec{u})\right) = \mathrm{div}\,(\vec{\vec{\sigma}}), \qquad \text{with} \qquad \sigma_{ij} = -p\delta_{ij} + \frac{1}{2}\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{1}$$

The total velocity vector $\vec{u}$ is written as the sum of a non-oscillating part $\vec{u}^s$ and an oscillating part $\vec{u}\,'$. The mean of eq. (1) is taken under the assumption that the typical

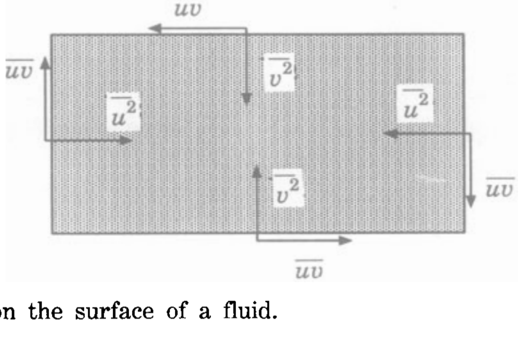

Fig. 6. – Reynolds stress on the surface of a fluid.

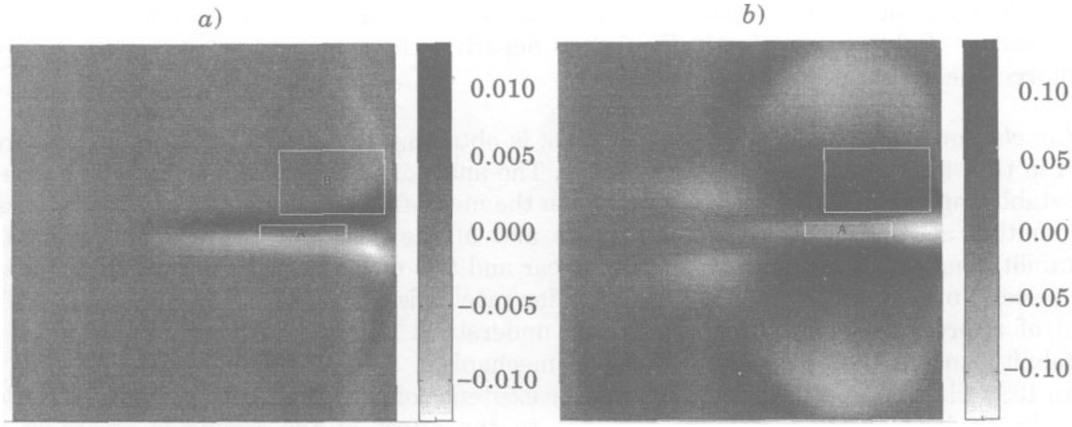

Fig. 7. – *a*) Graph of the Reynolds stress, responsible for the shear motions, *b*) mean-flow distortion for the u_x-velocity component.

oscillation period is very small compared with the typical time of the stationary motion, so that $\vec{u}' = \vec{0}$, and eq. (2) is obtained:

$$\varrho\left(\frac{\partial \vec{u}^s}{\partial t} + \operatorname{div}(\vec{u}^s \otimes \vec{u}^s)\right) = \operatorname{div}(\vec{\vec{\sigma}}^s + \vec{\vec{\sigma}}^r), \tag{2}$$

where $\sigma^s_{ij} = -p^s \delta_{ij} + (1/2)\mu(\partial u^s_i/\partial x_j + \partial u^s_j/\partial x_i)$ and $\vec{\vec{\sigma}}^r$ is the Reynolds tensor: $\sigma^r_{ij} = -\varrho\,\overline{u'_i u'_j}$. In the following, we denote u, v as the components of the oscillating-part velocity $\vec{u}'$. The Reynolds tensor $\vec{\vec{\sigma}}^r$ has the following expression:

$$\begin{bmatrix} -\overline{u^2} & -\overline{uv} \\ -\overline{uv} & -\overline{v^2} \end{bmatrix}.$$

Figure 6 represents the forces due to the development of the oscillatory perturbation on a control volume. The global action of these forces depends on the sign and on the intensity of the terms $\overline{uv}$, $\overline{u^2}$ and $\overline{v^2}$ on each side of the considered volume.

As seen previously, the oscillating part of the signal contains mainly the frequencies f and $2f$, so that u and v can be written as $u = u_1 + u_2$ and $v = v_1 + v_2$, where 1 and 2 represent the oscillating modes corresponding to f and $2f$ frequencies. It is easy to find that: $\overline{uv} = \overline{u_1 v_1} + \overline{u_2 v_2}$, $\overline{u^2} = \overline{u_1^2} + \overline{u_2^2}$ and $\overline{v^2} = \overline{v_1^2} + \overline{v_2^2}$.

The graph of fig. 7*a*) shows the term $\overline{uv}$ in the cavity, responsible for the shear motion and fig. 7*b*) the correction of the u_x-velocity component due to the development of the stationary mode.

We have represented in these figures two control volumes A and B on which the longitudinal forces apply. In the A-zone, corresponding to the axis line, we check from fig. 7*a*) that $\overline{uv}$ is positive on the top, negative on the bottom: the forces acting on this zone are in the opposite direction to the jet. This effect is in agreement with fig. 7*b*), where we observe that the longitudinal velocity corresponding to the distortion is negative on the axis: the jet velocity has decreased at this location.

In the B-zone, in the recirculations, $\overline{uv}$ is positive on the bottom, and $\overline{uv}$ is positive on the top but with a neglected intensity (see fig. 7*a*)). The action of the resulting forces on B is consequently positive. On the other hand, the action of the resulting forces in the top of the recirculation (light gray region in fig. 7*a*)) is negative: the combined actions of these forces reinforce the rotation of the recirculation.

Conclusion. – The main result of our work is obtaining and utilizing the Reynolds stress field in the study of an oscillating instability. The analysis of this field allows to conclude on the stabilizing effect of the Reynolds stress on the mean-flow development. We have stressed the mutual actions of the stationary mode and of the oscillating modes: the oscillating instability makes a mean-flow change to appear and this mean-flow correction, by stabilizing the velocity profile, contributes to the saturation mechanism of the oscillating instability. This kind of approach may be useful to better understand the non-linear development of an instability and in particular the saturation mechanism.

In 1989, Hegseth *et al.* [16] showed that the existence of a laminar zone close to a turbulent zone in Taylor-Couette instability was due to the effect of the Reynolds stress in the turbulent region. In 1921, Noether [17] noted in the wall-bounded shear flows an unstable mean-flow profile different from the undisturbed profile because of the presence of a steady wave and analysed this change with the Reynolds stress. She concludes that a self-consistent problem may be used to characterize the instability: the growth rate determines the unstable character of the system and the Reynolds tensor stabilizes the profile, so that the growth rate value diminishes until it reaches a zero value. For a review on shear flow instabilities, see for instance Bayly *et al.* [18]. Note that this kind of approach is similar to the «mean-field theory» used in various domains of physics, where it has given correct qualitative results. Our exploitation of numerical simulations to obtain the exact forms of the stationary mode and the field of the Reynolds stress is a first step towards quantitative studies.

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