

Dynamic response of a surface pod to seismic events on an asteroid (KW4-type)

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Abstract

Low ambient gravity on small asteroids is determining for the effect of an impact or a blast on the surface of such little bodies. We use a simple analytical model to compute the behaviour of a surface pod for BASiX mission. Any object sitting on the surface has the same response and is launched from the surface, as the mass has no effect on the path. Studying the influence of the amplitude and the frequency of the shock on the launch velocity, we use an analytical wave propagation theory compared to numerical simulations to find the attenuation factor of the asteroid (KW4) and compute the launch velocity–distance from the blast function in order to foresee the path of the pod from any position on the equator of the asteroid. Simulations of the path of the pod are performed and we isolate several zones according to the fact that the pod is either ejected definitively from the asteroid or comes back after a couple of revolutions. According to the characteristics of the shockwave, knowing the position of the pod before and after the blast, we are able to compute the attenuation factor. Also, as we are able to compute the movement of any particule on the surface, the shape reformation of the asteroid, as a consequence of the blast event, can be foreseen. Several proposals to improve the model are also given.

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Introduction

Small bodies like asteroids are affected by seismic waves generated by impacts on their surface as the acceleration of such shocks is greater than their local gravity (Robert *et al.* (2010) and Asphaug *et al.* (1996)). The present study tries to explain the effect of such an event on a small asteroid, like the quite well known 1999 KW4 (Scheeres *et al.*, 2010).

This study has been motivated by the BASiX mission proposal directed by D. J. Scheeres (Dissly *et al.* (2010)) within which seismometers are deployed on the surface of a small body and the seismic wave is created by a blast event located on the surface. Therefore in this work we try to model the dynamic response of a pod which contains the seismometers.

Here we propose a simple model of the asteroid: first we assume the ground to be perfectly rigid in order to model its sinusoidal movement, then we determine the launch velocity of the particle which would be sitting on the surface. Knowing the equations of motion, we use the ode45 Matlab function in order to determine the path of the particle around the asteroid. As the trajectory of the latter does not rely on its mass or either shape but only on the frequency and the amplitude of the shockwave, we can say that the path of any particle of the surface –even the pod which might be larger– would be the same. Thus, we are able to predict the path of any particle launched from the surface of the asteroid as soon as we know its launch velocity. As we assume the pod to be settled on the equator of the asteroid, we can only focus on the initial positions for which $z = 0$ (whether in the body fixed or the inertial frame).

Asphaug (2008) has developed a theoretical model of wave propagation in a continuous and homogenous field of regolith which gives the peak velocity of a particle sitting on the surface. We use numerical simulations based on finite elements method to determine the attenuation factor of the studied asteroid, according to Asphaug's theory. Thus, we use this results to foresee the launching velocity of a particle with respect to its distance from the seismic source.

In fine, we are now able to plot the path of any particle sitting on the surface including the pod, knowing just the amplitude and the frequency of the shockwave.

The plotting of such results brings new questions, particularly about the role of the shock frequency on the path of the pod (or any particle) and the shape model of the asteroid.

1 Is the pod launched by the moving ground?

The first question we need to answer is to determine whether or not the pod is launched by the shockwave. To do so we model the movement of the ground and we write the equation of motion of the pod.

1.1 Modeling the seismic wave and movement of the ground

The main issue is to get a simple –but realistic– model of the shake of the ground. This model has also to validate initial conditions (position, velocity...). The most simple model for such a seismic wave is a sin function wave because the formula is simple and the initial condition easy to define by adjusting several parameters (see section 1.1.1).

1.1.1 Importance of the initial conditions

At first we thought to model the movement of the ground as:

$$u_r(t) = \sin(\omega t) \quad (1)$$

But as we plot such a function we realize the initial conditions are not realistic. Indeed we have (see figure 1.1.1 page 4):

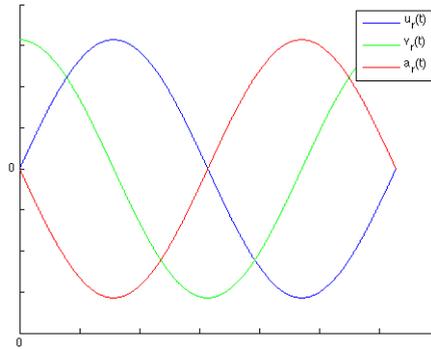


Figure 1: According to this model, $u_r(t) = U \sin(\omega t)$

$$u_r(0) = 0 \quad (2)$$

$$v_r(0) = U\omega \neq 0 \quad (3)$$

$$a_{rr}(0) = 0 \quad (4)$$

This is why, in order to have:

$$u_r(0) = 0 \quad (5)$$

$$v_r(0) = 0 \quad (6)$$

$$a_{rr}(0) = U\omega^2 = A \quad (7)$$

we choose to model the movement of the shaking ground as:

$$u_r(t) = U(1 - \cos(\omega t)) \quad (8)$$

from which we get:

$$v_r(t) = U\omega \sin(\omega t) = V \sin(\omega t) \quad (9)$$

$$a_r(t) = U\omega^2 \cos(\omega t) = A \cos(\omega t) \quad (10)$$

with $A = U\omega^2 \geq 0$.

This way the plot of the moving ground is:

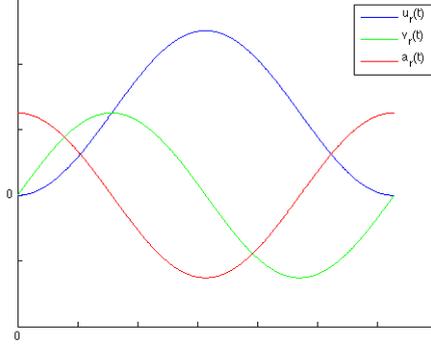


Figure 2: According to this model, $u_r(t) = U(1 - \cos(\omega t))$

1.1.2 The t_0 -time at which there is no more contact between the pod and the asteroid Dynamics of the vibrating ground: Equations of Motion

$$ma(t) = m\ddot{y}_{pod}(t) = -mg + N \quad (11)$$

With, in the inertia frame, $a(t) = A \cos(\omega t)$ the acceleration of the pod (which follows the sin movement of the ground: seismic wave), $g = g_{gravity} - \Omega^2 R$ (R is the radius of the asteroid and Ω is its rotating speed), $N = \max(0; mA \cos(\omega t) + mg)$ is the normal reaction of the surface on which the pod is.

Which means that there is no more contact with the ground when $N = 0$ (N.B.: we always have: $N \geq 0 \Rightarrow |\frac{g}{A}| \leq 1$):

$$N = mA \cos(\omega t) + mg = 0 \quad (12)$$

$$\Leftrightarrow -A \cos(\omega t) = g \quad (13)$$

$$\Rightarrow t_0 = \frac{1}{\omega} \arccos\left(\frac{-g}{A}\right) \quad (14)$$

1.1.3 Launching: determination of the initial speed v_0

The launching velocity of the pod is the velocity of the moving ground at the t_0 time, therefore $v_0 = v(t_0)$:

$$v_r(t) = U\omega \sin(\omega t) \quad (15)$$

$$\Rightarrow v_0 = v_r(t_0) = U\omega \sin\left(\arccos\left(\frac{-g}{A}\right)\right) \quad (16)$$

$$(17)$$

In order to have $v_0 \geq 0$ we choose the positive solution:

$$\Rightarrow v_0 = \frac{A}{\omega} \sqrt{1 - \left(\frac{g}{A}\right)^2} \quad (18)$$

1.2 Equations of motion of the pod in the rotating frame (body fixed), $t > t_0$

We know the equations of motion of a body orbiting around a planet (or here, the asteroid):

$$\begin{cases} \ddot{x} = 2\Omega\dot{y} + \Omega^2 x - \frac{\mu}{r^3} x \\ \ddot{y} = -2\Omega\dot{x} + \Omega^2 y - \frac{\mu}{r^3} y \end{cases} \text{ with } \mu = G \cdot M_{asteroid} \text{ and } \Omega \text{ the rotating speed}$$

1.3 First results, the pod is orbiting around the center of mass of the asteroid

Thus we have the path of the pod in the inertial frame, from the position $(x = 0; y = 750; z = 0)$, without modeling the asteroid itself (so there is no bounce for these first simulations).

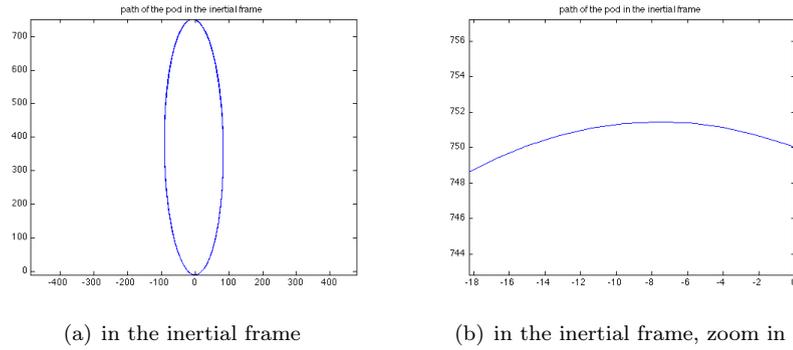


Figure 3: Path of the pod in the inertial frame

On figure 4 page 6 we can see what the result is in the rotating frame (body fixed).

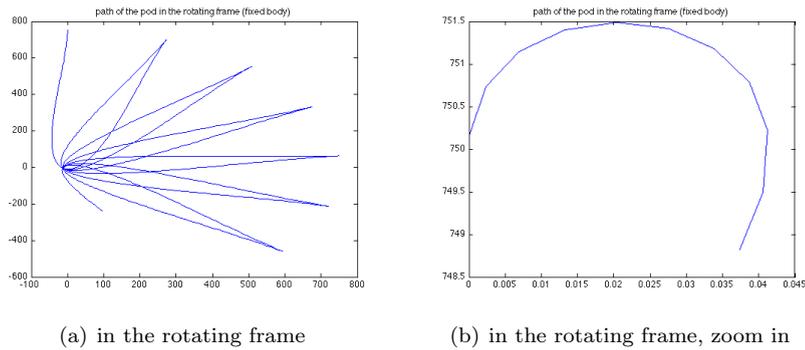


Figure 4: Path of the pod in the rotating frame (body fixed)

The pod seems to be in orbit around the center of mass of the asteroid $(0, 0, 0)$ which is consistent as we did not model the surface of the asteroid yet.

1.4 Verifying the simulations

In order to verify the results from the model and its simulation, we compute the conservative parameters: the angular momentum h and the energy of the system E . Even if the simulated paths we obtained seem to be correct, we chose to plot the energy of the system in order to make sure the energy is conserved (it has to be because all the forces applied on the system are conservative ; there is no friction because there is no atmosphere around the asteroid).

We can see that in both the rotating and the inertial frame the energy is conserved (see figures 5(b) and 5(a) page 7): we have to notice that the tolerance of the simulation is 10^{-9} which corresponds to the variation of the energy we can observe on these plots.

We can also plot the angular momentum to see if it is constant, as expected: see figure 6.

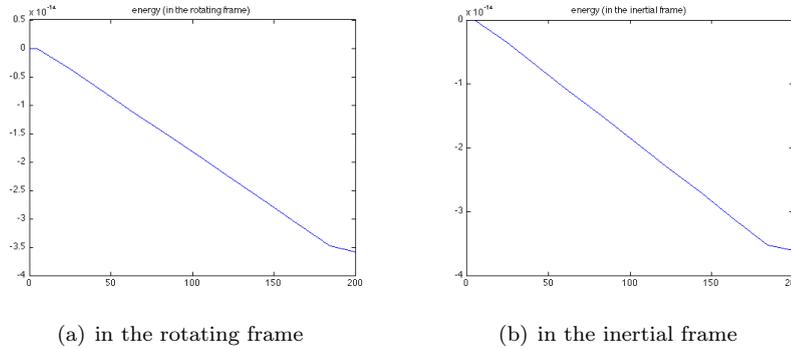


Figure 5: Energy of the system in different frames

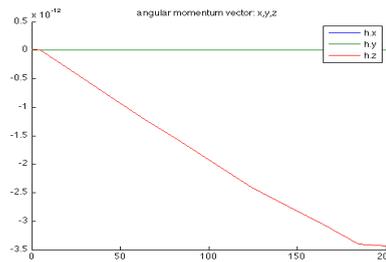


Figure 6: Angular momentum (*the tolerance is still 10^{-9} this is why we can conclude that the angular momentum is constant*)

2 Modeling the landing/bounce of the pod

2.1 Ode45 function on Matlab

We used ode45 Matlab function to solve the equation of motion of the pod. This routine is commonly used to solve differential equations numerically. It uses a variable step Runge-Kutta Method.

2.2 Event function on Matlab

As the equations of motion in the different frames seem to be correct, we can now add the model of the surface of the asteroid in order to see where the pod lands after having been launched by the seismic wave.

There are many options for the ode45 routine, such as the "event function". An event is a "user-initiated action that takes place in a server application"¹. For our use, we created a bounce_event.m file that stops the ode45 resolution when the condition $\{\text{Distance}(\text{center of the asteroid} - \text{pod}) \leq \text{Radius of the asteroid}\}$ is verified. Therefore, our model of the asteroid does not take in account the nature of the surface of the asteroid (granular field, friction...). Here the asteroid is a perfect spherical and not deformable solid.

3 Influence of the initial speed v_0 and the amplitude of the shockwave on the path of the pod

The pod describes a ballistic path around the asteroid. Thus the initial conditions, such as the direction (here we consider the pod to be launched vertically in respect with the orientation of the

¹cf. MathWorks.com

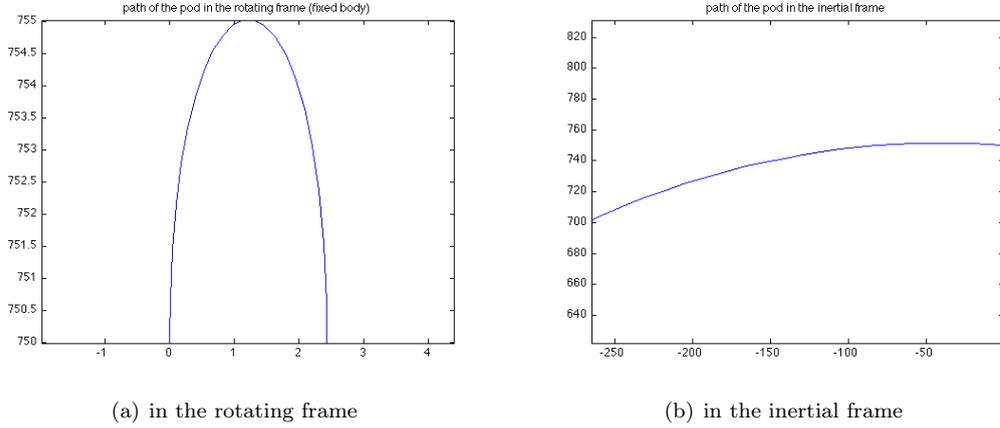


Figure 7: Bounce of the pod in different frames

ground) and the modulus of the initial speed of the pod, when it leaves the ground, are decisive for the whole following trajectory. That is why the expression of v_0 we obtained in section 1.1.3 is very useful to see which parameters of the seismic wave will be decisive for the nature of the path of the pod:

$$v_0 = \frac{A}{\omega} \sqrt{1 - \left(\frac{g}{A}\right)^2} \quad (19)$$

3.1 Influence of the amplitude A

By changing the code, we are able to plot the path of the pod in the several frames, for different values of A . As expected, the pod is launched further when the amplitude of the acceleration A (modulus) is greater (see figure 8 page 8).

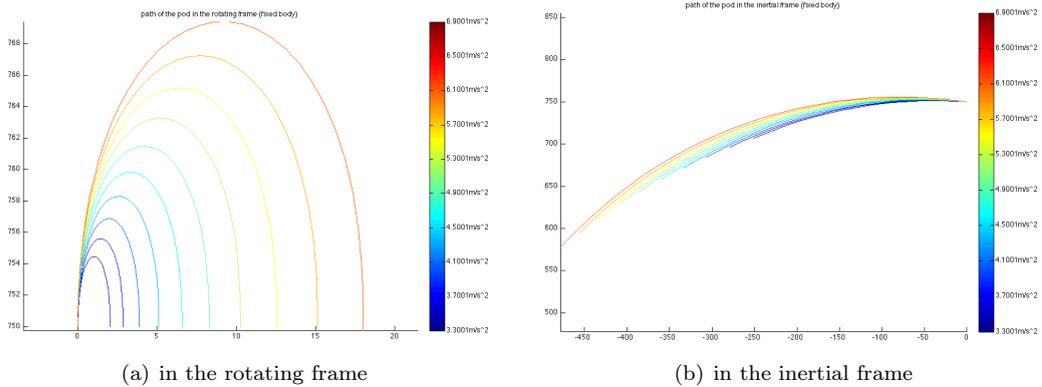


Figure 8: Path of the pod for different values of A , in different frames

3.2 Influence of the frequency of the seismic wave: $f = \frac{\omega}{2\pi}$

From the formula of the initial velocity v_0 we can guess the influence of the frequency of the seismic wave. Therefore we used Blitz's simulations (*Wave propagation simulations based on the spectral element method: application to the asteroid 65803 Didymos*, Blitz C., 2011) to apply an FFT computation in order to see the frequency of the peak acceleration on several points on the

asteroid² (see figures 9 & 10 pages 9 and 9).

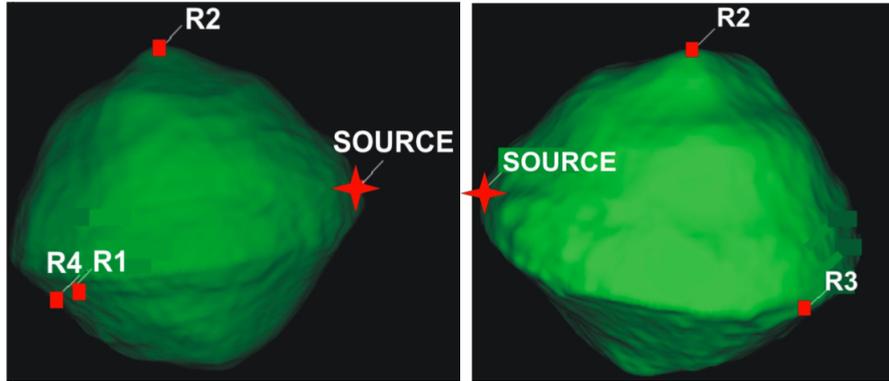


Figure 9: Locations of the source and the recorders on the shape model of asteroid Didymos

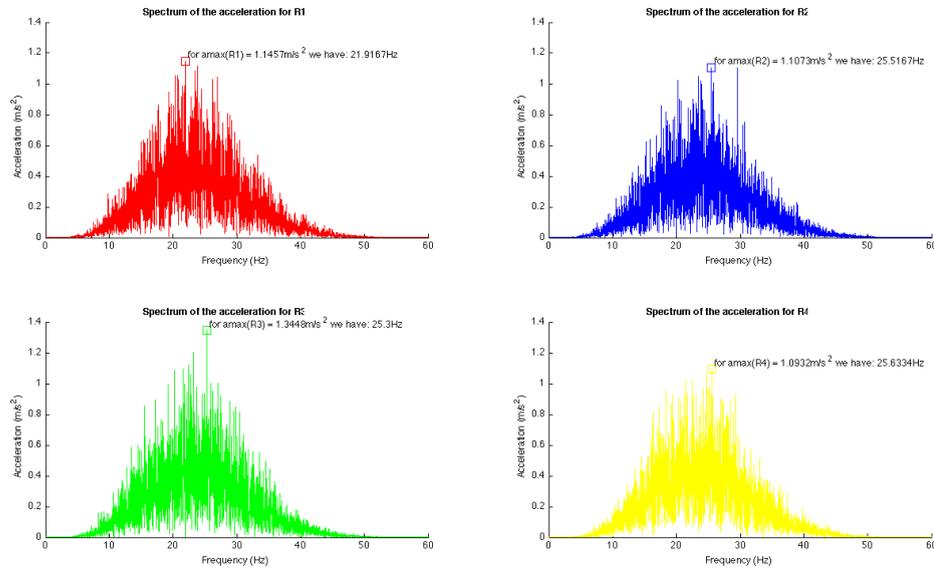


Figure 10: Spectrum of the seismic wave in several points of the surface of the asteroid

We can see that the frequency seems to be independant from the distance from the blast event ("source"). As Blitz uses a numerical model of the asteroid, we can assume that the frequency of the peak acceleration changes according to the shape of the asteroid rather than according to the distance from the source.

Thus, in our theoretical model, as we do not take in account the shape of the asteroid, we consider the frequency of the seismic wave to be constant and equal to $16Hz$ ³.

4 Relation between the initial velocity of the pod and it distance from the blast event

We just showed how the initial velocity v_0 and therefore the amplitude of the acceleration A are determining for the whole path of the pod. What we are looking for is a relation between this peak

²N. B.: the simulations were made for 65803 Didymos which we assume to have close characteristics to KW4.

³we took $16Hz$ because it was the frequency of the shockwave according to Blitz's report, but the results are very similar to the ones we get using $20Hz$

velocity v_0 and the distance from the blast event. That way, knowing the initial position of the pod, and thus its distance from the source, we can have v_0 and so compute the path of the pod around the asteroid.

As we can assume the frequency of the wave to be constant and not to change with the distance D , we can search for a relation $v_0 = v_0(D)$ or a relation $a_0 = a_0(D)$. Indeed, we can easily link v_0 to A (*c.f.* equation 18 and $A = \omega V$ in which $V = \omega U$).

4.1 Asphaug model: impact theory

Many papers deal with the modeling of the impact between two asteroids. Thus we had the idea to use such a theory to model the blast event and its effects on the asteroid.

4.1.1 Asphaug theoretical model

We have (Asphaug, 2008) the expression of the peak velocity in radial symmetry from a point source:

$$v_p(D) = v_i(D/r_i)^{-\alpha} \quad (20)$$

Where:

- α is the attenuation ($\alpha = 1$: energy is conserved = elastic material);
- D is the distance of the particle from the source;
- r_i is the radius of an impactor (a smaller asteroid which impacts the bigger asteroid of our study);
- v_i is the velocity of this impactor.

By using such a model and assuming that $v_p(D) = v_0(D)$, we now have a relation between the initial velocity of the pod v_0 and its distance D from the source, as soon as we have the characteristics of the equivalent impactor.

4.1.2 Characteristics of the impactor

Let us find the equivalent impactor to our 5kg PETN explosive load which creates the blast event, so that we will be able to find a relation between the velocity of a distant particule (v_p) and its distance from the blast event (source of the seismic wave).

We know the energy released by the blast event is 20MJ. We assume the impact of the "virtual impactor" converts all the kinetic energy into the mechanical energy of the seismic wave.

$$\frac{1}{2}m_i v_i^2 = 20MJ \quad (21)$$

with $v_i = 5km/s$ the impactor velocity and m_i its mass.

Thus:

$$\frac{1}{2}m_i v_i^2 = 20 \cdot 10^6 \quad (22)$$

$$\Leftrightarrow m_i = \frac{2 \cdot 20 \cdot 10^6}{(5 \cdot 10^3)^2} \Rightarrow \boxed{m_i = 1,6kg} \quad (23)$$

With a density of $\rho_g = 3500kg/m^3$ (ρ_g is the asteroid grain density) we have the radius of the impactor:

$$\frac{4}{3}\pi r_i^3 \rho_g = m_i \quad (24)$$

$$\Rightarrow r_i = \left(\frac{m_i}{\rho_g} \cdot \frac{3}{4\pi} \right)^{\frac{1}{3}} \Rightarrow \boxed{r_i = 2,2mm} \quad (25)$$

N.B.: we just presented a method to solve the problem for BASiX mission: the explosive load is chosen, we know the amount of energy released by such a blast, so we can "build" an equivalent impactor to use Asphaug theory. For her numerical simulation, Dr Blitz used an impactor with the following characteristics:

- impact velocity: $v_i = 3,2 km.s^{-1}$
- mass: $m_i = 5kg$
- radius (if $\rho_g = 3500kg/m^3$): $r_i = 69,9mm$

We used these characteristics for the following simulations presented in this report in order to be able to compare the theories.

4.2 Asphaug attenuation factor – Blitz numerical simulations

In order to define every parameter of Asphaug formula (*c.f.* equation 20), we need to choose a reasonable value for the attenuation factor α . To do so we assume the peak frequency to be the same in any simulated point from Blitz computation and we try to see for which α Asphaug model fits the best⁴ (see figure 11 page 11). For the following simulations (*c.f.* section 5) we chose $\alpha = 1, 27$.

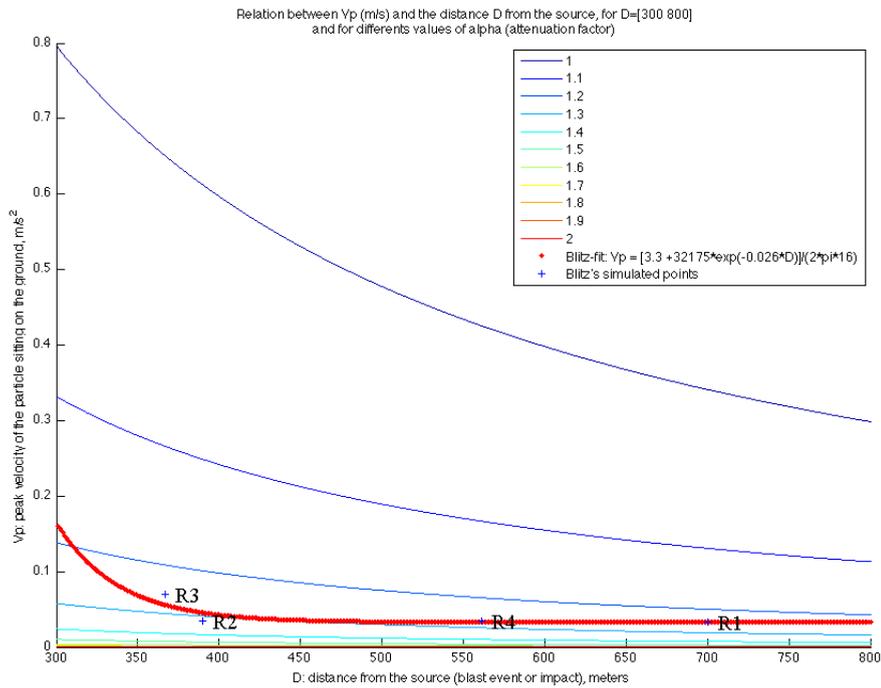


Figure 11: Asphaug model compared to Blitz numerical simulations

We can now predict the path of the pod and the place in which the pod bounces having its initial position on the asteroid regarding the location of the source.

4.3 Using Asphaug model to find a litteral expression of $v_0 = v_0(D)$

Knowing the attenuation factor and as we can assume the frequency does not change with the distance from the blast, we can have a relation between the path of the pod and its initial position from where it has been launched.

⁴we built a "Blitz-fit" exponential function in order to fit the simulated points

Thus, in this section we show how to find a litteral formula which describes the initial acceleration as a function of the distance from the source, in order for us to link the path of the particule to its initial distance from the blast.

Let us sum up: an asteroid (the impactor), with a radius r_i , hits our "KW4-type" asteroid and creates a $16Hz$ -frequency sin shockwave.

Knowing the attenuation factor, we can have the amplitude U of the movement of the ground $u_r(t)$.

We consider the peak velocity to be equal to the initial launch speed of the particule (*i.e.* the pod for the present study), v_0 . We have:

$$v_p(D) = v_i \left(\frac{D}{r_i} \right)^{-\alpha} = v_0 \quad (26)$$

but we also have:

$$v_0 = \omega U \sqrt{1 - \left(\frac{g}{\omega^2 U} \right)^2} \quad (27)$$

(*N.B.*: equation (18) with $A = \omega^2 U$)

So we can write:

$$\left(\frac{v_0}{\omega U} \right)^2 = 1 - \left(\frac{g}{\omega^2 U} \right)^2 \quad (28)$$

$$\Leftrightarrow \left(\frac{v_0}{\omega U} \right)^2 = \frac{U^2 \cdot \omega^4 - g^2}{U^2 \cdot \omega^4} \quad (29)$$

$$\Leftrightarrow v_0^2 \omega^2 = U^2 \cdot \omega^4 - g^2 \quad (30)$$

$$\Leftrightarrow U = \pm \frac{1}{\omega^2} \sqrt{(v_0 \omega)^2 + g^2} \quad (31)$$

And as we took $U \geq 0$:

$$\boxed{U = \frac{1}{\omega^2} \sqrt{(v_0 \omega)^2 + g^2}} \quad (32)$$

but:

$$v_0 = v_p = v_i \left(\frac{D}{r_i} \right)^{-\alpha} \quad (33)$$

so we have:

$$\boxed{U = \frac{1}{\omega^2} \sqrt{\omega^2 v_i^2 \left(\frac{D}{r_i} \right)^{-2\alpha} + g^2}} \quad (34)$$

That way we have $U = U(D)$. And we know that $A = \omega^2 U$, so:

$$\boxed{A(D) = a_0(D) = \sqrt{v_i^2 \left(\frac{D}{r_i} \right)^{-2\alpha} + g^2}} \quad (35)$$

5 Simulation of the path of any object sitting on the equator of the asteroid after a blast event

We made this study to foresee the response of a surface pod to a blast event and more precisely to see –for BASiX mission– if the pods which contain the instruments are not launched away from the studied body. But, we can notice that in our model of the path of the pod, the mass of the latter is not determining: indeed, we just studied the ballistic movement of an object in a gravitational field. Therefore, we can transpose this study to the movement of *any* particule sitting on the surface of the asteroid. *Any* boulder, rock, particule of dust will be launched and will have the same orbital behavior as the pod which would be launched from the same position regarding the blast.

This is why throughout the rest of the present paper, we will either speak of a particule or of the pod, as they have the same behaviour around the asteroid.

5.1 Observations and results

5.1.1 Global views of the path in the rotating and the inertial frames

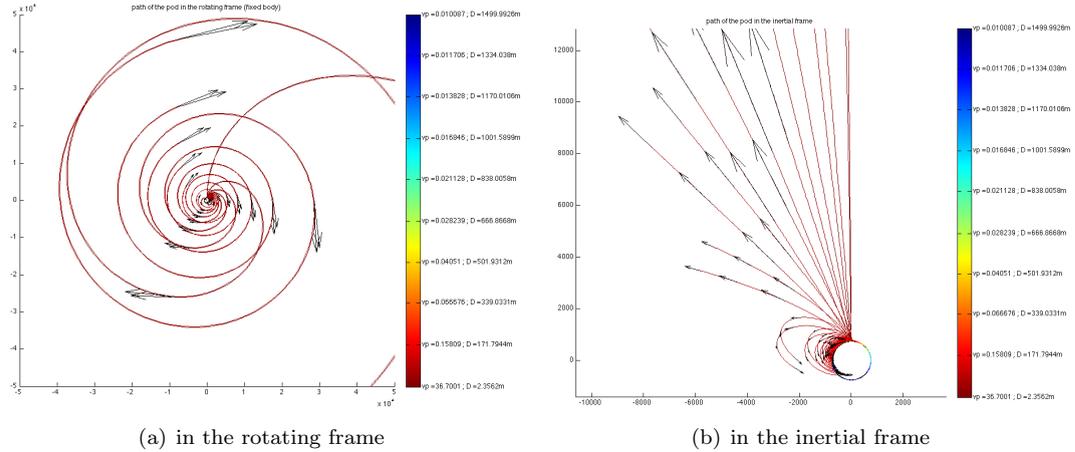


Figure 12: Global display of the path of the pod in different frames

As we can see on the figures 12 page 13, the particles located very closely to the blast (which is on the top of the asteroid on these images) are launched away with a very important velocity (see red color on the plots, which correspond to the highest computed speed). On these global views we can see that many of these particles have a divergent path and will not tend to come back on the asteroid.

The other particles, more distant from the explosion, are also ejected but come back on the asteroid, sometimes even after a couple of revolutions. Their path depends on their position from the source of the seismic wave.

5.1.2 Closer look at the asteroid

Landing particles Figure 13 page 14 shows more closely the path of the particles in the rotating frame. Only the particles located in a relatively close area around the blast are launched strongly enough to reach another region of the asteroid. As the asteroid rotates in the inertial frame, we can see that the particles migrate clockwise on the picture. Thus, on the plot, we understand well that the area located on the right side of the asteroid is where most of the particles land, whereas the left and bottom sides are areas where the dust barely moves.

The particles located just on the left of the blast, if not ejected, seem to be sent to the "landing area" on the right of the asteroid. However it is important to notice that the simulation was done with a definite number of points, which means nothing proves that no particle will migrate from the blast area to the left side of the asteroid. We can only make a quantitative statement: most of the particles located next to the blast migrate to the right of the body. We can see on figure 14 page 15 a diagram which shows the correspondence between the launching angular position (x-axis) and the landing position (y-axis) and confirms what we just assumed by looking at figure 13. Moreover, figure 15 page 15 gives an idea of the scatter of the particles around the asteroid after their landing: we made the computation on 1000 points, and we see that most of the landing position are located in the area the angles of which are between 0 and $-20deg$ (0 taken in position of the blast, according to the center of the asteroid, and the angles are measured from 0 to 360 degrees).

Escape zone As we said in the beginning of section 5.1.2, some particles, located near the blast, seem to be launched and escape to the attraction of the asteroid (Their initial velocity is superior to the escape velocity). Thus, we can give a closer look at figure 14 page 15 and zoom in to isolate this area more precisely (see figure 16 page 16). According to this plot, the "escape zone" is defined by angles between -4 and 4 degrees (it is symmetric, *i.e.* we see the same peak between 356 and

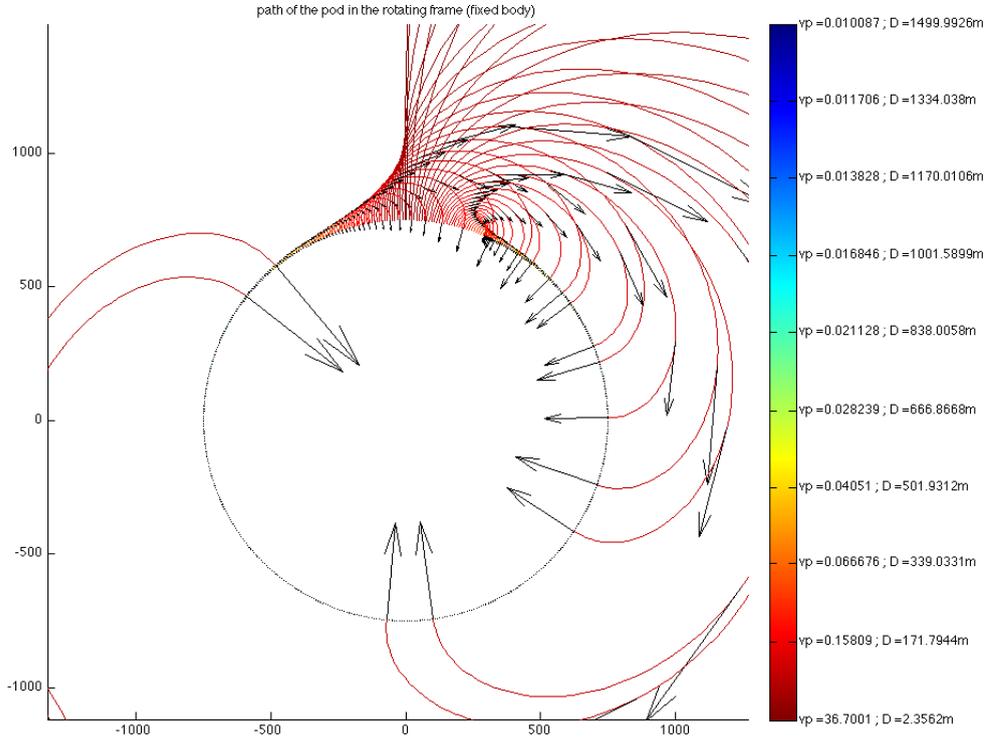


Figure 13: Path of a particle (*e.g.* the pod) around the asteroid according to its position regarding the blast (which is at the top of the asteroid on this image), in the body fixed frame (*i.e.* rotating frame)

360 degrees). However, it is important to notice that –as shown on figure 11 page 11– the model is not very realistic when we get close to the source of the seismic wave: it is a propagation model and when we get close to the explosion, the most determining phenomenon is the explosion itself rather than the resulting wave.

Regarding BASiX mission, within which we want the seismeters not to fly away from the asteroid, we will not want the "geopods" (the pods which carry the instruments) to land in such an area.

5.2 Opening for this study

Surface reformation of the asteroid As a consequence to the movement initiated by the moving ground, *any* particule sitting of the surface seems to be launched according to its position from the source of the wave. Therefore, we can predict a global movement of the surface dust (and "rocks") and therefore a shape reformation of the asteroid.

Isolating differents area on the asteroid As we did when giving a closer look at figure 16 page 16, we can divide the asteroid in different zones according to the evolution of the surface, the sensitivity to the wave (are the particules easily sent away, how far...), and the amount of new materials arriving from another region.

Reversing the process Knowing the launching position and knowing the characteristics of the blast (amplitude and frequency of the shockwave), we can have an estimation of the attenuation factor α of the asteroid.

In another way, if we know the $\alpha_{asteroid}$ and the landing and launching positions of the pod/particule, we can have a good estimation of the amplitude of the seismic wave (assuming its frequency is not decisive for the propagation).

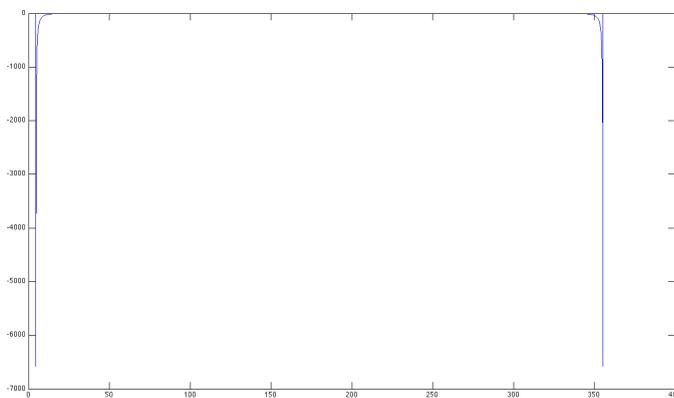


Figure 14: Correspondence between the launching angular position (x-axis) and the landing position (y-axis)

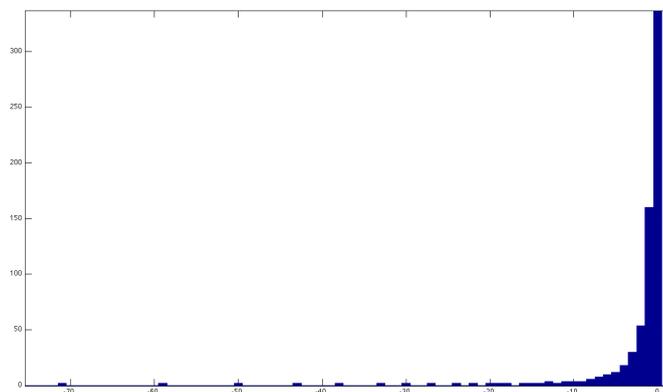


Figure 15: Scatter diagram of the landing position of the particle (x-axis is in degrees, y-axis is the number of individuals)

5.3 Critics, proposals in order to improve the model and new questions

Shape and frequency dependance Our model tries to fit a theoretical model for the wave propagation (model within which the asteroid is a continuous medium with no limit and therefore no particular shape) to a numerical model which takes in account the shape and the internal structure of the asteroid⁵. We assume the shape, the "irregularities" of the surface and the layers of the asteroid to have an effect on the propagation and the resonance of the seismic wave. Therefore, a next step in the study would be to formally take in account the spherical shape of the asteroid and the porosity of the material (Asphaug, 2007). We could also take in account the several internal layers and the reflexion of the wave on them, in order to understand better the influence of the irregularities of the surface and the subsoil.

To carry-on this study, it would be interesting to have numerical simulation results from a spherical model such as Dr Blitz provided us on a real shaped asteroid.

Interaction between the flying objects Our model only focuses on one object at a time. We study the path of a particle –the pod or a particule of the surface– launched at a precise initial velocity, in a gravitational field. What we do not take in account is the interaction between the

⁵see Blitz report for more details and specially the different layered models of the KW4 asteroid.

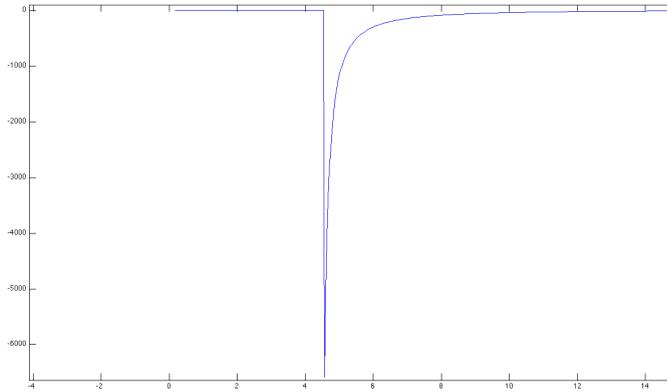


Figure 16: The "escape zone" is defined for angular position between -4 and 4 degrees around the blast.

different materials: between the particules themselves or between the pod and the dust during the flight. The trajectories may interfere, and shocks and deviation of the path may occur.

Modeling the pod A next step of the study would be to model the pod itself: in the present work we considered the pod as a punctual body, with no shape and therefore no interaction with the ground during the seismic event, no resonance phenomenon, etc.

Other questions We simulated the path of the pod from an idealistic equatorial position. What if the pod where not perfectly on the equator? We made some quick simulations, by shifting the pod a little from the equator (couple meters) and we guess it just gives a slight z-component to the path, and therefore changes a lot the place in which the particule lands. Nevertheless, according to the shape of Didymos and its gravitational field, we can assume the pod to roll up the hill located on the equator: the inertial acceleration caused by the rotation of the asteroid compensates the gravity on the equator and therefore the resultant gravity is very low.

In order to have a simple model, we only focus on the first bounce –when the asteroid lands–, but it would be interesting to be able to foresee the other bounces in order to have a more realistic idea of the position of the pod after the blast (and maybe find a relation between this position and the characteristics of the shockwave and/or the asteroid).

Conclusion

This work shows the insight into the response of a surface pod on a small asteroid and its usefulness. We used KW4 characteristics as we can find quite a lot of informations about it in the literature. This first study shows how to use the ballistic flight of the ejected body (the pod) in order to have information about the asteroid and/or the shockwave.

The model used is quite simple –and we give some leads to improve it– but gives a first insight into the phenomenum of the surface response to a seismic event. A next step would be to complete this theory with a finite elements method and a numerical model of the KW4 asteroid.

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